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NONLINEAR CURRENT DENSITY IN SEMICONDUCTORS

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A theory describing the nonlinear current density in semiconductors is proposed. The calculations are carried out starting from the Boltzmann transport equation with the relaxation time approximation expanded to include both a momentum relaxation time τ_m and energy relaxation time τ_ε. The current density is derived for a strongly degenerate electron gas, with the energy dependence of the momentum relaxation time, and for a nondegenerate electron gas, with the momentum relaxation time assumed a constant phenomenological parameter.

1. Boltzmann equation

A review of high-field d. c. conductivity and other high-field effects in semiconductors may be found in Refs [1, 2].

The earlier elementary phenomenological theories have not involved analytic Bolzmann solutions. A great deal of theoretical work has been done on the solution of the Boltzmann equation which is approximated by a Maxwell-Bolzmann distribution function with an electron temperature T_e higher than the lattice temperature T, but this is by no means generally the case. Many authors use the diffusion approximation even for higher fields, but f_0 has also to be determined by the Boltzmann equation. It is usually difficult to solve the Boltzmann equation for cases of practical interest.

In the present paper the addition of a phenomenological energy relaxation term to the Boltzmann equation provides analytical expressions for the conductivity in a high d. c. electric field.

To get an explicit form for the hot electron distribution function, we have had to use the standard relaxation time approximation expanded to include both a momentum τ_m and energy τ_ε relaxation time.

The energy relaxation time τ_ε may be introduced as some measure of the time for the excited electronic system to come into thermal equilibrium with the lattice [4, 5].

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The application of a large electric field to semiconductors can produce symmetric and antisymmetric changes in the electron distribution function f [3]

$$f = f_0 + f_S + f_A, \tag{1}$$

where f_0 is the equilibrium distribution function, f_S is the change in the symmetric part of the distributions function $f_S(-\vec{k}) = f_S(\vec{k})$ and f_A is the antisymmetric change in the distribution function $f_A(-\vec{k}) = -f_A(\vec{k})$, $\vec{k}$ is the wave vector of the carriers.

In the absence of a temperature gradient and in the case of a homogeneous electric field $\vec{E}$ the Boltzmann equation can be written [4]

$$\frac{\partial f}{\partial t} + \frac{1}{\hbar} \vec{E} \nabla_{\vec{k}} f = -\frac{f_A}{\tau_m} - \frac{f_S}{\tau_\varepsilon}. \tag{2}$$

If the applied electric field is d. c. $\partial f/\partial t = 0$. The symmetric and antisymmetric parts of Eq. (2) can be identified and we can obtain two equations.

Now we shall expand the function f_A and f_S in a power series with respect to the electric field $\vec{E}$. This gives

$$\frac{q}{\hbar} \vec{E} \frac{df_0}{d\vec{k}} + \frac{q}{\hbar} \vec{E} \frac{df_S^{(n-1)}}{d\vec{k}} = -\frac{f_A^{(n)}}{\tau_m}, \tag{3}$$

$$\frac{q}{\hbar} \vec{E} \frac{df_A^{(n-1)}}{d\vec{k}} = -\frac{f_S^{(n)}}{\tau_\varepsilon}, \tag{4}$$

where n and $(n-1)$ refer to the iteration order. The initial conditions are $f_A^{(0)} = 0, f_S^{(0)} = 0$.

In a linear approximation we obtain $f_S^{(1)} = 0$ and

$$f_A^{(1)} = -q\vec{E}\vec{v}\tau_m \frac{df_0}{d\varepsilon}. \tag{5}$$

It is seen from Eqs (5), (15) that in a linear approximation Ohm's law is satisfied.

In further calculations it is necessary to make assumption regarding the shape of the energy bands and the momentum relaxation time τ_m. We assume that the energy bands are parabolic and the energy surfaces are spherically symmetric. The momentum relaxation time in this case is expressed in the following form [6]

$$\tau_m(\varepsilon) = \tau_{mr}\varepsilon^{r/2}, \quad \tau_{mr} = \text{const}. \tag{6}$$

The index r describes the type of the scattering mechanism.

The velocity of an electron for simple bands can be expressed in the form

$$\vec{v} = \frac{\hbar \vec{k}}{m^*}, \tag{7}$$

where m^* is the momentum effective mass at $\vec{k} = 0$. For a nonparabolic band the momentum effective mass is a function of energy.

Now we calculate the electric current density in the third order approximation.

Putting $n = 3$ in Eq. (3) we obtain

$$\frac{q}{\hbar}\vec{E}\frac{df_0}{d\vec{k}} + \frac{q}{\hbar}\vec{E}\frac{df_S^{(2)}}{d\vec{k}} = -\frac{f_A^{(3)}}{\tau_m} \tag{8}$$

and putting $n = 2$ in Eq. (4) we have

$$\frac{q}{\hbar}\vec{E}\frac{df_A^{(1)}}{d\vec{k}} = -\frac{f_S^{(2)}}{\tau_\varepsilon}. \tag{9}$$

Using the tensor notation one can calculate the terms $f_S^{(2)}$ and $f_A^{(3)}$

$$f_S^{(2)} = \frac{q^2\tau_\varepsilon}{\hbar}E_\alpha E_\beta \frac{d}{dk_\beta}\left[\tau_m(\varepsilon)v_\alpha \frac{df_0}{d\varepsilon}\right], \tag{10}$$

$$f_A^{(3)} = -q\tau_m E_\alpha v_\alpha \frac{df_0}{d\varepsilon} - \frac{q\tau_m}{\hbar}E_\gamma \frac{df_S^{(2)}}{dk_\gamma}. \tag{11}$$

The detailed calculations of the current density associated with the deviation from Ohm's law shall be given in this paper for the two limiting cases: a strongly degenerate and a nondegenerate electron gas.

2. *Strongly degenerate electron gas*

For a strongly degenerate electron gas we set $f_0 = 1/2$ in the calculations. In such a case we find

$$\frac{df_S^{(2)}}{dk_\gamma} = \frac{\hbar^2}{m^*}\frac{d\tau_m}{d\varepsilon}\frac{df_0}{d\varepsilon}[\delta_{\beta\gamma}v_\alpha+\delta_{\alpha\gamma}v_\beta+\delta_{\alpha\beta}v_\gamma]$$

$$+\left[\hbar^2\frac{d^2\tau_m}{d\varepsilon^2}\frac{df_0}{d\varepsilon}+\frac{2\hbar^2}{k_0T}\tau_m\left(\frac{df_0}{d\varepsilon}\right)^2\right]v_\alpha v_\beta v_\gamma, \tag{12}$$

because

$$\frac{df_0}{d\varepsilon} = -\frac{1}{k_0T}f_0(1-f_0), \tag{13}$$

$$\frac{d^2f_0}{d\varepsilon^2} = \frac{1}{k_0T}(2f_0-1)\frac{df_0}{d\varepsilon}. \tag{14}$$

where f_0 is the Fermi-Dirac function, k_0 is Boltzmann's constant.

Electric current density in the th rd order approximation is given by

$$J_\varrho = \frac{2}{(2\pi)^3}q\int f_A^{(3)}v_\varrho d_3k \tag{15}$$

where

$$d_3k = k^2 dk\, d\Omega. \tag{16}$$

Substituting Eqs (11), (12) into Eq. (15) we obtain

$$J_\varrho = \frac{2q^2}{(2\pi)^3}\frac{\hbar^2}{m^{*2}}E_\alpha\int\left(-\frac{df_0}{d\varepsilon}\right)\tau_m k^4 dk\times I_1$$
$$+\frac{2q^4}{(2\pi)^3}\frac{\hbar^2}{m^{*3}}\tau_\varepsilon E_\alpha E_\beta E_\gamma\int\left(-\frac{df_0}{d\varepsilon}\right)\tau_m\frac{d\tau_m}{d\varepsilon}k^4 dk\times 3\delta_{\beta\gamma}I_1$$
$$+\frac{2q^4}{(2\pi)^3}\frac{\hbar^4}{m^{*4}}\tau_\varepsilon E_\alpha E_\beta E_\gamma\int\left(-\frac{df_0}{d\varepsilon}\right)\tau_m\left[\frac{d^2\tau_m}{d\varepsilon^2}+\frac{2\tau_m}{k_0T}\frac{df_0}{d\varepsilon}\right]k^6 dk\times I_2 \tag{17}$$

where

$$I_1 = \int\frac{k_\alpha k_\varrho}{k^2}d\Omega = \tfrac{4}{3}\pi\delta_{\alpha\gamma} \tag{18}$$

$$I_2 = \int\frac{k_\alpha k_\beta k_\gamma k_\varrho}{k^4}d\Omega = \tfrac{4}{15}\pi[\delta_{\alpha\beta}\delta_{\gamma\varrho}+\delta_{\alpha\gamma}\delta_{\beta\varrho}+\delta_{\alpha\varrho}\delta_{\beta\gamma}]. \tag{19}$$

Integrating over the solid angle Ω we have

$$J_\varrho = \frac{q^2}{3\pi^2}\frac{\hbar^2}{m^{*2}}E_\varrho\int\left(-\frac{df_0}{d\varepsilon}\right)\tau_m k^4 dk$$
$$+\frac{q^4}{\pi^2}\frac{\hbar^2}{m^{*3}}\tau_\varepsilon E^2 E_\varrho\int\left(-\frac{df_0}{d\varepsilon}\right)\tau_m\frac{d\tau_m}{d\varepsilon}k^4 dk$$
$$+\frac{q^4}{5\pi^2}\frac{\hbar^4}{m^{*4}}\tau_\varepsilon E^2 E_\varrho\int\left(-\frac{df_0}{d\varepsilon}\right)\tau_m\left[\frac{d^2\tau_m}{d\varepsilon^2}+\frac{2\tau_m}{k_0T}\frac{df_0}{d\varepsilon}\right]k^6 dk. \tag{20}$$

In the case of the strongly degenerate electron gas the derivative of the distribution function f_0 can be replaced by the delta function $(-f_0') = \delta(\varepsilon-\xi)$ and we obtain

$$J_\varrho = \frac{q^2}{3\pi^2}\frac{1}{m^*}E_\varrho\tau_m(\varepsilon)k^3(\varepsilon)|_{\varepsilon=\xi}+\frac{q^4}{\pi^2}\frac{1}{m^{*2}}\tau_\varepsilon E^2E_\varrho\tau_m(\varepsilon)\frac{d\tau_m(\varepsilon)}{d\varepsilon}k^3(\varepsilon)|_{\varepsilon=\xi}$$
$$+\frac{q^4}{5\pi^2}\frac{\hbar^2}{m^{*3}}\tau_\varepsilon E^2E_\varrho\tau_m(\varepsilon)k^6(\varepsilon)\left[\frac{d^2\tau_m}{d\varepsilon^2}+\frac{2\tau_m}{k_0T}\frac{df_0}{d\varepsilon}\right]\Bigg|_{\varepsilon=\xi}. \tag{21}$$

Substituting Eq. (6) in Eq. (21) the current density is obtained in the form

$$J_\varrho = \frac{q^2}{3\pi^2}\frac{1}{m^*}E_\varrho\tau_m k^3\Bigg[1-\tfrac{3}{5}\frac{q^2}{m^*}\frac{\tau_\varepsilon\tau_m}{k_0T}\xi E^2$$
$$+\tfrac{3}{2}\frac{q^2}{m^*}r\tau_\varepsilon\tau_m\xi^{-1}+\tfrac{3}{5}\frac{q^2}{m^*}r\left(\frac{r}{2}-1\right)\tau_\varepsilon\tau_m\xi^{-2}E^2\Bigg]. \tag{22}$$

The momentum relaxation time τ_m corresponding to the energy of the Fermi level ξ is equal to $\tau_m(\xi) = \tau_{mr}\,\xi^{r/2}$. The current density Eq. (22) in the third order approximation associated with the energy dependence of the momentum relaxation time $\tau_m(\varepsilon)$ depends on the scattering mechanism through the index r. It is seen that Eq. (22) depends on the statistics and is satisfied dependently of the degree of degeneracy of the electron gas.

Let us make the assumption that the momentum relaxation time is independent of energy, i. e. τ_{m0} = const for $r = 0$. The electric conductivity in this case is described by

$$\sigma(E) = \frac{q^2\tau_{m0}}{m^*}\, n\left[1-\tfrac{3}{5}\,\frac{q^2}{m^*}\,\frac{\tau_\varepsilon\tau_{m0}}{(k_0T)^2}\,\xi E^2\right], \tag{23}$$

where the carrier concentration n for a degenerate gas is equal to

$$n = \frac{1}{3\pi^2}\,k^3(\xi), \tag{24}$$

Making use of the above equation we can describe the Fermi level ξ for the simple band structure as follows

$$\xi = \frac{\hbar^2(3\pi^2 n)^{2/3}}{2m^*}. \tag{25}$$

Substituting into Eq. (23) the expression for ξ Eq. (25) we find

$$\mu(E) = \frac{q\tau_{m0}}{m^*}\left[1-\tfrac{3}{10}\,\frac{q^2}{m^*}\,\frac{\hbar^2(3\pi^2 n)^{2/3}}{(k_0T)^2}\,\tau_\varepsilon\tau_{m0}E^2\right]. \tag{26}$$

It is seen that Eq. (26) determines the energy relaxation time $\tau_\varepsilon(E)$ dependent on the electric field for $\vec{E} \neq 0$.

3. *Nondegenerate electron gas*

In the case of a nondegenerate electron gas the distribution function f_0 can be descriebed by the Boltzmann function

$$f_0 = e^{\xi/k_0T}e^{-\varepsilon/k_0T}, \tag{27}$$

where ξ is the Fermi level. By neglecting the effect of the energy dependence of the momentum relaxation time, the current density for τ_{m0} = const and simple band can be calculated.

According to Eq. (10) we have

$$\frac{df_S^{(2)}}{dk_\gamma} = \frac{q^2\hbar}{k_0T}\,\tau_\varepsilon\tau_{m0}E_\alpha E_\beta\,\frac{df_0}{d\varepsilon}\left\{\frac{(2f_0-1)^2-2f_0(1-f_0)}{k_0T}\,v_\alpha v_\beta v_\gamma + \frac{(2f_0-1)}{m^*}\left[\delta_{\alpha\beta}v_\gamma+\delta_{\alpha\gamma}v_\beta+\delta_{\beta\gamma}v_\alpha\right]\right\} \tag{28}$$

where

$$\frac{dv_\alpha}{dk_\beta} = \frac{\hbar}{m^*}\delta_{\alpha\beta}. \tag{29}$$

Substituting Eq. (28) into Eq. (11) we obtain

$$f_{\rm A}^{(3)} = -q\tau_{\rm m0}v_\alpha E_\alpha\frac{df_0}{d\varepsilon} - \frac{q^3}{k_0T}\tau_\varepsilon\tau_{\rm m0}E_\alpha E_\beta E_\gamma\frac{df_0}{d\varepsilon}$$

$$\times\left\{\frac{2f_0-1}{k_0T}\left[\delta_{\alpha\beta}v_\gamma+\delta_{\alpha\gamma}v_\beta+\delta_{\beta\gamma}v_\alpha\right]+\frac{(2f_0-1)^2-2f_0(1-f_0)}{k_0T}v_\alpha v_\beta v_\gamma\right\}. \tag{30}$$

Now we are able to calculate the electric current density in the third order approximation. This current is given by Eq. (15).

Substituting into Eq. (15) the expression for $f_{\rm A}^{(3)}$ Eq. (30) and integrating over the solid angle Ω we find

$$J_\varrho = \frac{q^2}{3\pi^2}\left(\frac{\hbar}{m^*}\right)^2\tau_{\rm m0}E_\varrho\int\left(-\frac{df_0}{d\varepsilon}\right)k^4dk$$

$$+\frac{q^4}{\pi^2}\left(\frac{\hbar}{m^*}\right)^2\frac{\tau_\varepsilon}{(k_0T)^2}\tau_{\rm m0}^2E^2E_\varrho\int\left(-\frac{df_0}{d\varepsilon}\right)\frac{2f_0-1}{m^*}k^4dk$$

$$+\frac{q^4}{5\pi^2}\left(\frac{\hbar}{m^*}\right)^4\frac{\tau_\varepsilon}{(k_0T)^2}\tau_{\rm m0}^2E^2E_\varrho\int\left(-\frac{df_0}{d\varepsilon}\right)\left[(2f_0-1)^2-2f_0(1-f_0)\right]k^6dk. \tag{31}$$

It is seen from Eq. (31) that the general expression of the conductivity tensor in linear approximation is given by

$$\sigma_0 = \frac{q}{3\pi^2}\mu_0\frac{\hbar^2}{m^*}\int\left(-\frac{df_0}{d\varepsilon}\right)k^4dk, \tag{32}$$

where $\mu_0 = q\tau_{\rm m0}/m^*$ is the mobility of electrons. The carrier concentration can be given by

$$n = \frac{1}{3\pi^2}\frac{\hbar^2}{m^*}\int\left(-\frac{df_0}{d\varepsilon}\right)k^4dk. \tag{33}$$

For the nondegenerate electron system we have $k_0T \gg \xi$ and $f_0 \ll 1$. Making use of the last inequality the following expression for the conductivity can be found

$$\sigma(E) = q\mu_0 n\left[1-\frac{q^2}{5\pi^2}\frac{\hbar^4}{m^{*3}}\frac{\tau_{\rm m0}\tau_\varepsilon}{(k_0T)^2}E^2\frac{1}{n}\int 2f_0\left(-\frac{df_0}{d\varepsilon}\right)k^6dk\right], \tag{34}$$

where

$$\int 2f_0\left(-\frac{df_0}{d\varepsilon}\right)k^6dk = \frac{1}{k_0T}\left(\frac{2m^*}{\hbar^2}\right)^{7/2}e^{2\xi/k_0T}\int e^{-2\varepsilon/k_0T}\varepsilon^{5/2}d\varepsilon. \tag{35}$$

According to Eq. (33) the carrier concentration of electrons in the nondegenerate case is given by

$$n = \frac{\sqrt{\pi}}{4\pi^2}\left(\frac{2m^*k_0T}{\hbar^2}\right)e^{\xi/k_0T}. \tag{36}$$

Making use of the expression for e^{ξ/k_0T} from Eq. (36) we obtain

$$\mu(E) = \mu_0\left[1-\frac{3\pi^{3/2}q^2\hbar^3 n\tau_\varepsilon\tau_{m0}}{4m^{*5/2}(k_0T)^{5/2}}E^2\right]. \tag{37}$$

To analyse the effect associated with $\tau_m(\varepsilon)$ it is necessary to take into account $\tau_m = \tau_{mr}\varepsilon^{r/2}$, where the index r describes the kind of scattering mechanism.

In many-valley semiconductors the conduction band minima may be approximated by ellipsoids. The band of ellipsoidal structure depends on the effective mass tensor and therefore on the carrier mobility. The mobility anisotropy is further affected through an anisotropy in the momentum relaxation time [5].

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