Vol. A53 (1978) ACTA PHYSICA POLONICA No 1

THE STATIC SPIN SUSCEPTIBILITY OF THE SUPERFLUID ^{3}He-B

By R. Gonczarek and L. Jacak

Institute of Physics, Technical University of Wrocław*

(*Received April 28, 1977*)

The exact result for the static spin susceptibility of the phase B of superfluid ^{3}He at nonzero temperatures is obtained. The agreement of the result in microscopic and phenomenological approaches is shown.

1. Introduction

An incessant development of experiments on superfluid ^{3}He and permanent improvement of the measurement technique [1–3] induced the authors of this paper to the complete investigation of the static spin susceptibility of the phase B of superfluid ^{3}He [4]. This problem seems to be well known. The exact result for the static spin susceptibility for $T = 0$ has been already derived in 1967 by Czerwonko [5]. The generalization of this formula for nonzero temperatures has been performed by Leggett [6] in 1975, but with restriction to only zeroth Landau's parameter. In 1976 Wölfle [7] tried to give more general formula with two even Landau's parameters. It seems that the obtained result solves this problem completely, because the static spin susceptibility of superfluid ^{3}He-B contains only zeroth and second Landau's parameter (see below or Ref. [6]). Nevertheless, the result obtained by Wölfle [7] was not correct, (it is correct only for s-pairing!). This is easy to verify by comparison with the well known Czerwonko formula [5]. The analysis of the method used by Wölfle [7] allows one to state that the author [7] made some misleading intuitive assumptions. Namely, he assumed that distribution spin matrix has a diagonal form. This simplification is not correct, because the off-diagonal elements play a significant role (see below).

In this paper we consider the linear response of the superfluid system with BW pairing [4] to the static and slightly unhomogeneous magnetic field. Our purpose is to obtain the tensor of the spin susceptibility χ_{ij}, without any restrictions imposed on the Fermi

* Address: Instytut Fizyki, Politechnika Wrocławska, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

liquid interactions (i.e. the ordinary Landau function). On the other hand, we assume that the effective quasiparticle interaction in the particle-particle channel is spin-symmetric. Two possible approaches to such formulated subject will be presented below.

2. *Microscopic approach*

We apply the theory developed by Czerwonko [5] being a continuation of the Larkin–Migdal theory [8]. The approach of [5] was confined to $T = 0$ but, without significant difficulties, could be extended to nonzero temperatures. Namely, equations (2.27) of [5] remain still valid, but now the kernels L, M, N, O (cf. [5]) have slightly different form, obtained firstly by Leggett [9].

Our notation is almost identical to that used in [5] particularly in Eqs. (2.27). Let us list its main symbols. We denote: σ^i — the Pauli matrices, $\hat{e}_j$ — the unit vectors directed along the j-th axis in the momentum space, $\hat{k}$, $\hat{q}$ — the unit vectors parallel to the momentum k and the wave vector q of the external magnetic field, respectively, Δ — the energy gap, $\langle ... \rangle$ — the average over spherical angles connected with k, ν — the density of states on the Fermi surface.

The normal vertex function can be written as follows:

$$\hat{\mathcal{T}}(\hat{k}) = \mathcal{T}^i_j(\hat{k})\sigma^i\hat{e}_j,$$

$$\overset{s}{\mathcal{T}}{}^i_j = \tfrac{1}{2}[\mathcal{T}^i_j(\hat{k})+\mathcal{T}^i_j(-\hat{k})], \tag{1}$$

whereas the anomalous vertex function as

$$\hat{\tau}(\hat{k}) = \tau^i_j(\hat{k})\sigma^i\hat{e}_j,$$

$$\hat{\tau}(\hat{k}) = -\hat{\tau}(-\hat{k}). \tag{2}$$

The exchange part of dimensionless effective interaction in the particle-hole channel, i. e. the exchanges Landau function, is written as

$$B(\hat{k}\hat{k}') = \sum_{l=0}^{\infty}(2l+1)b_l P_l(\hat{k}\hat{k}'), \qquad \overset{s}{B} = \tfrac{1}{2}[B(\hat{k}\hat{k}')+B(-\hat{k}\hat{k}')]. \tag{3}$$

According to the results obtained by Leggett [9], for static fields ($\omega = 0$) and $qv \ll \Delta$, where v is the velocity on the Fermi sphere, we have

$$L-O = -1, \qquad 2\times O = 1-Y, \tag{4}$$

where:

$$Y = \int_0^{\infty}\frac{d\xi}{2T}\,\mathrm{ch}^{-2}\left(\frac{\sqrt{\xi^2+\Delta^2}}{2T}\right)$$

is the Yosida function, having the following properties:

$$(i)\ 0 \leqslant Y \leqslant 1, \quad (ii)\ Y(T=0) = 0, \quad (iii)\ Y(T = T_c) = 1.$$

Moreover, it is sufficient to know that for $\omega = 0$ M is an odd function of $\hat{k}$.

Taking into account the above formulae, we obtain from Eqs. (2.27) of [5], that

$$\overset{s}{\mathscr{T}}{}^i_j = a\delta_{ij} + \langle B\{-\overset{s}{\mathscr{T}}{}^i_j + (1-Y)\overset{s}{\mathscr{T}}{}^k_j \hat{k}_k \hat{k}_i\}\rangle, \tag{5}$$

whereas expression (3.31) of [5] now has the form:

$$\chi_{ij} = a^{-1}\mu_B^2 \nu\{\langle \overset{s}{\mathscr{T}}{}^i_j\rangle - (1-Y)\langle \overset{s}{\mathscr{T}}{}^k_j \hat{k}_k \hat{k}_i\rangle\}. \tag{6}$$

Such important simplification of the equations is caused by the fact that averages $\langle \ldots \rangle$ of odd functions with respect to $\hat{k}$ vanish. Eqs (5) and (6) are sufficient to determine χ_{ij}. For this purpose, it is necessary to average Eq. (5) and Eq. (5) multiplied by $\hat{k}_k \hat{k}_i$. The obtained system of two equations can be easily solved and, using (6), we obtain:

$$\chi_{ij} = \frac{\mu_B^2 \nu[\frac{2}{3}(1+b_2 Y) + \frac{1}{3} Y(1+b_2)]}{\frac{2}{3}(1+b_0)(1+b_2 Y) + \frac{1}{3}(1+b_0 Y)(1+b_2)} \delta_{ij}. \tag{7}$$

3. *Kinetic equation approach*

The kinetic equation has the following form ([10, 11]):

$$\omega \delta n_k = \delta n_k \overset{0}{\varepsilon}_{k+q/2} - \overset{0}{\varepsilon}_{k-q/2} \delta n_k + \overset{0}{n}_{k-q/2} \delta\varepsilon_k - \delta\varepsilon_k \overset{0}{n}_{k+q/2}. \tag{8}$$

For matrices δn_k and $\delta \varepsilon_k$, we use the standard notation (cf. [11])

$$\delta n_k = \begin{pmatrix} \delta n_e & \delta n_+ \\ \delta n_- & \partial n_t \end{pmatrix}, \quad \delta\varepsilon_k = \begin{pmatrix} M_k^{\mathrm{tr}} & \delta\Delta^+ \\ \partial\Delta & -M_{-k} \end{pmatrix}. \tag{9}$$

We perform our calculations in the static limit, therefore, for the spin matrix δn_+ the following equation is valid:

$$\delta n_+ = \frac{1}{2\xi}\Bigg(\delta n_e \Delta^+ - \Delta^+ \delta n_t + 2\frac{\varphi}{E}\xi\delta\Delta^+ - \frac{\varphi}{E}(\Delta^+ M_{-k} + M^{\mathrm{tr}}\Delta^+) + \frac{\overline{vq}}{2}\frac{d}{d\xi}\left(\frac{\varphi}{E}\right)(\Delta^+ M_{-k} - M^{\mathrm{tr}}\Delta^+)\Bigg), \tag{10}$$

where "tr" denotes the transposed matrix in the spin space. In this notation we have:

$$\xi_k = \frac{k^2}{2m^*} - \mu, \quad E_k^2 = \xi_k^2 + |\Delta_k|^2, \quad \varphi_k = -\tfrac{1}{2}\,\mathrm{tgh}\,\frac{\beta E_k}{2}. \tag{11}$$

Taking into account equations for δn_+ and δn_- ($\delta n_- = \delta n_+^+$, where + denotes the hermitian conjugation), we obtain from (8) in the static limit, the relation:

$$\delta n_e = \frac{d}{d\xi}\left(\frac{\varphi}{E}\xi\right)M^{\mathrm{tr}} - \frac{1}{2\xi}\frac{d}{d\xi}\left(\frac{\varphi}{E}\right)(\Delta^+ M_{-k}\Delta - M^{\mathrm{tr}}|\Delta|^2) + \frac{d}{d\xi}\left(\frac{\varphi}{E}\right)(\delta\Delta^+\Delta + \Delta^+\delta\Delta), \tag{12}$$

where M_k denotes

$$M_k = \Big(\sum_{k'} f^s_{kk'}\delta \overset{0}{n}_{e,k'}\Big)\sigma^0 + \Big(\sum_{k'} f^a_{kk'}\delta\overline{n}_{e,k'} - \mu_B \overline{H}\Big)\overline{\sigma}. \tag{13}$$

f^s and f^a are the spin symmetric and antisymmetric parts of the Landau kernel, respectively, $\nu f^a_{kk'} = B(\hat{k}\cdot\hat{k}')$ (cf. (3)), ν is the density of states on the Fermi surface. $\overline{H}$ is the external magnetic field.

Each spin matrix w can be expressed in the form

$$w = w^0\sigma^0 + \overline{w}\overline{\sigma}^{tr} \tag{14}$$

and, using (14) the following simple relation can be formulated,

$$\Delta^+\overline{w}\overline{\sigma}\Delta = -2\overline{d}(\overline{d}^*\cdot\overline{w})\overline{\sigma}^{tr} + |\Delta|^2\overline{w}\overline{\sigma}^{tr}, \tag{15}$$

where

$$\Delta = i(\overline{\sigma}\cdot\overline{d}_k)\sigma_2.$$

Hence, for the magnetization $\overline{m} = \sum_k \mu_B\delta\overline{n}_{ek}$, the following equation will be obtained:

$$m_i = -b_0 m_i + \mu_B^2[\nu + \tfrac{1}{3}\nu(Y-1)]H_i$$
$$-\tfrac{1}{3}(b_0-b_2)(Y-1)m_i - \mu_B(Y-1)b_2\sum_{k'}\hat{k}'_i\hat{k}'_j\delta n^j_{e,k'}, \tag{16}$$

where Y is the Yosida function. The last term in (16) has to be derived independently. After some calculations we find

$$\mu_B\sum_{k'}\hat{k}'_i\hat{k}'_j\delta n^j_{e,k'} = \frac{-\frac{1}{3}(b_0-b_2)Ym_i + \frac{1}{3}\mu_B^2\nu YH_i}{1+b_2Y}. \tag{17}$$

Using the definition of the susceptibility tensor

$$m_i = \chi_{ij}H_j, \tag{18}$$

and taking into account equations (17) and (16), we obtain the result (7).

4. Conclusions

It is easy to see that (7) is positive for $i = j$, as a result of Pomeranchuk inequalities [12] and the property (*i*) of the Yosida function. Now for $T = 0$ (7) passes into the expression given by [5], whereas at $T = T_c$ we obtain the formula for susceptibility of the normal system. The same result as for $T = T_c$ is obtained if $\omega = 0$ and $\mu \gg qv \gg \Delta$; cf. Landau [13] and Larkin [14].

The approaches presented in this paper can be, of course, applied to the other systems. For the system with *s*-pairing they are much simpler and then obtained formula is the same as Leggett's [9] one from 1965.

It has been stated, during the Karpacz School of Theoretical Physics, 1977, that Professor J. W. Serene participating in this School obtained formula (7) in an independent way.

The authors are greatly indebted to Professor J. Czerwonko for suggesting the theme as well as for helpful and valuable discussions.

REFERENCES

[1] J. C. Wheatley, *Rev. Mod. Phys.* **47**, 415 (1975).
[2] W. P. Halperin, C. N. Archie, F. B. Rasmussen, T. A. Alvesalo, R. C. Richardson, *Phys.Rev.* **B13**, 2124 (1976).
[3] A. I. Ahonen, M. Krusius, M. A. Paalanen, *J. Low Temp. Phys.* **25**, 421 (1976).
[4] P. W. Anderson, F. Brinkman, *Phys. Rev. Lett.* **30**, 1108 (1973).
[5] J. Czerwonko, *Acta Phys. Pol.* **32**, 335 (1967).
[6] A. J. Leggett, *Rev. Mod. Phys.* **47**, 331 (1975).
[7] P. Wölfle, *J. Low Temp. Phys.* **22**, 157 (1976).
[8] A. I. Larkin, A. B. Migdal, *Zh. Eksp. Teor. Fiz.* **44**, 1703 (1963).
[9] A. J. Leggett, *Phys. Rev.* **A140**, 1869 (1965).
[10] O. Betbeder-Matibet, P. Noziéres, *Ann. Phys.* (USA) **51**, 932 (1969).
[11] R. Combescot, *Phys. Rev.* **A10**, 1709 (1974).
[12] T. Yu. Pomeranchuk, *Zh. Eksp. Teor. Fiz.* **35**, 524 (1958).
[13] L. D. Landau, *Zh. Eksp. Teor. Fiz.* **30**, 1058 (1956).
[14] A. I. Larkin, *Zh. Eksp. Teor. Fiz.* **46**, 2188 (1964).