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FRAUNHOFER DIFFRACTION IN A MEDIUM WITH UNIAXIAL ELECTRIC ANISOTROPY

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A plane electromagnetic wave diffracted by an arbitrary aperture in a medium with uniaxial electric anisotropy is considered. The optic axis is perpendicular to the aperture. The diffracted field is examined in the Fraunhofer region. TM and TE type fields are considered as incident fields. In both cases the diffracted field at the Fraunhofer region possesses properties of a field obtained by the superposition of TM and TE type fields.

1. Introduction

The tensorial dependence of the electric and the magnetic induction vectors on the electric and the magnetic field strengths vectors is a very distinctive feature of an anisotropic medium

$$D_i = \varepsilon_0 \varepsilon_{ij} E_j, \tag{1.1}$$

$$B_i = \mu_0 \mu_{ij} H_j, \tag{1.2}$$

where ε_{ij} is the dielectric permittivity tensor, μ_{ij} — the magnetic permittivity tensor of a medium.

We shall limit ourself to a medium with uniaxial electric anisotropy. This medium has one distinguished symmetry axis called in optics — the optic axis. The tensors ε_{ij} and μ_{ij} are of the form:

$$\varepsilon_{ij} = \varepsilon^0(\delta_{ij} - c_i c_j) + \varepsilon^e c_i c_j, \tag{1.3}$$

$$\mu_{ij} = \delta_{ij}, \tag{1.4}$$

where ε^0 and ε^e are the dielectric permittivity constants in the direction perpendicular and parallel to the optic axis respectively, $\boldsymbol{c}$ — the unit vector along the direction of the optic axis.

One of the main dielectric permittivity tensor ε_{ij} axes has been choosen as the optic axis, which in the cartesian coordinate frame x_1, x_2, x_3 coincides with the ox_3 axis.

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2. The electromagnetic Huygens principle for a medium with uniaxial electric anisotropy

The electromagnetic Huygens principle for a medium with uniaxial anisotropy has been derived by Wünsche [7].

In our considerations in which the optic axis coincides with the ox_3 axis of the cartesian coordinate frame, we shall write it in the following form

$$E_j(P) = \frac{i}{4\pi\sqrt{\varepsilon^0}} \oint_S d^2x'_Q \Big\{ k_0\varepsilon^e\varepsilon^0\alpha^{-2}\varepsilon_{jkl}\varepsilon_{kpq}N_pE_qn_l$$

$$+\omega\mu_0\varepsilon^e\alpha^{-3}\kappa_{jk}n_kn_l\varepsilon_{lpq}N_pH_q\left(1-\frac{3}{ik_0\alpha R}-\frac{3}{k_0^2\alpha^2R^2}\right)$$

$$+\omega\mu_0(\varepsilon^0-\varepsilon^e)\alpha^{-3}\kappa_{jk}n_k\varepsilon_{3pq}N_pH_q\left(1-\frac{3}{ik_0\alpha R}-\frac{3}{k_0^2\alpha^2R^2}\right)$$

$$-\omega\mu_0\alpha^{-1}\kappa_{jk}\varepsilon_{kpq}N_pH_q\Big\}\frac{e^{ik_0\alpha R}}{R}+\frac{ik_0\sqrt{\varepsilon^0}}{4\pi}\frac{\varepsilon_{j3k}n_k}{(1-n_3^2)}\oint_S d^2x'_Q\{n_3n_l\varepsilon_{lpq}N_pE_q$$

$$-\varepsilon_{3pq}N_pE_q\}\left(\frac{\varepsilon^e}{\alpha^2}\frac{e^{ik_0\alpha R}}{R}-\frac{e^{ik^0R}}{R}\right)-\frac{i\omega\mu_0}{4\pi\sqrt{\varepsilon^0}}\frac{\varepsilon_{j3k}n_k}{(1-n_3^2)}\oint_S d^2x'_Q\Big\{\varepsilon_{3lm}\varepsilon_{lpq}N_pH_qn_m$$

$$\times\left(\frac{\varepsilon^e}{\alpha}\frac{e^{ik_0\alpha R}}{R}-\sqrt{\varepsilon^0}\frac{e^{ik^0R}}{R}\right)+\frac{1}{4\pi\sqrt{\varepsilon^0}}\oint_S d^2x'_Q\Big\{\frac{\varepsilon^e\varepsilon^0}{\alpha^3}\varepsilon_{jkl}\varepsilon_{lpq}n_kN_pE_q$$

$$+\sqrt{\frac{\mu_0}{\varepsilon_0}}\alpha^{-2}\kappa_{jk}\varepsilon_{kpq}N_pH_q\Big\}\frac{e^{ik_0\alpha R}}{R^2}-\frac{\sqrt{\varepsilon^0}\,\varepsilon_{j3k}n_k}{4\pi(1-n_3^2)}\oint_S d^2x'_Q\{n_3n_l\varepsilon_{lpq}N_pE_q-\varepsilon_{3pq}N_pE_q\}$$

$$\times\left(\frac{\varepsilon^e}{\alpha^3}\frac{e^{ik_0\alpha R}}{R^2}-\frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right)$$

$$-\frac{\sqrt{\varepsilon^0}\,\varepsilon_{j3k}n_kn_3}{4\pi(1-n_3^2)^2}\oint_S d^2x'_Q\{n_l\varepsilon_{lpq}N_pE_q-n_3\varepsilon_{3pq}N_pE_q\}\left(\frac{1}{\alpha}\frac{e^{ik_0\alpha R}}{R^2}-\frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right)$$

$$+\frac{\sqrt{\varepsilon^0}\,n_jn_3}{4\pi(1-n_3^2)^2}\oint_S d^2x'_Q\varepsilon_{3kl}\varepsilon_{kpq}N_pE_qn_l\left(\frac{1}{\alpha}\frac{e^{ik_0\alpha R}}{R^2}-\frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right)_{j=1,2}$$

$$+\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon^0}}\frac{n_j}{4\pi(1-n_3^2)^2}\oint_S d^2x'_Q\{n_l\varepsilon_{lpq}N_pH_q-n_3\varepsilon_{3pq}N_pH_q\}\left(\frac{e^{ik_0\alpha R}}{R^2}-\frac{e^{ik^0R}}{R^2}\right)_{j=1,2}$$

$$+\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\varepsilon_{jk3}n_k}{4\pi\sqrt{\varepsilon^0}(1-n_3^2)^2}\oint_S \varepsilon_{3ml}\varepsilon_{mpq}N_pH_qn_l\left(\frac{e^{ik_0\alpha R}}{R^2}-\frac{e^{ik^0R}}{R^2}\right)d^2x'_Q$$

$$+\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{i}{4\pi\alpha^3k_0\sqrt{\varepsilon^0}}\oint_S \kappa_{jk}\varepsilon_{kpq}N_pH_q\frac{e^{ik_0\alpha R}}{R^3}\,d^2x'_Q, \tag{2.1}$$

$$H_j(P)=\frac{i\sqrt{\varepsilon^0}}{4\pi}\oint_S d^2x'_Q\Big\{k_0\varepsilon_{jkl}\varepsilon_{kpq}N_pH_qn_l$$

$$-\omega\varepsilon_0\sqrt{\varepsilon^0}\,n_jn_k\varepsilon_{kpq}N_pE_q\left(1-\frac{3}{ik_0\sqrt{\varepsilon^0}R}-\frac{3}{k_0^2\varepsilon^0R^2}\right)+\omega\varepsilon_0\sqrt{\varepsilon^0}\,\varepsilon_{jpq}N_pE_q\Big\}\frac{e^{ik^0R}}{R}$$

$$-\frac{ik_0\sqrt{\varepsilon^0}}{4\pi}\frac{\varepsilon_{j3k}n_k}{(1-n_3^2)}\oint_S d^2x'_Q\{n_3n_l\varepsilon_{lpq}N_pH_q-\varepsilon_{3pq}N_pH_q\}\left(\frac{\varepsilon^e}{\alpha^2}\frac{e^{ik_0\alpha R}}{R}-\frac{e^{ik^0R}}{R}\right)$$

$$-\frac{i\omega\varepsilon_0\sqrt{\varepsilon^0}}{4\pi}\frac{\varepsilon_{j3k}n_k}{(1-n_3^2)}\oint_S d^2x'_Q\varepsilon_{3lm}\varepsilon_{lpq}N_pE_qn_m\left(\frac{\varepsilon^e}{\alpha}\frac{e^{ik_0\alpha R}}{R}-\sqrt{\varepsilon^0}\frac{e^{ik^0R}}{R}\right)$$

$$+\frac{1}{4\pi}\oint_S d^2x'_Q\left\{\varepsilon_{jkl}\varepsilon_{lpq}n_kN_pH_q-\sqrt{\frac{\varepsilon_0\varepsilon^0}{\mu_0}}\,\varepsilon_{jpq}N_pH_q\right\}\frac{e^{ik^0R}}{R^2}$$

$$+\frac{\sqrt{\varepsilon^0}}{4\pi}\frac{\varepsilon_{j3k}n_k}{(1-n_3^2)}\oint_S d^2x'_Q\{n_3n_l\varepsilon_{lpq}N_pH_q-\varepsilon_{3pq}N_pH_q\}\left(\frac{1}{\alpha}\frac{e^{ik_0\alpha R}}{R^2}-\frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right)$$

$$-\frac{\sqrt{\varepsilon^0}\,n_3n_j}{4\pi(1-n_3^2)^2}\oint_S d^2x'_Q\varepsilon_{3kl}\varepsilon_{kpq}N_pH_qn_l\left(\frac{1}{\alpha}\frac{e^{ik_0\alpha R}}{R^2}-\frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right)_{j=1,2}$$

$$+\frac{1}{4\pi}\sqrt{\frac{\varepsilon_0\varepsilon^0}{\mu_0}}\frac{n_j}{(1-n_3^2)}\oint_S d^2x'_Q\{n_k\varepsilon_{kpq}N_pE_q-n_3\varepsilon_{3pq}N_pE_q\}\left(\frac{e^{ik_0\alpha R}}{R^2}-\frac{e^{ik^0R}}{R^2}\right)_{j=1,2}$$

$$+\frac{1}{4\pi}\sqrt{\frac{\varepsilon_0\varepsilon^0}{\mu_0}}\frac{\varepsilon_{j3k}n_k}{(1-n_3^2)^2}\oint_S d^2x'_Q\varepsilon_{3lm}\varepsilon_{lpq}N_pE_qn_m\left(\frac{e^{ik_0\alpha R}}{R^2}-\frac{e^{ik^0R}}{R^2}\right)$$

$$-\frac{i}{4\pi k_0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\oint_S \varepsilon_{jpq}N_pE_q\frac{e^{ik^0R}}{R^3}\,d^2x'_Q, \tag{2.2}$$

where $\boldsymbol{n} = \{n_1, n_2, n_3\}$ is the unit vector in the direction of the vector $\boldsymbol{R}$ which connects the integration point Q on the surface with point P in which we calculate the electromagnetic field

$$\boldsymbol{n} = \frac{\boldsymbol{R}}{R}, \quad \boldsymbol{R} = \{x_1 - x_1', x_2 - x_2', x_3 - x_3'\}, \tag{2.3}$$

$$\alpha = (\varepsilon^e n_1^2 + \varepsilon^e n_2^2 + \varepsilon^0 n_3^2)^{1/2}, \tag{2.4}$$

$$k_0 = \omega\sqrt{\varepsilon_0 \mu_0}, \quad k^0 = \sqrt{\varepsilon^0 k_0}, \tag{2.5}$$

$$\kappa_{ji} = \varepsilon^e(\delta_{ij} - c_i c_j) + \varepsilon^0 c_i c_j, \quad \boldsymbol{c} = \{0, 0, 1\}. \tag{2.6}$$

3. *The Kirchhoff integral*

Let us consider a plane screen with an arbitrary aperture fully immersed in a medium with uniaxial anisotropy in such a way that the left and right half-space are filled with this medium. Let us further assume that Kirchhoff's boundary conditions are satisfied on the screen B and the aperture A (Fig. 2).

$$\varepsilon_{jpq} N_p H_q(Q) = \varepsilon_{jpq} N_p H_q^{(i)}(Q) \quad \text{on the aperture,} \tag{3.1}$$

$$\varepsilon_{jpq} N_p E_q(Q) = \varepsilon_{jpq} N_p E_q^{(i)}(Q) \quad \text{on the aperture,} \tag{3.2}$$

$$\varepsilon_{jpq} N_p E_q(Q) = \varepsilon_{jpq} N_p H_q(Q) = 0 \quad \text{on the screen.} \tag{3.3}$$

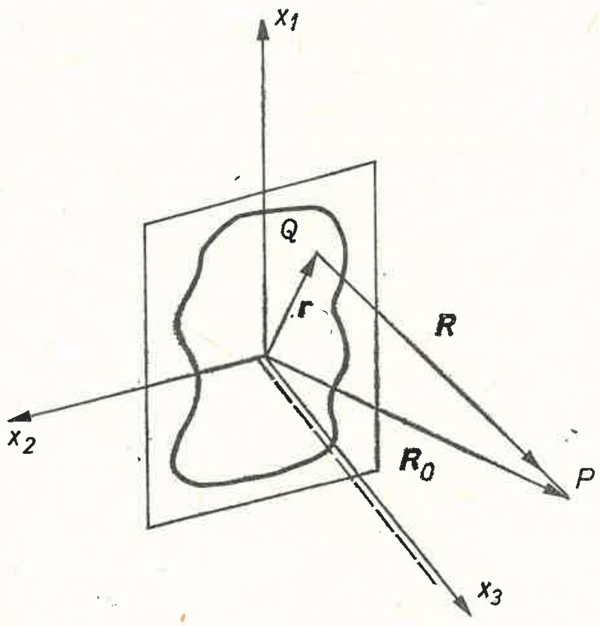

Fig. 1. The geometry and notation used in the derivation of Eq. (2.1) and (2.2). The dashed line indicates the direction of the optic axis

In accordance with (3.1), (3.2) and (3.3) we assume that the tangential components of the field on the aperture A are equal to the tangential components of the incident field. The tangential components on the unradiated part of the screen B vanish.

In diffraction problems such as the diffraction on the aperture or on the half plane we must integrate (2.1) and (2.2) over an infinite surface which includes the screen and

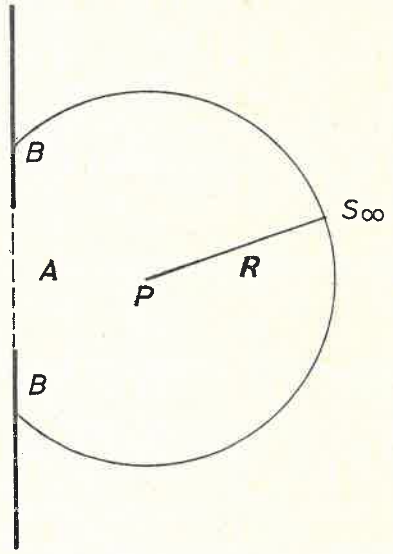

Fig. 2. Geometry and notation used for the study of Kirchhoff's diffraction by apertures in plane screens

aperture surfaces S_1 and closed hemisphere S_R with the radius R extending to infinity. This hemisphere is further designated as S_∞ (Fig. 1, 2).

The integration over the surface S_∞ gives the zeroth-contribution in the observation point because the electromagnetic field in a medium with uniaxial anisotropy satisfies infinity the radiation conditions [8].

Kirchhoff's conditions (3.1)–(3.3) and the radiation conditions thus limit the integration surface in the integrals (2.1) and (2.2) only to an aperture surface A.

In such a formulation the integrals (2.1) and (2.2) are often called in literature Kirchhoff's integrals, and the diffraction problem solved using this formalism is known as the saltus problem [6].

4. *The electromagnetic field in the Fraunhofer region*

If we remove the observation point P to infinity i. e. if we consider the field in a distant region, we can assume the following approximations called Fraunhofer's approximations

$$R \approx R_0 - \frac{\boldsymbol{R}_0 \boldsymbol{r}'}{R_0} = R_0 - \boldsymbol{n}\boldsymbol{r}', \tag{4.1}$$

$$R^{-1} \approx R_0^{-1}, \tag{4.2}$$

$$x_1 - x_1' \approx n_1 R_0, \quad x_2 - x_2' \approx n_2 R_0, \quad x_3 - x_3' \approx n_3 R_0, \tag{4.3}$$

where

$$\boldsymbol{R}_0 = \{x_1, x_2, x_3\}, \tag{4.4}$$

$$\boldsymbol{r}' = \{x'_1, x'_2\}, \tag{4.5}$$

and vector $\boldsymbol{n}$ has here meaning of unit vector in the direction

$$\boldsymbol{n} = \frac{\boldsymbol{R}_0}{R_0}. \tag{4.6}$$

Using the approximations (4.1)–(4.3) to Kirchhoff's integrals and after a few simple transformations we obtain the electromagnetic field in the Fraumhofer region in the following form:

$$\begin{aligned} E_j^{\mathrm{F}} \approx \; & \frac{iC}{4\pi\sqrt{\varepsilon^0}} \int_A d^2x'_Q \{ k_0 \varepsilon^e \varepsilon^0 \alpha^{-2} \varepsilon_{jkl} \varepsilon_{kpq} N_p E_q^{(i)}(Q) n_l + \omega\mu_0 \varepsilon^e \alpha^{-3} \kappa_{jk} n_k n_l \varepsilon_{lpq} N_p H_q^{(i)}(Q) \\ & + \alpha^{-3} \omega\mu_0 (\varepsilon^0 - \varepsilon^e) \kappa_{jk} n_k \varepsilon_{3pq} N_p H_q^{(i)}(Q) - \omega\mu_0 \alpha^{-1} \kappa_{jk} \varepsilon_{kpq} N_p H_q^{(i)}(Q) \} e^{-ik_0 \alpha n_k r'_k} \\ & + \frac{ik_0\sqrt{\varepsilon^0}}{4\pi(1-n_3^2)} \varepsilon_{j3k} \varepsilon_{lpq} n_k \int_A d^2x'_Q \{ n_3 n_l N_p E_q^{(i)}(Q) - N_p E_q^{(i)}(Q) \} \left(C\varepsilon^e \alpha^{-2} e^{-ik_0 \alpha n_k r_k'} - D e^{-ik^0 n_k r_k'} \right) \\ & + \frac{i\omega\mu_0}{4\pi\sqrt{\varepsilon^0}(1-n_3^2)} \varepsilon_{jk3} \varepsilon_{3lm} \varepsilon_{lpq} n_k n_m \int_A N_p H_q^{(i)}(Q) \left(C\varepsilon^e \alpha^{-1} e^{-ik_0 \alpha n_k r_k'} - D\sqrt{\varepsilon^0}\, e^{-ik^0 n_k r_k'} \right) d^2x'_Q, \end{aligned} \tag{4.7}$$

and

$$\begin{aligned} H_j^{\mathrm{F}} \approx \; & \frac{i\sqrt{\varepsilon^0}\,D}{4\pi} \int_A d^2x'_Q \{ k_0 \varepsilon_{jkl} \varepsilon_{kpq} N_p H_q^{(i)}(Q) \\ & - \omega\varepsilon_0 \sqrt{\varepsilon^0}\, \varepsilon_{kpq} n_j n_k N_p E_q^{(i)}(Q) + \omega\varepsilon_0 \sqrt{\varepsilon^0}\, \varepsilon_{jpq} N_p E_q^{(i)}(Q) \} e^{-ik^0 n_k r_k'} \\ & + \frac{i\sqrt{\varepsilon^0}\,k_0}{4\pi(1-n_3^2)} \varepsilon_{jk3} n_k \int_A d^2x'_Q \{ \varepsilon_{lpq} n_3 n_l N_p H_q^{(i)}(Q) \\ & - \varepsilon_{3pq} N_p H_q^{(i)}(Q) \} \left(C\varepsilon^e \alpha^{-2} e^{-ik_0 \alpha n_k r_k'} - D e^{-ik^0 n_k r_k'} \right) \\ & + \frac{i\omega\sqrt{\varepsilon^0}\,\varepsilon_0}{4\pi(1-n_3^2)} \varepsilon_{jk3} \varepsilon_{3lm} n_k n_m \varepsilon_{lpq} \int_A N_p E_q^{(i)}(Q) \left(C\varepsilon^e \alpha^{-1} e^{-ik_0 \alpha n_k r_k'} - D\sqrt{\varepsilon^0}\, e^{-ik^0 n_k r_k'} \right) d^2x'_Q, \end{aligned} \tag{4.8}$$

where C and D are constants

$$C = \frac{e^{ik_0 \alpha R_0}}{R_0}, \qquad D = \frac{e^{ik^0 R_0}}{R_0}.$$

5. *The properties of the diffracted field in a medium with uniaxial anisotropy in the Fraunhofer region*

In the Fraunhofer approximation for the diffracted field some terms appear, which are multiplied by two different exponential factors, $\exp[-ik_0\alpha n_k r'_k]$ and $\exp[-ik^0 n_k r'_k]$. Using these approximations in the form of a sum of two components

$$E_j^{\mathrm{F}} = E_j^{\mathrm{I}} + E_j^{\mathrm{II}}, \tag{5.1}$$

$$H_j^{\mathrm{F}} = H_j^{\mathrm{I}} + H_j^{\mathrm{II}}, \tag{5.2}$$

where E_j^{I}, H_j^{I} contain exponential factors $\exp[-ik_0\alpha n_k r'_k]$, E_j^{II}, H_j^{II} factors $\exp[-ik^0 n_k r'_k]$, we can demonstrate the following properties of the field (5.1) and (5.2):
(a) transversality towards the optic axis $\boldsymbol{c} = \{0, 0, 1\}$

$$c_j E_j^{\mathrm{I}} \neq 0, \quad c_j H_j^{\mathrm{I}} = 0, \tag{5.3}$$

$$c_j E_j^{\mathrm{II}} = 0, \quad c_j H_j^{\mathrm{II}} = 0, \tag{5.4}$$

(b) transversality towards the direction of wave propagation n (the direction of the observation is given by the vector $-\boldsymbol{n}$)

$$n_j E_j^{\mathrm{I}} \neq 0, \quad n_j H_j^{\mathrm{I}} = 0, \tag{5.5}$$

$$n_j E_j^{\mathrm{II}} = 0, \quad n_j H_j^{\mathrm{II}} = 0. \tag{5.6}$$

The electric field strength vector $\boldsymbol{E}^{\mathrm{I}}$ has longitudinal components in the direction of the optic axis and in the direction of diffracted wave propagation as well. The magnetic field strength vector $\boldsymbol{H}^{\mathrm{II}}$ has the component in the direction of the optic axis but is perpendicular to the direction of wave propagation $\boldsymbol{n}$.

Comparing the properties of the field (5.3)--)5.6) with well known properties of TM and TE type plane waves in a medium with uniaxial anisotropy we can state that the field with index I, in the Fraunhofer region assumes a character of the TM type plane wave, the field with index II assumes a character of the TE type plane wave. These two waves propagate in the same direction $\boldsymbol{n}$ but with two different phase velocities.

The properties (a) and (b) have been obtained without any restrictions in relation to the incident field. They are thus valid for an arbitrary incident field.

6. *The diffracted field for a given incident field*

Let us now consider the diffracted field in the case when the incident field takes the shape of the plane wave [9]. The incident TM field

$$E_j^{\mathrm{TM}} = k_0^2 \varepsilon^0 \delta_{j3} u + \nabla_j \nabla_3 u, \tag{6.1}$$

$$H_j^{\mathrm{TM}} = -i\omega\varepsilon_0\varepsilon^0 \varepsilon_{jk3} \nabla_k u, \tag{6.2}$$

$$u = \exp[i(k^e l_1 x_1 + k^e l_2 x_2 + k^0 l_3 x_3)], \tag{6.3}$$

where

$$k^e = \sqrt{\varepsilon^e}\, k_0, \quad k^0 = \sqrt{\varepsilon^0}\, k_0, \quad l_1^2 + l_2^2 + l_3^2 = 1.$$

After some simple transformations of (4.7) and (4.8) we obtain

$$E_j^{\mathrm{F}} \approx \frac{ik_0 k^0 \varepsilon^e C}{4\pi\alpha^2} \Big\{ \varepsilon_{jkp}\varepsilon_{pqr} N_q n_k (k^0 \delta_{r3} - l_3 \tau_r) + \frac{\alpha}{\varepsilon^e}\, \kappa_{jp}(\tau_p - \delta_{p3} l_3 k^0) - k^e \alpha^{-1} \kappa_{jp} n_p (l_1 n_1 + l_2 n_2)$$

$$+ k^e (1-n_3^2)^{-1} \varepsilon_{jk3} n_k (n_1 l_2 - n_2 l_1)\, (\sqrt{\varepsilon^0}\, n_3 l_3 - \alpha^2) \Big\} \int\limits_A e^{i(k^e l_1 - k_0 \alpha n_1) x_1' + i(k^e l_2 - k_0 \alpha n_2) x_2'} d^2 x_Q'$$

$$+ \frac{ik_0 k^0 k^e \sqrt{\varepsilon^0}\, D}{4\pi(1-n_3^2)}\, \varepsilon_{jk3} n_k (n_2 l_1 - n_1 l_2)\, (l_3 n_3 - 1) \int\limits_A e^{i(k^e l_1 - k^0 n_1) x_1' + i(k^e l_2 - k^0 n_2) x_2'} d^2 x_Q', \tag{6.4}$$

and

$$H_j^{\mathrm{F}} \approx \frac{ik^0 k^e \omega \varepsilon_0 \varepsilon^0 D}{4\pi} \{ \varepsilon_{jk3} l_k (n_3 - l_3) + (l_1 n_2 - l_2 n_1)\, (N_j - n_j l_3)$$

$$+ \varepsilon_{jk3} n_k (l_1 n_1 + l_2 n_2)\, (1-n_3^2)^{-1} (n_3 - l_3) \} \int\limits_A e^{i(k^e l_1 - k^0 n_1) x_1' + i(k^e l_2 - k^0 n_2) x_2'} d^2 x_Q'$$

$$+ \frac{ik_0 k^e \omega \varepsilon_0 \varepsilon^0 \varepsilon^e C}{4\pi\alpha^2 (1-n_3^2)}\, \varepsilon_{jk3} n_k (l_1 n_1 + l_2 n_2)\, (\alpha l_3 - \sqrt{\varepsilon^0}\, n_3)$$

$$\times \int\limits_A e^{i(k^e l_1 - k_0 \alpha n_1) x_1' + i(k^e l_2 - k_0 \alpha n_2) x_2'} d^2 x_Q'. \tag{6.5}$$

The incident TE field

$$E_j^{\mathrm{TE}} = i\omega\mu_0 \varepsilon_{jk3} \nabla_k \vartheta, \tag{6.6}$$

$$H_j^{\mathrm{TE}} = k_0^2 \varepsilon^0 \delta_{j3} \vartheta + \nabla_j \nabla_3 \vartheta, \tag{6.7}$$

$$\vartheta = \exp.\, [ik^0 (l_1 x_1 + l_2 x_2 + l_3 x_3)]. \tag{6.8}$$

For the diffracted field in the Fraunhofer region we have then the following expressions:

$$E_j^{\mathrm{F}} \approx \frac{ik_0 k^0 \omega \mu_0 \varepsilon^e}{4\pi\alpha^3}\, C\{ \varepsilon_{jk3} [l_k - n_k (l_1 n_1 + l_2 n_2)\, (1-n_3^2)^{-1}]$$

$$\times (\alpha \sqrt{\varepsilon^0}\, n_3 + \alpha^2 l_3) + (l_1 n_2 - l_2 n_1)\, (\alpha \sqrt{\varepsilon^0}\, N_j - \kappa_{jk} n_k l_3) \}$$

$$\times \int\limits_A e^{i(k^0 l_1 - k_0 \alpha n_1) x_1' + i(k^0 l_2 - k_0 \alpha n_2) x_2'} d^2 x_Q'$$

$$+ \frac{ik^0 k^0 \omega \mu_0 D}{4\pi(1-n_3^2)}\, \varepsilon_{jk3} n_k (l_1 n_1 + l_2 n_2)\, (n_3 + l_3) \int\limits_A e^{ik^0 (l_1 - n_1) x_1' + ik^0 (l_2 - n_2) x_2'} d^2 x_Q', \tag{6.9}$$

and

$$H_j^{\mathrm{F}} \approx \frac{ik^0k^0k^0D}{4\pi}\{\varepsilon_{jkl}\varepsilon_{lpq}N_pn_k(\delta_{q3}-l_3l_q)-n_j(l_1n_1+l_2n_2)+(l_j-\delta_{j3}l_3)$$

$$+\varepsilon_{jk3}n_k(n_1l_2-n_2l_1)(1-n_3^2)^{-1}(1+l_3n_3)\}\int_A e^{ik^0(l_1-n_1)x_1'+ik^0(l_2-n_2)x_2'}d^2x_Q'$$

$$+\frac{ik^0k^0k^0\varepsilon^e}{4\pi\alpha^2(1-n_3^2)}C\varepsilon_{jk3}n_k(\alpha\sqrt{\varepsilon^0}+l_3n_3)(l_1n_2-l_2n_1)$$

$$\times\int_A e^{i(k^0l_1-k_0\alpha n_1)x_1'+i(k^0l_2-k_0\alpha n_2)x_2'}d^2x_Q', \tag{6.10}$$

where

$$\tau_i = k^{(i)}l_i, \quad k^{(1)} = k^{(2)} = k^e, \quad k^{(3)} = k^0, \quad N = \{001\}.$$

7. *Conclusions*

If we study the diffracted field in the medium under discussion at a sufficiently large distance from the diffracting screen, i.e. in the Fraunhofer region, then the total field can be parted into the TM-type field an TE-type field. These two types of the field are generated by an arbitrary incident field. In this region these two fields propagate with different phase velocities and they are the plane waves.

Analysing carefully the expressions (6.9) (and 6.10) it appears that in the direction of wave propagation $\boldsymbol{n}$ which coincides with the direction $\boldsymbol{l}$ of the incident wave on the aperture plane, we find only one type of field, namely this, which is represented in the incident field.

The polarization state of the diffracted field can be found to be conserved because the direction $\boldsymbol{l}$ is the ray direction of the incident wave i. e. the direction of energy propagation.

It is very easy to transform the results described here into the well known results for the isotropic case [4, 5] by putting in (4.7) and (4.8) $\varepsilon^e = \varepsilon^0 = 1$.

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