STATISTICS OF SUPERPOSITION OF MULTIFOLD INDEPENDENT PARTIALLY POLARIZED SPECKLE FIELDS AND COHERENT BACKGROUND

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The present paper provides formulae, following the close relationship between the photocounting statistics and speckle, for the probability distribution of the intensity (integrated intensity) in a multifold speckle pattern with a coherent background and for its moments of arbitrary order, including partial polarization and taking into account arbitrary temporal and spatial spectra of light and arbitrary detection times and areas of the photocathode. Corrections are obtained to the correlation area of the speckle caused by the interference of the coherent background with the speckle pattern in the fourth order and by partial polarization. A number of formulae for special cases are also deduced, which have not been used in the speckle theory till now. A possibility of introducing the statistical properties of the incident light is outlined. The obtained formulae may be applied to exclude the microstructure from the macrostructure for a rough surface.

1. *Introduction*

Recently, great attention has been paid to the statistical properties and correlation properties of the speckle which can serve as a source of important information about surface roughness and can be used in image processing as well as in metrology and stellar interferometry [1–3]. As a model of speckle statistics, the negative exponential intensity distribution and its multifold generalization have been used [4, 5]. Sometimes the probability distribution of the intensity has been calculated exactly on the basis of the exact characteristic function and the Fredholm integral equation for finding eigenvalues of the correlation function [4, 5]. Such multifold formulae are useful for analyzing the speckle produced by light composed of a number of spectral components [5] or of a finite spectral bandwidth as well as for analyzing the integrated speckle with a detector of a finite aperture and finite resolving time. As a consequence of the fact that rough surfaces are depolarizing light during the scattering process, the effect of partial polarization must be included in the description [4]. Also the question of the correlation area of the speckle pattern has

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been discussed in [4]. As to the case of the speckle pattern with a coherent background, the model of narrow-band noise with one coherent component (the modified Rician density) has been used in the speckle literature till now and it is stated in [4] (p. 29) that even this model has not been generalized before to describe arbitrary polarization states.

In this paper we would like to show that there are formulae in the literature describing this as well as much more general situations obtained in connection with studies of the photocounting statistics of the superposition of coherent and chaotic fields (for a review of contributions to this field, see [7, 8]). Following the analogy between the photocounting statistics and the speckle, we are able (*i*) to give approximate as well as exact formulae for the description of multifold independent partially as well as fully polarized speckle fields with a coherent background, (*ii*) to give exact and approximate formulae to describe the multifold partially polarized speckle pattern (without a coherent background), and (*iii*) to propose some corrections to the correlation area of the speckle as a result of the interference (in the fourth order) between the speckle pattern and the coherent background and also caused by partial polarization. In general, we can describe effects of finite resolving time and detection area of a photodetector as well as arbitrary temporal and spatial spectra of light.

2. *Formulae describing the superposition of M-fold polarized speckle fields and a coherent background*

We start with a simpler case when the speckle fields are assumed to be fully polarized, also the coherent component is fully polarized and both polarization planes are in coincidence. For simplicity we consider stationary and homogeneous fields.

It has been found (e.g. [9]) that the characteristic function describing such a superposed field can be written in the form

$$\langle \exp isI \rangle = \prod_{\lambda=1}^{M} (1 - is\langle I_{ch\lambda}\rangle)^{-1} \exp\left[\frac{is\langle I_{c\lambda}\rangle \kappa_\lambda^2}{1 - is\langle I_{ch\lambda}\rangle} + is\langle I_{c\lambda}\rangle (1-\kappa_\lambda^2)\right], \quad (2.1)$$

where *is* represents a parameter, I is the intensity in the speckle (or rather the integrated intensity if a detection process with real detectors having finite detection area and resolving time is considered), $\langle I_{ch\lambda}\rangle$ is the chaotic mean intensity in mode λ (eigenvalue of the Fredholm integral equation with the second-order correlation function as the kernel [9, 4, 5]), $\langle I_{c\lambda}\rangle$ is the mean intensity of the coherent component in the mode λ and κ_λ describe shifts of corresponding frequencies of the chaotic and coherent components, e.g. for the Lorentzian spectrum of chaotic light and one coherent component it is $\kappa^2 = \sin^2(\Omega/2)/(\Omega/2)^2$, Ω being $(\omega_c - \omega_0)T$ (ω_c and ω_0 are the frequency of the coherent component and the mean frequency of the chaotic component, T is the photodetection time). Such formulae containing the frequency shifts can be applied to heterodyning a chaotic source with a coherent local oscillator. The intensities $\langle I_{ch\lambda}\rangle$ and $\langle I_{c\lambda}\rangle$ play the role of mean numbers of photons $\langle n_{ch\lambda}\rangle = \langle I_{ch\lambda}\rangle TS$ and $\langle n_{c\lambda}\rangle = \langle I_{c\lambda}\rangle TS$ if I is interpreted as the integrated intensity as mentioned above, S being the detection area.

The probability distribution of I is then given by the Fourier transformation applying the residuum theorem

$$P(I) = (2\pi)^{-1} \int_{-\infty}^{\infty} \langle \exp isI' \rangle \exp(-isI) ds$$

$$= \exp\left(-\sum_{\lambda=1}^{M} \frac{\langle I_{c\lambda}\rangle}{\langle I_{ch\lambda}\rangle}\right) \sum_{\{n_\lambda\}=0}^{\infty} \prod_{\lambda=1}^{M} \frac{1}{n_\lambda!} \frac{\langle I_{c\lambda}\rangle^{n_\lambda}}{\langle I_{ch\lambda}\rangle^{2n_\lambda+1}}$$

$$\times \sum_{\lambda'=1}^{M} \exp\left(-\frac{I-B}{\langle I_{ch\lambda'}\rangle}\right) \sum_{\sum_{\mu\neq\lambda'} m_\mu+\beta=n_{\lambda'}} \prod_{\mu\neq\lambda'}^{M} \frac{(-1)^{m_\mu}(n_\mu+m_\mu)!}{m_\mu! n_\mu!}$$

$$\times \left(\frac{\langle I_{ch\mu}\rangle \langle I_{ch\lambda'}\rangle}{\langle I_{ch\lambda'}\rangle - \langle I_{ch\mu}\rangle}\right)^{n_\mu+m_\mu+1} \frac{(I-B)^\beta}{\beta!}, \quad I \geqslant B, \quad P(I) = 0, \quad I < B \tag{2.2}$$

where

$$B = \langle I_c\rangle (1-\kappa^2), \quad \langle I_c\rangle = \sum_{\lambda=1}^{M} \langle I_{c\lambda}\rangle \text{ and } \kappa^2 = \sum_{\lambda=1}^{M} \langle I_{c\lambda}\rangle \kappa_\lambda^2 / \sum_{\lambda=1}^{M} \langle I_{c\lambda}\rangle.$$

For chaotic light, $\langle I_{c\lambda}\rangle = 0$ and the well-known result is obtained (e.g. [4])

$$P(I) = \sum_{\lambda=1}^{M} \exp\left(-\frac{I}{\langle I_{ch\lambda}\rangle}\right) \frac{\langle I_{ch\lambda}\rangle^{M-2}}{\prod_{\mu\neq\lambda}^{M} (\langle I_{ch\lambda}\rangle - \langle I_{ch\mu}\rangle)}, \quad I \geqslant 0,$$

$$P(I) = 0, \quad I < 0. \tag{2.3}$$

The moments of $P(I)$ are expressed in terms of the Laguerre polynomials $L_n^s(x)$ [9]

$$\langle I^k\rangle = k! \sum_{j=0}^{k} \frac{B^{k-j}}{(k-j)!} \sum_{\sum_{\lambda=1}^{M} m_\lambda=j} \prod_{\lambda=1}^{M} \frac{\langle I_{ch\lambda}\rangle^{m_\lambda}}{m_\lambda!} L_{m_\lambda}^0\left(-\frac{\langle I_{c\lambda}\rangle \kappa_\lambda^2}{\langle I_{ch\lambda}\rangle}\right) \tag{2.4a}$$

are for $\kappa_\lambda = 1 \quad (\omega_c = \omega_0)$

$$\langle I^k\rangle = k! \sum_{\sum_{\lambda=1}^{M} m_\lambda=k} \prod_{\lambda=1}^{M} \frac{\langle I_{ch\lambda}\rangle^{m_\lambda}}{m_\lambda!} L_{m_\lambda}^0\left(-\frac{\langle I_{c\lambda}\rangle}{\langle I_{ch\lambda}\rangle}\right). \tag{2.4b}$$

Of course, in all cases $\langle I\rangle = \sum_{\lambda=1}^{M} (\langle I_{c\lambda}\rangle + \langle I_{ch\lambda}\rangle) = \langle I_c\rangle + \langle I_{ch}\rangle$. For chaotic light we have $\langle I_{c\lambda}\rangle = 0$ and

$$\langle I^k\rangle = k! \sum_{\sum_{\lambda=1}^{M} m_\lambda = k} \prod_{\lambda=1}^{M} \langle I_{ch\lambda}\rangle^{m_\lambda} \tag{2.4c}$$

and if $M = 2$ [4]

$$\langle I^k\rangle = k! \sum_{m_1+m_2=k} \langle I_{ch1}\rangle^{m_1} \langle I_{ch2}\rangle^{m_2} = k! \frac{\langle I_{ch1}\rangle^{k+1} - \langle I_{ch2}\rangle^{k+1}}{\langle I_{ch1}\rangle - \langle I_{ch2}\rangle}. \tag{2.4d}$$

For $k = 2$, $\langle I^2\rangle = 2\langle I\rangle^2$, where $\langle I\rangle = \langle I_{ch1}\rangle + \langle I_{ch2}\rangle$.

These rather complex formulae can be simplified in a broadband limit where $\langle I_{ch\lambda}\rangle = \langle I_{ch}\rangle / M$ are the same, which leads to [10, 9]

$$P(I) = \frac{M}{\langle I_{ch}\rangle} \left(\frac{I-B}{\langle I_c\rangle}\right)^{(M-1)/2} \exp\left(-\frac{I + \langle I_c\rangle \kappa^2 - B}{\langle I_{ch}\rangle} M\right)$$
$$\times I_{M-1}\left(2|\kappa| M \frac{[\langle I_c\rangle (I-B)]^{1/2}}{\langle I_{ch}\rangle}\right), \quad I \geqslant B, \quad P(I) = 0, \quad I < B \tag{2.5}$$

or to the simplified expression if $\kappa = 1$ ($\omega_c = \omega_0$) when $B = 0$ [11]. Here I_M is the modified Bessel function. The corresponding moments are obtained in the form [10, 9]

$$\langle I^k\rangle = k! \sum_{j=0}^{k} \frac{B^{k-j}}{(k-j)!\,\Gamma(j+M)} \left(\frac{\langle I_{ch}\rangle}{M}\right)^j L_j^{M-1}\left(-\frac{\langle I_c\rangle \kappa^2 M}{\langle I_{ch}\rangle}\right) \tag{2.6a}$$

and for $\kappa = 1$ [11]

$$\langle I^k\rangle = \frac{k!}{\Gamma(k+M)} \left(\frac{\langle I_{ch}\rangle}{M}\right)^k L_k^{M-1}\left(-\frac{\langle I_c\rangle M}{\langle I_{ch}\rangle}\right), \tag{2.6b}$$

$\Gamma(k)$ being the gamma function. These formulae follow directly from (2.4a, b) if an identity for the Laguerre polynomials is applied [9]. For $k = 1, 2$

$$\langle I\rangle = \langle I_c\rangle + \langle I_{ch}\rangle, \quad \langle I^2\rangle = \langle I\rangle^2 + \frac{1}{M}(\langle I_{ch}\rangle^2 + 2\langle I_{ch}\rangle \langle I_c\rangle \kappa^2). \tag{2.6c}$$

All these expressions are generalizing the well-known expressions following from the Rician distribution [4] if $M = \kappa = 1$ (in [4] (2.6b) for $M = 1$ is given in terms of the confluent hypergeometric function). They can be applied to describe multifrequency or finite spectral bandwidth speckles with a coherent background or speckles with a coherent background integrated by a detector with a finite aperture and resolving time (a definition of M for this purpose will be given in Sec. 4).

For the speckle alone, $\langle I_c \rangle = 0$, and we arrive at the expressions used before

$$P(I) = \left(\frac{M}{\langle I_{ch}\rangle}\right)^M \frac{I^{M-1}}{\Gamma(M)} \exp\left(-\frac{IM}{\langle I_{ch}\rangle}\right), \quad I \geqslant 0, \quad P(I) = 0, \quad I < 0, \tag{2.7a}$$

$$\langle I^k \rangle = \langle I_{ch}\rangle^k \frac{\Gamma(k+M)}{\Gamma(M)M^k}, \tag{2.7b}$$

$$\langle I^2 \rangle = \langle I_{ch}\rangle^2 \left(1+\frac{1}{M}\right). \tag{2.7c}$$

3. *The superposition of M-fold partially polarized speckle fields and a coherent background*

Denoting ϕ as the angle between the polarization direction of the coherent component and the positive direction of the x-axis of the main polarization system of axes (x, y), we can deduce all formulae describing this case from the characteristic function [12–14]

$$\langle \exp isI \rangle = \prod_{\lambda=1}^{M} (1-is/E_\lambda)^{-1}(1-is/F_\lambda)^{-1}$$

$$\times \exp\left[\frac{is\langle I_{c\lambda 1}\rangle \kappa_\lambda^2}{1-is/E_\lambda} + \frac{is\langle I_{c\lambda 2}\rangle \kappa_\lambda^2}{1-is/F_\lambda} + isB/M\right], \tag{3.1}$$

where $\langle I_{c\lambda 1}\rangle = \langle I_{c\lambda}\rangle \cos^2\phi$, $\langle I_{c\lambda 2}\rangle = \langle I_{c\lambda}\rangle \sin^2\phi$, $E_\lambda = 2/(1+P)\langle I_{ch\lambda}\rangle$, $F_\lambda = 2/(1-P)\langle I_{ch\lambda}\rangle$ and P is the degree of polarization. For $P = 1$ and $\phi = 0$, we have (2.1). We also assume the polarization cross-spectral purity expressing independence of the coherence and polarization properties [12, 14].

An analogous expression to (2.2) for this case has been obtained in [13]. It is of rather complicated structure and therefore we do not repeat it here. Calculating the moments, we arrive at

$$\langle I^k \rangle = k! \sum_{j=0}^{k} \frac{B^{k-j}}{(k-j)!} \sum_{\sum_{\lambda}^{M} (m_\lambda+n_\lambda)=j} \prod_{\lambda=1}^{M} \frac{1}{m_\lambda! n_\lambda!} E_\lambda^{-m_\lambda} E_\lambda^{-n_\lambda}$$

$$\times L_{m_\lambda}^0(-\langle I_{c\lambda 1}\rangle \kappa_\lambda^2 E_\lambda) L_{n_\lambda}^0(-\langle I_{c\lambda 2}\rangle \kappa_\lambda^2 F_\lambda) \tag{3.2a}$$

or

$$\langle I^k \rangle = k! \sum_{\sum_{\lambda}^{M} (m_\lambda+n_\lambda)=k} \prod_{\lambda=1}^{M} \frac{1}{m_\lambda! n_\lambda!} E_\lambda^{-m_\lambda} F_\lambda^{-n_\lambda} L_{m_\lambda}^0(-\langle I_{c\lambda 1}\rangle E_\lambda) L_{n_\lambda}^0(-\langle I_{c\lambda 2}\rangle F_\lambda) \tag{3.2b}$$

if $\kappa_\lambda = 1$. For $P = 1$ and $\phi = 0$ we obtain (2.4a, b).

For chaotic light we derive

$$P(I) = \sum_{\lambda=1}^{M} \prod_{\lambda' \neq \lambda}^{M} \frac{\langle I_{ch\lambda}\rangle}{\langle I_{ch\lambda}\rangle - \langle I_{ch\lambda'}\rangle} \left\{ E_\lambda \exp(-IE_\lambda) \prod_{\lambda''}^{M} \frac{F_{\lambda''}}{F_{\lambda''} - E_\lambda} \right.$$

$$\left. + F_\lambda \exp(-IF_\lambda) \prod_{\lambda''}^{M} \frac{E_{\lambda''}}{E_{\lambda''} - F_\lambda} \right\}, \quad I \geqslant 0, \quad P(I) = 0, \quad I < 0 \tag{3.3}$$

providing (2.3) for fully polarized light. The moments are

$$\langle I^k \rangle = k! \sum_{\sum_\lambda^M (m_\lambda + n_\lambda) = k} \prod_{\lambda=1}^{M} E_\lambda^{-m_\lambda} F_\lambda^{-n_\lambda} \tag{3.4a}$$

giving (2.4c) for $P = 1$. If $M = 1$, we have

$$\langle I^k \rangle = k! \sum_{m+n=k} E^{-m} F^{-n} = k! \frac{(1/E)^{k+1} - (1/F)^{k+1}}{1/E - 1/F}, \tag{3.4b}$$

and $\langle I^k \rangle = k!\, \langle I_{ch} \rangle^k$ for $P = 1$.

In the broad-band limit we obtain by using the residuum theorem [12]

$$P(I) = \frac{E^M}{F^{M-1}} \left(\frac{(I-B)^{1/2}}{|\kappa|\, \langle I_{c2}\rangle^{1/2}} \right)^{2M-1} \exp(-F(I-B) - E\kappa^2 \langle I_{c1}\rangle - F\kappa^2 \langle I_{c2}\rangle)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{\Gamma(n+M)} \left[\left(1 - \frac{E}{F}\right) \frac{(I-B)^{1/2}}{|\kappa|\, \langle I_{c2}\rangle^{1/2}} \right]^n L_n^{M-1} \left(\frac{E^2 \kappa^2 \langle I_{c1}\rangle}{E-F} \right)$$

$$\times I_{n+2M-1}(2F|\kappa|\, (\langle I_{c2}\rangle\, (I-B))^{1/2}), \quad I \geqslant B, \quad P(I) = 0, \quad I < B, \tag{3.5a}$$

or for $\phi = 0$ [13]

$$P(I) = (EF)^M (I-B)^{2M-1} \exp(-E\langle I_c\rangle \kappa^2 - F(I-B))$$

$$\times \sum_{n=0}^{\infty} \frac{1}{\Gamma(n+M)} \left[\left(1 - \frac{E}{F}\right) F(I-B) \right]^n \frac{1}{\Gamma(n+2M)} L_n^{M-1} \left(\frac{E^2 \kappa^2 \langle I_c\rangle}{E-F} \right),$$

$$I \geqslant B, \quad P(I) = 0, \quad I < B; \tag{3.5b}$$

here $E = 2M/\langle I_{ch}\rangle(1+P)$, $F = 2M/\langle I_{ch}\rangle(1-P)$ and in (3.5b) $\langle I_{c2}\rangle = 0$, $\langle I_{c1}\rangle = \langle I_c\rangle$. Expression (3.5a) for $M = 1$ just provides that formula asked for in [4].

The corresponding moments are [12]

$$\langle I^k \rangle = k! \sum_{j=0}^{k} \frac{1}{(k-j)!} B^{k-j} F^{-j} \sum_{i=0}^{j} \frac{1}{\Gamma(i+M)\Gamma(j+M-i)} \left(\frac{F}{E}\right)^i$$

$$\times L_i^{M-1}(-E\kappa^2\langle I_{c1}\rangle) L_{j-i}^{M-1}(-F\kappa^2\langle I_{c2}\rangle) \quad (3.6a)$$

and for $\phi = 0$ [13]

$$\langle I^k \rangle = \frac{k!}{\Gamma(M)} \sum_{j=0}^{k} \frac{1}{(k-j)!} B^{k-j} F^{-j} \sum_{i=0}^{j} \frac{\Gamma(j+M-i)}{(j-i)!\Gamma(i+M)} \left(\frac{F}{E}\right)^i L_i^{M-1}(-E\kappa^2\langle I_c\rangle). \quad (3.6b)$$

From (3.6a) for $k = 2$

$$\langle I^2 \rangle = \langle I \rangle^2 + \frac{1}{M}\left(\langle I_{ch}\rangle^2 \frac{1+P^2}{2} + 2\kappa^2\langle I_{ch}\rangle \langle I_c\rangle \frac{1+P\cos 2\varphi}{2}\right). \quad (3.6c)$$

For $\omega_c = \omega_0$ we have also from (3.6a)

$$\langle I^k \rangle = k! F^{-k} \sum_{i=0}^{k} \frac{1}{\Gamma(i+M)\Gamma(k+M-i)} \left(\frac{F}{E}\right)^i L_i^{M-1}(-E\langle I_{c1}\rangle) L_{k-i}^{M-1}(-F\langle I_{c2}\rangle). \quad (3.6d)$$

Equations (3.5) and (3.6) provide the generalization of (2.5) and (2.6) following from $P = 1$ and $\phi = 0$.

Excluding the coherent background by putting $\langle I_c \rangle = 0$, we arrive at [12, 13]

$$P(I) = \frac{(EF)^M}{\Gamma(2M)} I^{2M-1} \exp(-FI)\, {}_1F_1(M, 2M; (F-E)I)$$

$$= \frac{\pi^{1/2}(EF)^M}{\Gamma(M)} \left(\frac{I}{F-E}\right)^{M-1/2} \exp\left(-\frac{E+F}{2} I\right) I_{M-1/2}\left(\frac{F-E}{2} I\right),$$

$$I \geqslant 0, \quad P(I) = 0, \quad I < 0 \quad (3.7)$$

with the following moments

$$\langle I^k \rangle = \frac{1}{\Gamma^2(M)} F^{-k} \sum_{j=0}^{k} \binom{k}{j} \Gamma(j+M)\Gamma(k+M-j) \left(\frac{F}{E}\right)^j; \quad (3.8)$$

here ${}_1F_1$ is the confluent hypergeometric function. For $P \to 1$ we arrive at (2.7a, b) and for $M = 1$

$$P(I) = \frac{EF}{F-E}(\exp(-EI) - \exp(-FI))$$

$$= \frac{1}{P\langle I_{ch}\rangle}\left[\exp\left(-\frac{2I}{(1+P)\langle I_{ch}\rangle}\right) - \exp\left(-\frac{2I}{(1-P)\langle I_{ch}\rangle}\right)\right],$$

$$I \geqslant 0, \quad P(I) = 0, \quad I < 0, \tag{3.9a}$$

$$\langle I^k\rangle = k!F^{-k}\sum_{j=0}^{k}\left(\frac{F}{E}\right)^j = k!\,\frac{(1/F)^{k+1}-(1/E)^{k+1}}{1/F-1/E} \tag{3.9b}$$

giving $\langle I^k\rangle = k!\langle I_{ch}\rangle^k$ for $P = 1$ (in agreement with (3.4b)).

4. *Determination of M*

As has been mentioned, M can be chosen as a number of lines in the spectrum of laser light used for producing the speckle [5]. However, taking into account the influence of the detector, i.e. its finite resolving time and detection area, then I should be interpreted as the integrated intensity $\int_S \int_0^T I(\vec{x}, t) d^2\vec{x} dt$ and M can be determined by the comparison of the exact moment $\langle I^2\rangle$ with the approximate one given by (3.6c). The exact second moment has the form [16, 9, 12, 13, 17]

$$\langle I^2\rangle = \langle I\rangle^2 + \langle I_{ch}\rangle^2 \frac{1+P^2}{2}\mathcal{I}_1 + 2\langle I_{ch}\rangle\langle I_c\rangle\frac{1+P\cos 2\varphi}{2}\bar{\mathcal{I}}_1, \tag{4.1}$$

where

$$\mathcal{I}_1 = \frac{1}{T^2S^2}\iint_S\iint_0^T |\gamma_{ch}(\vec{x}_1-\vec{x}_2, t_1-t_2)|^2 d^2\vec{x}_1 d^2\vec{x}_2 dt_1 dt_2,$$

$$\bar{\mathcal{I}}_1 = \frac{1}{T^2S^2}\,\mathrm{Re}\iint_S\iint_0^T \gamma_{ch}(\vec{x}_1-\vec{x}_2, t_1-t_2)\gamma_c(\vec{x}_2-\vec{x}_1, t_2-t_1) d^2\vec{x}_1 d^2\vec{x}_2 dt_1 dt_2; \tag{4.2}$$

here γ_{ch} and γ_c are degrees of coherence of the chaotic and coherent components respectively and $\langle I_{ch}\rangle$ and $\langle I_c\rangle$ represent the corresponding mean intensities multiplied by TS (for a more general case of non-homogeneous fields, see [17]). Thus comparing (4.1) with (3.6c) we arrive at [16, 12, 13]

$$M = \frac{1+2\kappa^2(\langle I_c\rangle/\langle I_{ch}\rangle)(1+P\cos 2\phi)/(1+P^2)}{\mathcal{I}_1 + 2(\langle I_c\rangle/\langle I_{ch}\rangle)\bar{\mathcal{I}}_1(1+P\cos 2\phi)/(1+P^2)}. \tag{4.3}$$

For $\langle I_c \rangle = 0$, $M = 1/\mathscr{J}_1$ in agreement with [20, 21, 18, 19, 4]. It can be shown that in general $M \geqslant 1$. The dependence of M on T/τ_c ($\tau_c = 1/\Gamma$ being the coherence time and Γ the half-width of the spectrum) for the superposition of coherent and chaotic fields has been demostrated in [16] while its dependence on S/S_c (S_c being the coherence area) has been obtained in [18, 19, 4].

The expression (4.3) enables us to define the terms of the coherence time, area and volume more generally than it is usual for chaotic light [17] and consequently to define the correlation area of the speckle pattern more generally. Assuming the so-called cross-spectral purity of light enabling us to separate the temporal and spatial coherence ($\gamma(\vec{x}_1 - \vec{x}_2, \tau) = \gamma(\vec{x}_1 - \vec{x}_2)\gamma(\tau)$), we obtain for the speckle correlation area

$$S_c = \frac{S}{M} = S \frac{\mathscr{J}_{1S} + 2(\langle I_c \rangle / \langle I_{ch} \rangle)\bar{\mathscr{J}}_{1S}(1 + P\cos 2\phi)/(1+P^2)}{1 + 2\kappa^2(\langle I_c \rangle / \langle I_{ch} \rangle)(1 + P\cos 2\phi)/(1+P^2)}, \tag{4.4}$$

where

$$\mathscr{J}_{1S} = \frac{1}{S^2} \iint_{-\infty}^{\infty} |\gamma_{ch}(\vec{x}_1 - \vec{x}_2)|^2 d^2\vec{x}_1 d^2\vec{x}_2 \approx \frac{1}{S} \iint_{-\infty}^{+\infty} |\gamma_{ch}(x, y)|^2 dxdy,$$

$$\bar{\mathscr{J}}_{1S} = \frac{1}{S^2} \mathrm{Re} \iint_{-\infty}^{+\infty} \gamma_{ch}(\vec{x}_1 - \vec{x}_2)\gamma_c(\vec{x}_2 - \vec{x}_1) d^2\vec{x}_1 d^2\vec{x}_2 \approx \frac{1}{S} \iint_{-\infty}^{+\infty} \mathrm{Re}\,\{\gamma_{ch}(x, y)\gamma_c^*(x, y)\} dxdy, \tag{4.5}$$

being $(x, y) = (\vec{x}_1 - \vec{x}_2)$. Thus (4.4) provides corrections from the interference between the coherent and chaotic components and from partial polarization to the usual definition $S_c = \iint_{-\infty}^{+\infty} |\gamma_{ch}(x, y)|^2 dxdy$ [18, 19, 4] following from (4.4) for $\langle I_c \rangle = 0$. A detailed discussion of the corrections arising from the interference can be found in [17].

The influence of the finite resolving time of a photodetector can be discussed quite similarly [20, 9, 17]. In this case M is determined by (4.3), where

$$\mathscr{J}_{1T} = \frac{1}{T} \int_{-\infty}^{+\infty} |\gamma_{ch}(\tau)|^2 d\tau,$$

$$\bar{\mathscr{J}}_{1T} = \frac{1}{T} \int_{-\infty}^{+\infty} \mathrm{Re}\,\{\gamma_{ch}(\tau)\gamma_c^*(\tau)\} d\tau \tag{4.6}$$

stand for $\mathscr{J}_1$ and $\bar{\mathscr{J}}_1$ and $\tau_c = T/M$ similarly as for S_c given in (4.4) [17]. Also combined temporal-spatial effects, especially when the cross-spectral purity condition is not fulfilled, can be considered [17].

It should be noted that the above given formulae are exact in the narrow- and broad-band limits when $M = 1$ and $M \to \infty$ respectively. Nevertheless they can be used as approximate for all T and S, all temporal and spatial spectra and arbitrary polarization states [21, 22, 16, 23, 5], the effect of which is included in M through $\mathscr{I}_1$ and $\overline{\mathscr{I}}_1$ in the form (4.3) ($\mathscr{I}_1 = \mathscr{I}_{1S}\mathscr{I}_{1T}, \overline{\mathscr{I}}_1 = \overline{\mathscr{I}}_{1S}\overline{\mathscr{I}}_{1T}$). Particularly, it has been found in [23] that the accuracy of these formulae increases as the ratio $\langle I_c \rangle / \langle I_{ch} \rangle$ increases and P decreases and it is better than 1% if $\langle I_c \rangle / \langle I_{ch} \rangle > 4$ and $P = 1$. However, for purely chaotic light the maximal error is about 15%. In these studies the photocounting distribution $p(n, T)$ and its factorial moments $\langle I^k \rangle$ have been discussed. The relationship of $p(n, T)$ for chaotic light (for the superposition, cf. [23]) based on the approximate formulae and more exact calculation published in [22] is in agreement with the relationship of $P(I)$ based on the approximate formula and the exact calculation as given in [5]. More accurate results should be based on the recursion formulae [24, 25, 14] obtained for $p(n, T)$ and its factorial moments.

Till now the mostly used tool of investigation of the statistical properties of the speckle has been $P(I)$. However, the more convenient quantity should be the photocounting distribution $p(n, T)$. The all formulae for it corresponding to those given above for $P(I)$ and $\langle I^k \rangle$ can be found in the literature quoted here, particularly in [9, 10, 12, 13, 15].

Finally we note that we have considered coherent light to be incident on a rough surface. Assuming fluctuating incident light it is sufficient to substitute $\langle I_{ch} \rangle \to \langle I_{ch} \rangle I'$ (respectively $\langle I_c \rangle \to \langle I_c \rangle I'$) [26–28] with the additional average over I' with the probability distribution $P'(I')$ corresponding to the incident light. This generally leads to rather complicated expressions, but for the chaotic limit we easily obtain from (2.7b) for $M = 1$ if also chaotic light is incident that $\langle I^k \rangle = (k!)^2 \langle I \rangle^k$ [27–29]. Other approaches to describe the effect of a random medium have been proposed in [30–31].

If the temporal and spatial spectra of light together with T and S are known, we can determine M and the equations (2.6c) (or (3.6c) if P and ϕ were known) provide a quadratic equation for $\langle I_c \rangle$ after elimination of $\langle I_{ch} \rangle$ (the speckle) knowing $\langle I \rangle$ and $\langle I^2 \rangle$ from an experiment, i.e. measuring the photocounting distribution $p(n, T)$ or the intensity (integrated intensity) probability distribution $P(I)$ and calculating the first two moments or performing a correlation experiment with the help of two photodetectors, we can exclude the microstructure of a rough surface from its macrostructure, which is an important technical problem. As additional literature relevant to tackling this problem, the monograph [32] can be given. At present these questions are under consideration.

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