Vol. A52 (1977) ACTA PHYSICA POLONICA No 4

REMARKS ON THE MICROSCOPIC PROOF OF STABILITY CONDITIONS FOR NORMAL FERMI LIQUIDS

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(Received March 14, 1977)

It is assumed that the expansion coefficients of the ground-state autocorrelation functions with respect to inverse squares of the frequency, should be positive. Then it is shown, that for normal Fermi liquids, the first two of these coefficients are positive if the Pomeranchuk inequalities for Landau amplitudes hold. These inequalities allow for an independent microscopic proof of the stability conditions for normal Fermi liquids, for comparison with Leggett's proof. The method of calculating autocorrelation functions in the quasihomogeneous limit is also presented.

1. Introduction. Statement of the problem

The Pomeranchuk inqualities [1], for Legendre amplitudes of the effective quasiparticle interaction (i.e. Landau parameters), were obtained in the phenomenological Landau approach [2], from the stability condition of the ground state. This simply means that the ground state has the lowest energy. Unfortunately, this energy cannot be expressed immediately in terms of quasiparticle quantities in the microscopic approach to Fermi liquids [3–5]. Hence, the inequalities [1] cannot be obtained there, in the same way as for phenomenological approach. Leggett [6] demonstrated how to obtain these inequalities in the microscopic approach. They follow from the negativity of the quasiparticle part of static autocorrelation functions, which is caused by their spectral representation and the stability of the ground state.

The ground-state autocorrelation function, [6], can be written as follows

$$K_\xi(k\omega) = \sum_n \frac{2\omega_{n0}|\xi_{kn0}|^2}{(\omega+i\delta)^2-\omega_{n0}^2}. \tag{1}$$

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Here ω_{n0} is the excitation energy of the n-th state, ξ_{kn0} is the transition element between the ground (0) and the n-th state of the k-th Fourier transform of the operator ξ, $\delta = 0^+$ and ω is the external frequency. According to (1), all terms near ω^{-2s}, $s = 1, 2, \ldots$ in the series expansion of (1) have to be positive. These inequalities, as we shall show, also lead to the Pomeranchuk inequalities [1], and do not impose, at least for $s = 1$ and 2, any other constraint on the Landau parameters.

According to paper [4], the autocorrelation function for normal degenerate Fermi liquids, for an arbitrary ξ, can be represented for $|\omega|, kv \ll E_F$, where E_F denotes the Fermi energy and v-the velocity on the Fermi sphere, as follows

$$K_\xi(k\omega) = K_\xi^\omega - \sum_p \xi_p^{\omega *}\delta_p(k\omega)\,[\delta_{p,p'} - \sum_{p'} f_{p,p'}(k\omega)\delta_{p'}(k\omega)]\xi_{p'}^\omega. \quad (2)$$

Let us now define the symbols used (for details see e.g. papers [4] and [6]). K_ξ^ω — is the nonquasiparticle part of the autocorrelation function, which is equal to the (uncommutative) "ω limit" $\lim_{\omega\to 0}\lim_{k\to 0}$ of $K_\xi(k\omega)$;

$$\delta_p(k\omega) = a^2\delta(E_p - E_F)v(\hat{p}k)\,[v(\hat{p}k)-\omega]^{-1},$$

where E_p denotes the one-particle energy, $\hat{p}$ — the unit vector directed along the momentum p and a — the discontinuity of the density of particles on the Fermi sphere, δ is the Dirac delta function. The p-summation in (2) is over momentum space and the spin variable, ξ_p^ω is the vertex ξ in the ω-limit and $f_{p,p'}(k\omega)$ is the four-point function for the energy--momentum transfer ω, k, [4], cf. also [6]. If the system is invariant with respect to spin rotation, i. e. if the total spin is conserved, then the spin, i. e. traceless in spin space, vertices appear together with the spin-exchange part of $f_{p,p'}$ $(k\omega)$ whereas spinless ξ — with the spin-direct part of this function. If the system considered is invariant under rotation in momentum space, then for $\xi_p = f(|p|)\,Y_{lm}\,(\hat{p})\tau$, where τ is the unit matrix, or one of the Pauli matrices in spin space, then we have on the Fermi sphere, $\xi_p^\omega = \tau Y_{lm}(\hat{p})/a_{l\tau}[f]$, where a depends on l and τ, and functionally on f, but does not depend on m, as well $K_\xi^\omega \to K_{l\tau}[f]$, as a consequence of the Wigner-Eckart theorem. For conserved quantities ξ, $a_{lm} = a$, $K_\xi^\omega = 0$, [4]. It should be noted that formula (2) is a result of transformations of a purely algebraic character, [4]. Note also that f has to depend on $|p| = p$ such, that f can be changed appreciably only if the p — variation is, at least, of the order of the Fermi momentum, p_0, in order not to introduce a correlation length much greater than $\hbar/p_0$. Note also that for $l \gg 1$ the function $f(|p|)\,Y_{lm}(\hat{p})$ varies very quickly on the Fermi sphere and that the conditions of applicability of the formula (3) can be stronger than $|\omega|$, $kv \ll E_F$.

Using the relation between the function $f_{p,p'}$, $(k\omega)$ in the ω — limit and this function itself and applying the relation for ξ^ω for above ξ, one can rewrite (2) as follows (cf. [4, 6, 7]).

$$K_{flm}(k\omega) = K_{fl}^\omega + (va^2/a_{fl\tau}^2)\,\langle Y_{lm}^*(\hat{p})Q(k\omega)$$

$$\times\{[1-FQ(k\omega)]^{-1}\}_{p,p'}Y_{lm}(\hat{p}')\rangle_{p,p'} \equiv K_{fl}^\omega + \left(\frac{va^2}{a_{fl\tau}^2}\right)S_{lm}(k\omega). \quad (3)$$

Here v denotes the density of states on the Fermi sphere, F is the spin-exchange or spin--direct part of the dimensionless effective interaction for the spin and spinless vertices ξ

respectively, the bracket $\langle\ldots\rangle_{p,p'}$, denotes a double average over spherical angles connected with the momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$. Note that the expression in the curly bracket denotes the $\boldsymbol{p}$, $\boldsymbol{p}'$ — matrix element of the operator inverse to $1-FQ(k\omega)$, where $Q(k\omega)$ is defined by $Q(k\omega)g(p) = \{v(\hat{p}k)\,[\omega-v(\hat{p}k)]^{-1}\}\,g(p)$, whereas F is a nondiagonal operator in this representation, depending on $F(\hat{p}\,\hat{p}')$; the multiplication of operators assumes here the average over spherical angles connected with an intermediate momentum. The operator F is determined by its Legendre amplitudes, called in this case Landau parameters. They are defined as follows

$$F(\hat{p}\hat{p}') = \sum_{l=0}^{\infty} (2l+1)F_l P_l(\hat{p}\hat{p}'). \tag{4}$$

Note that the symmetry of this problem allows the choice of $\boldsymbol{k}$ along the z-th axis, without any loss of generality; in such a case $(\hat{p}\,\boldsymbol{k}) \equiv kz$. Such a choice will be applied by us further in this paper.

According to (2), for normal liquids, $K_\xi(k\omega)$ is a homogeneous function of zeroth degree of the variables ω and kv, i. e. a function of kv/ω as a single variable. Putting there $k = 0$ we find that

$$K_\xi^\omega = \sum_n \frac{2\omega_{n0}|\xi_{k=0,n0}|^2}{(\omega-i\delta)^2-\omega_{n0}^2}.$$

Since K_ξ^ω has to be ω-independent, we get that if $\xi_{k=0,n0} \neq 0$ then either $\omega_{n0} = 0$ or $\omega_{n0} \gtrsim E_F$. Hence

$$K_\xi^\omega = -\sum_n{}' \frac{2|\xi_{k=0,n0}|^2}{\omega_{n0}} \leqslant 0, \tag{5}$$

where the prime denotes that the sum was restricted to such states with $\boldsymbol{k} = 0$ that $\omega_{n0} > 0$. The above relation for $\xi_{k=0,n0}$ could serve as an additional characteristic of admissible ξ-operators. It seems that our previous characteristic and the present one lead to the same class of operators.

Now, putting $\omega = 0$ in $K_\xi(k\omega)$ we find that

$$K_\xi(k0) = -\sum_n \frac{2|\xi_{kn0}|^2}{\omega_{n0}} = -\sum_n{}' \frac{2|\xi_{k=0,n0}|^2}{\omega_{n0}}$$

$$-\widetilde{\sum_n} \lim_{k\to 0} \frac{2|\xi_{kn0}|^2}{\omega_{n0}} \equiv K_\xi^\omega + J_\xi, \qquad J_\xi < 0, \tag{6}$$

where the tilde confines states n such that $\lim_{k\to 0} \omega_{n0} = 0$. Note that in transformation (6) the homogeneity of the function $K_\xi(k\omega)$ has been used. Passing to the autocorrelation functions (3) and applying the fact that $J_\xi < 0$ one can obtain all Pomeranchuk inequalities [1] in a microscopic way [6].

2. Transformation of autocorrelation functions; inequalities of the first rank

Let us transform the function $S_{lm}(\boldsymbol{k}\omega)$, defined by the identity in formula (3). Expanding $Q(\boldsymbol{k}\omega)$ in $S_{lm}(\boldsymbol{k}\omega)$ into a power series with respect to $R \equiv kv/\omega$, next expanding $[1-FQ(\boldsymbol{k}\omega)]^{-1}$ and taking into account our rules of operator multiplication, after some algebra one can find that

$$S_{lm}(\boldsymbol{k}\omega) = \sum_{s=1}^{\infty} R^{2s}\{\langle|Y_{lm}(\hat{\boldsymbol{p}})|^2 z^{2s}\rangle_p + \sum_{t=1}^{2s-1} \sum_{k=1}^{t} (\sum_{n_1=1}^{t} \dots \sum_{n_k=1}^{t})'$$

$$\times\langle z^{2s-t} Y_{lm}(\hat{\boldsymbol{p}})F(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}_1)z_1^{n_1}F(\hat{\boldsymbol{p}}_1\hat{\boldsymbol{p}}_2)z_2^{n_2} \dots F(\hat{\boldsymbol{p}}_{k-1}\hat{\boldsymbol{p}}_k)z_k^{n_k} Y_{lm}(\hat{\boldsymbol{p}}_k)\rangle_{p,p_1\dots p_k}\} \equiv \sum_{s=1}^{\infty} R^{2s} Q_{slm}. \quad (7)$$

Here the prime over the sums over $n_1 \dots n_k$ denotes the restriction to $n_1+n_2+ \dots n_k = t$, $\hat{\boldsymbol{p}}_i = [x_i, y_i, z_i]$, $x_i^2+y_i^2+z_i^2 = 1$, and the symbol $\langle\dots\rangle_{p,p_1\dots p_k}$ denotes the average over spjerical angles of all momenta: $\boldsymbol{p}, \boldsymbol{p}_1 \dots \boldsymbol{p}_k$; as we see, the operator notation was rejected by us in this formula. Note that the terms near odd powers of R vanished in (7) as a result of the odd parity of the function under the symbol $\langle\dots\rangle$.

According to formulae (1) and (3), (7) $Q_{slm} > 0$ for all $s = 1, 2, \dots$ and $l = 0, 1, 2, \dots |m| \leqslant l$. Let us discuss this inequality for $s = 1$. In this case we have

$$\langle|zY_{lm}(\hat{\boldsymbol{p}})|^2\rangle_p + \langle zY_{lm}^*(\hat{\boldsymbol{p}})F(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}')z'Y_{lm}(\hat{\boldsymbol{p}}')\rangle_{p,p'} > 0. \quad (8)$$

Taking into account the well — known recurrence properties for associate Legendre polynomials. P_l^m, (cf. e. g. [8]) one can write for normalized spherical functions

$$zY_{lm}(\hat{\boldsymbol{p}}) = C_{lm}Y_{l+1,m}(\hat{\boldsymbol{p}}) + C_{l-1,m}Y_{l-1,m}(\hat{\boldsymbol{p}}), \quad (9)$$

where

$$C_{lm} = \left[\frac{(l+1)^2-m^2}{(2l+1)(2l+3)}\right]^{1/2}, \quad (10)$$

and the phases of spherical functions are chosen here in such a way that $Y_{lm} = e^{im\varphi}\bar{P}_l^m$, with $\bar{P}_l^m$-normalized associate Legendre polynomials. Substituting (9) into (8) and applying the addition theorem for spherical functions one finds that (8) is equivalent to

$$(1+F_{l+1})C_{lm}^2 + (1+F_{l-1})C_{l-1,m}^2 > 0, \quad (11)$$

which does not impose any other constraint on the Landau parameters in comparison to inequalities [1], i. e.

$$1+F_l > 0, \quad l = 0, 1, 2, \dots \quad (12)$$

The conditions (11) together with the positivity of compressibility and spin susceptibility are equivalent to (12). In order to show this statement, let us observe that inequality (11) for $l = m$ passes into (12), but for $l = 1, 2, \dots$, and apply the formulae for the compressibility and spin susceptibility.

Let us verify whether the consideration, that ξ is a linear combination of Y_{lm} with different l of the same parity, does not lead to inequalities for F_l stronger than (12). Note that without any loss of generality one can consider only linear combinations of Y_{lm} with fixed m, based on the Wigner-Eckart theorem, because m describes irreducible representations of the rotation group around the $\boldsymbol{k}$-axis. Moreover, the application of this theorem to the full rotation group shows that the autocorrelation function in the ω-limit splits into the sum of autocorrelation functions of ξ-operators with definite l, and that these functions are m-independent. On the other hand, the quasiparticle part of the autocorrelation function, an analogue of $S_{lm}(\boldsymbol{k}\omega)$ in (3), does not split into the sum of autocorrelation functions with definite l, there are also some transition terms. Taking ξ in the form

$$\sum_{l} X_l f_l(p) Y_{lm}(\hat{\boldsymbol{p}}), \tag{13}$$

with l restricted to odd or even numbers, and performing transformations similar to the previous ones, we find that the quadratic form

$$\sum_{l,l'} U_{ll'}(ms)\,(X_l X_{l'}/a_{l\tau}a_{l'\tau}), \tag{14}$$

where

$$U_{ll'}(ms) = \langle Y_{lm}(\hat{\boldsymbol{p}}) z^{2s} Y_{l'm}(\hat{\boldsymbol{p}})\rangle_p + \sum_{t=1}^{2s-1} \sum_{k=1}^{t} \Big(\sum_{n_1=1}^{t} \cdots \sum_{n_k=1}^{t}\Big)'$$
$$\times \langle z^{2s-t} Y^*_{lm}(\hat{\boldsymbol{p}}) F(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}_1) z_1^{n_1} \ldots F(\hat{\boldsymbol{p}}_{k-1}\hat{\boldsymbol{p}}_k) z_k^{n_k} Y_{l'm}(\hat{\boldsymbol{p}}_k)\rangle_{p,p_1\cdots p_k} \tag{15}$$

has to be positive definite for any m and s. The application of the addition theorem for spherical functions and formula (9) shows that (15) depends on m only through $C_{l''m}$ and that there appear $C_{l''m}$ with varying l'' but with fixed m. This allows for a slight simplification of the notation, namely: to omit m's in $C_{l''m}$, keeping in mind that all inequalities have to be fulfilled for any m.

For $s = 1$ (15) takes the form

$$U_{ll'}(m1) = \langle Y^*_{lm}(\hat{\boldsymbol{p}}) z^2 Y_{l'm}(\hat{\boldsymbol{p}})\rangle_p + \langle z Y_{lm}(\hat{\boldsymbol{p}}) F(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}') z' Y_{l'm}(\hat{\boldsymbol{p}}')\rangle_{p,p'}. \tag{16}$$

The application of the formula (9) together with the addition theorem for spherical functions shows that $U_{ll'}(m1) = 0$ unless $|l-l'| = 0$ or 2. The term $U_{ll}(m1)$ has been calculated by us; using the same methods one can also obtain $U_{l,l+2}(m1) = U_{l+2,l}(m1)$. The inequality corresponding to the positive definiteness of the quadratic form (14) can be rewritten as follows

$$\sum_{l=|m|} \{V_l^2[(1+F_{l+1})C_l^2 + (1+F_{l-1})C_{l-1}^2]$$
$$+ V_l V_{l+2}(1+F_{l+1})C_l C_{l+1} + V_l V_{l-2}(1+F_{l-1})C_{l-1}C_{l-2}\} > 0, \tag{17}$$

where $V_l \equiv X_l/a_{l\tau}$. If the summation variable is changed in such way that all terms under the sum are proportional to $(1+F_{l+1})$, one can rewrite (17) as follows

$$\sum_{l=|m|} (1+F_{l+1})\,(V_l C_l + V_{l+2} C_{l+1})^2 > 0, \tag{18}$$

which does not impose any new restrictions on the Landau parameters. On the other hand, if $V_{l+2} = -V_l C_l / C_{l+1}$, then (18) vanishes even though the Pomeranchuk inequalities hold. This means that (18) is only positive semidefinite even though $1+F_l > 0$. Since $\lim_{l\to\infty} (C_{l+1}/C_l) = 1$ for any finite m, thus the modulae of all V_l' s, fulfilling the above condition, are asymptotically equal each other, for $l \gg 1$. This corresponds to the operators ξ^{ω} being some distributions of the variable $\hat{p}$ rather than its regular functions. The consideration of such ξ is rather physically meaningless. On the other hand, for ξ^{ω} not represented by convergent series, the quasiparticle part of the autocorrelation function as well as its series expansion do not exist in the mathematical sense.

3. The inequalities of the second rank and concluding remark

Let us discuss the expression (14) for $s = 2$. It is clear that the special choice of $X_l \neq 0$ only for $l = l_0$ leads to the second-rank analogue of the ineqality (8). Simultaneous application of formula (9) and the addition theorem for spherical functions shows that $U_{ll'}(m2) = 0$ unless $l-l' = 0$, 2 or 4. Moreover $U_{ll'}(m2) = U_{l'l}(m2)$. From the formula (15) we find that for $s = 2$ the following terms of the sum are possible

(a) with $t = 1$, $k = 1$, $n_1 = 1$,
(b) with $t = 2$, $k = 1$, $n_1 = 2$,
(c) with $t = 2$, $k = 2$, $n_1 = n_2 = 1$,
(d) with $t = 3$, $k = 1$, $n_1 = 3$,
(e) with $t = 3$, $k = 2$,, $n_1 = 2$,, $n_2 = 1$,
(*f*) with $t = 3$, $k = 2$, $n_1 = 1$, $n_2 = 2$,
(g) with $t = 3$, $k = 3$, $n_1 = n_2 = n_3 = 1$.

All these terms could be written down explicitly without any serious difficulty. In order to obtain $U_{ll'}(m2)$ one should add the terms (a)–(g) to the term $\langle Y_{lm}(\hat{p})\, z^4 Y_{l'm}(\hat{p})\rangle_p$. The list (a)–(g) and the prescription for the average values $\langle \dots \rangle_{p,p_1-p_k}$ in (15) give an almost explicit expression for all above terms. Hence, we will not reproduce here all terms (a)–(g) in such an explicit form. Let us write the term (*f*) as an illustrative example. We have:

$$\langle z Y^*_{lm}(\hat{p}) F(\hat{p}\hat{p}_1) z_1 F(\hat{p}_1\hat{p}_2) z_2^2 Y_{l'm}(\hat{p}_2)\rangle_{p,p_1,p_2}, \tag{19}$$

The calculation of integrals such as above, needs the subsequent application of the formula (9) and the addition theorem for spherical functions. This last, together with (4), give that the eigenvalue of the operator F in the state with definite l is F_l. In order to systematize calculations of the type (19) let us exploit the analogy between formula (9) and the random walk process. Namely, the multiplication of Y_{lm} by z corresponds to transition to $l+1$ with the weight C_l and to $l-1$ with the weigth C_{l-1}. The subsequent multiplication by z corresponds to the next step of the random walk. From this point of view all integrals (a)–(g) correspond to the four-step random walk, because $n_1 + \dots n_k + 2s - t = 4$ for $s = 2$. If $l' = l+4$ then there is only one possible four-step way from l to $l+4$, with the statistical weight $C_l C_{l+1} C_{l+2} C_{l+3}$ and, for the integral (19), with the multiplier $F_{l+1} F_{l+2}$. Hence, the contribution of integral (19) to $U_{l,l+4}$ $(m2)$ is given by $C_l C_{l+1} C_{l+2} C_{l+3} F_{l+1} F_{l+2}$. Let us analyze the contribution of integral (19) to $U_{l,l+2}$ $(m2)$. It is easy to see that there are four possible paths leading from l to $l+2$. They can be denoted as follows

$$(-+++),\ (+-++),\ (++-+),\ (+++-), \tag{20}$$

where $(+)$ denotes a forward step, whereas $(-)$ denotes a backward step. Assuming that initial step of each path is on the left-hand side of each bracket (...), and that we start from the state l, one can obtain the statistical weight for each path. For the paths listed above by (20) we have respectively

$$C_{l-1}^2 C_l C_{l+1},\ C_l^3 C_{l+1},\ C_l C_{l+1}^3,\ C_l C_{l+1} C_{l+2}^2 \tag{21}$$

The F_l-dependent factor is determined simultaneously by the l-state near the operator F in each integral (a)–(g) on each of the above listed paths. For integral (19) we have respectively

$$F_{l-1}F_l,\ F_{l+1}F_l,\ F_{l+1}F_{l+2},\ F_{l+1}F_{l+2}. \tag{22}$$

Multiplying now the statistical weights (21) by the corresponding factors of (22) and taking the sum of such products, we obtain the contribution of the integral (f) (19) to $U_{l,l+2}$ $(m2)$.

Let us list, for completeness, the paths beginning at l and ending at the same point after 4 steps. We have the following paths:

$$\begin{gathered}(--++),\ (-+-+),\ (-++-),\\ (+--+),\ (+-+-),\ (++--),\end{gathered} \tag{23}$$

with the following corresponding statistical weights:

$$\begin{gathered}C_{l-2}^2 C_{l-1}^2,\ C_{l-1}^4,\ C_{l-1}^2 C_l^2\\ C_{l-1}^2 C_l^2,\ C_l^4,\ C_l^2 C_{l+1}^2.\end{gathered} \tag{24}$$

The factors corresponding to these weights in the integral (17) will have the form

$$F_{l-1}F_{l-2},\ F_{l-1}F_l,\ F_{l-1}F_l,\ F_{l+1}F_l,\ F_{l+1}F_l,\ F_{l+1}F_{l+2}. \tag{25}$$

In order to obtain the contribution of (19) to U_{ll} $(m2)$ one should multiply (24) by the corresponding factors (25) and form the sum of products. It is clear, from the above considerations, that the contribution of the first right-hand side term in (13) to $U_{l,l+4}$ $(m2)$ is equal to $C_l C_{l+1} C_{l+2} C_{l+3}$, whereas the contribution to $U_{l,l+2}$ $(m2)$ and $U_{l,l}$ $(m2)$ are equal to the sum of weights (21) and (24), respectively. The contributions of the terms (a)–(e) end (g) to the mentioned matrix elements can be calculated quite similarly as for the term (f). Note that the listed paths and their statistical weights have a universal character for the integrals (a)–(g). The developed methods can also be useful for the calculation of $U_{ll'}$ (ms) for $s > 2$.

Performing the calculation sketched above, one can rewrite the condition of positive definiteness of the quadratic form (14) at $s = 2$ as follows

$$\sum_{l=|m|} \{[C_l^2 C_{l+1}^2 T_l^2 T_{l+2} V_l^2 + C_{l-2}^2 C_{l-1}^2 T_{l-2} T_{l-1}^2 V_l^2 + T_l (T_{l+1} C_l^2 + T_{l-1} C_{l-1}^2)^2 V_l^2]$$

$$+2C_l C_{l+1} T_l T_{l+1} (T_{l+1} C_l^2 + T_{l-1} C_{l-1}^2) V_l V_{l+2}$$

$$+2C_l C_{l+1} T_{l+1} T_{l+2} (T_{l+3} C_{l+2}^2 + T_{l+1} C_{l+1}^2) V_l V_{l+2}$$

$$+2C_l C_{l+1} C_{l+2} C_{l+3} T_{l+1} T_{l+2} T_{l+3} V_l V_{l+4}\} > 0, \quad (26)$$

if the sum over l of V_l^2 is greater than zero. In formula (26) T_l denotes the Pomeranchuk variable, $1+F_l$. Changing the variable l into $l+4$ and $l+2$ in the second and third term in the square bracket respectively, and into $l+2$ in the first term proportional to $V_l V_{l+2}$, one can transform (26) into

$$\sum_{l=|m|} T_{l+2}[C_l C_{l+1} T_l V_l + (T_{l+3} C_{l+2}^2 + T_{l+1} C_{l+1}^2) V_{l+2} + C_{l+2} C_{l+3} T_{l+3} V_{l+4}]^2 > 0. \quad (27)$$

This inequality does not impose any additional constraint on the variables T_l.

In such a way we have proved that Pomeranchuk inequalities are sufficient to preserve the positivity of terms proportional to R^2 and R^4 in the autocorrelation functions. It is rather impossible to prove in a general way that these inequalities are sufficient to preserve the positivity of terms proportional to R^{2s}, $s = 3, 4, \ldots$, even though it is rather doubtful that inequalities of rank greater than 2 impose any additional constraint on Pomeranchuk variables. Hence, one can assert that the phenomenological and microscopic approach to Fermi liquids, equivalent from the point of view of equations, cf. [3, 5], are also equivalent from the point of view of inequalities, with the exception of Leggett's inequality, [6]. Note that this could be connected with the fact that we deal with normal paramagnetic liquids. Hence, it is interesting to compare the relations between the inequalities $J_\xi < 0$ and $Q_{slm} > 0$, cf. (6), for the simplest model of ferromagnetic Fermi liquids, i. e. with spherical Fermi surfaces for both spins, cf. e. g. [9], and for the isotropic models of superfluid [10, 11]. The consequences of the inequality $J_\xi < 0$ for the first system have been investigated in our paper [12]. On the other hand, Landau parameters for ferromagnetic liquids are symmetric spin matrices, cf. e. g. [12]. These matrices can be uncommutative for different l and this is the reason why the inequalities for the quadratic form (14) could, in principle, lead to constraints for Landau parameters uncoinciding with those discussed in [12].

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