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ANTICORRELATION EFFECT IN PARAMETRIC AMPLIFICATION PROCESSES

By L. Mišta and J. Peřina

Laboratory of Optics, Palacký University, Olomouc*

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It is shown that the anticorrelation effect can also occur in the non-degenerate parametric process as a result of the coupling of signal and idler modes. The obtained initial phase condition for the anticorrelation is a simple generalization of the Stoler condition for the degenerate case. We compare the results for non-degenerate as well as degenerate cases with classical and quantum pumping, which provides other possibilities of obtaining the anticorrelation regardless of the initial phase condition. For instance it always occurs for some time interval after switching on the interaction in the pumping mode during the second harmonic generation starting with zero amplitude for the second harmonic. It can also be observed in a correlation experiment between the pumping and signal (or idler) modes or the pumping and second harmonic modes.

1. Introduction

It is well-known that the interaction of light with matter changes the statistical properties of light. The nature of the change depends on the kind of interaction process and on the statistical properties of the incident electromagnetic field. Recently, much attention has been paid to the so-called anticorrelation (antibunching) effect (ACE) [1–7]. This effect is characterized by the fact that (*i*) the variance of the photon number is less than the average photon number and (*ii*) the photocounting distribution becomes narrower than the corresponding Poisson distribution for a coherent state. The ACE has been demonstrated for two-photon absorption [1, 3–5], multi-photon absorption [6], degenerate parametric amplification process [2, 7] and two-photon stimulated emission [8].

In the present paper we show that the ACE can also occur in the non-degenerate parametric process if both the signal and idler modes are simultaneously detected, which is a result of their coupling. The obtained initial phase condition for the anticorrelation is simple generalization of the Stoler condition for the degenerate case [2]. Further we compare results for quantum pumping obtained on the basis of the iterative solution of

* Address: Laboratory of Optics, Palacký University, Leninova 26, 771 46 Olomouc, Czechoslovakia.

the Heisenberg-Langevin equations for the non-degenerate as well as degenerate cases with those for classical pumping and we find some corrections in higher powers of the field intensity. As a consequence the ACE occurs regardless of the initial phase condition if two modes of various frequences are in interaction producing radiation of the sum frequency with the zero initial amplitude detecting both the incident modes beyond the medium or if the second harmonic is generated from the zero initial amplitude and the pumping radiation beyond the medium is detected. The ACE occurs in some time interval after switching on the interaction.

2. Non-degenerate case

This process is described by the Hamiltonian [9]

$$H = \sum_{j=1}^{2} \hbar\omega_j a_j^\dagger a_j - \hbar\kappa[a_1^\dagger a_2^\dagger \exp(-i\omega t + i\phi) + a_1 a_2 \exp(i\omega t - i\phi)], \tag{1}$$

where $\omega = \omega_1 + \omega_2$, ω_j ($j = 1, 2$) are frequencies of the signal and idler modes, ω is the frequency of the pumping mode, ϕ is the initial phase of the pumping and κ is a real positive coupling constant. First we assume the pumping to be so intense that it can be treated classically ($\sim \exp(-i\omega t + i\phi)$). We neglect any losses.

The equations of motion for the annihilation operators a_j($a_j^\dagger$ being the creation operators) in modes 1 and 2 corresponding to (1) are

$$i\dot{a}_1 = \omega_1 a_1 - \kappa a_2^\dagger \exp(-i\omega t + i\phi),$$

$$i\dot{a}_2 = \omega_2 a_2 - \kappa a_1^\dagger \exp(-i\omega t + i\phi). \tag{2}$$

Their solution is well-known [9],

$$a_1(t) = \exp(-i\omega_1 t)(a_1 \operatorname{ch}\kappa t + i a_2^\dagger \exp i\phi \operatorname{sh}\kappa t),$$

$$a_2(t) = \exp(-i\omega_2 t)(a_2 \operatorname{ch}\kappa t + i a_1^\dagger \exp i\phi \operatorname{sh}\kappa t), \tag{3}$$

where $a_j = a_j(0)$. Suppose the field consisting of modes 1 and 2 to be in the coherent state $|\xi_1, \xi_2\rangle$ ($\xi_j = |\xi_j| \exp i\phi_j$) [10] at $t = 0$. We introduce the photon number operator $n(t) = a_1^\dagger(t)a_1(t) + a_2^\dagger(t)a_2(t)$ and calculate the quantity $\langle(\Delta W)^3\rangle = \langle\mathcal{N}n^2(t)\rangle - \langle n(t)\rangle^2$ using (3) ($\mathcal{N}$ denotes the normally ordering operator in $a_j^\dagger(t)$, $a_j(t)$ and the brackets mean the average in the coherent state $|\xi_1, \xi_2\rangle$). Thus

$$\langle(\Delta W)^2\rangle = 2 \operatorname{sh}\kappa t \left[\frac{\operatorname{sh} 3\kappa t - \operatorname{sh}\kappa t}{2} + \operatorname{sh}(3\kappa t)(|\xi_1|^2 + |\xi_2|^2)\right.$$

$$\left. + 2 \operatorname{ch}(3\kappa t)|\xi_1||\xi_2| \sin(\phi_1 + \phi_2 - \phi)\right]. \tag{4}$$

This expression can be negative for some finite time interval after the switching on of the interaction if $\sin(\phi_1 + \phi_2 - \phi) < 0$ and then the field displays the ACE. The ACE is greatest when $\sin(\phi_1 + \phi_2 - \phi) = -1$, i. e. $\phi_1 + \phi_2 - \phi = -\pi/2$.

The parametric amplification process with quantum pumping is described by the Hamiltonian [11]

$$H = \sum_{j=1}^{3} \hbar\omega_j a_j^\dagger a_j - \hbar g(a_1 a_2 a_3^\dagger + a_1^\dagger a_2^\dagger a_3), \tag{5}$$

$\omega_3 = \omega_1 + \omega_2$ and a_3 is the annihilation operator of a photon in the pumping mode 3 with frequency ω_3 and g is the real positive coupling constant. The corresponding Heisenberg equations of motion for $a_j(t), j = 1, 2, 3$ cannot be solved in a closed form. Taking into account terms up to the second order in t, we can write an approximate solution in the form [11]

$$a_1(t) = \exp(-i\omega_1 t)\left[a_1 + igta_2^\dagger a_3 - \frac{g^2t^2}{2}(a_1 a_2^\dagger a_2 - a_1 a_3^\dagger a_3)\right],$$

$$a_2(t) = \exp(-i\omega_2 t)\left[a_2 + igta_1^\dagger a_3 - \frac{g^2t^2}{2}(a_1^\dagger a_1 a_2 - a_2 a_3^\dagger a_3)\right],$$

$$a_3(t) = \exp(-i\omega_3 t)\left[a_3 + igta_1 a_2 - \frac{g^2t^2}{2}(a_2^\dagger a_2 a_3 + a_1 a_1^\dagger a_3)\right]. \tag{6}$$

Supposing the field to be in the coherent state $|\xi_1, \xi_2, \xi_3\rangle$ $(\xi_j = |\xi_j| \exp i\phi_j, j = 1, 2, 3)$ at $t = 0$ and making use of (6), we obtain

$$\langle(\Delta W)^2\rangle = 4\,gt\,|\xi_1|\,|\xi_2|\,|\xi_3| \sin(\phi_1+\phi_2-\phi_3)$$
$$+2g^2t^2(3|\xi_2|^2|\xi_3|^2 + 3|\xi_1|^2|\xi_3|^2 + |\xi_3|^2 - |\xi_1|^2|\xi_2|^2), \tag{7}$$

where $\langle(\Delta W)^2\rangle = \langle\mathcal{N}n^2(t)\rangle - \langle n(t)\rangle^2$, $n(t) = a_1^\dagger(t)a_1(t) + a_2^\dagger(t)a_2(t)$.

Expanding (4) into a power series in t, taking into account only terms up to the second order in t and putting $\kappa = g|\xi_3|$, $\phi_3 = \phi$, we obtain (7) except for the term $-2g^2t^2|\xi_1|^2|\xi_2|^2$, which is generally small compared with the others. Since it is of higher order in the field intensities, it cannot follow from (4). The difference is due to the quantum description of pumping in (5). Thus considering such a process in which two radiation modes with frequencies ω_1 and ω_2 ($|\xi_1|, |\xi_2| \neq 0$) are in interaction producing radiation of frequency ω_3 with $|\xi_3| = 0$ at $t = 0$, we have from (7) $\langle(\Delta W)^2\rangle = -2g^2t^2|\xi_1|^2|\xi_2|^2$ and the ACE occurs regardless of the initial phase condition. In order to observe this effect we need to detect simultaneously both the modes 1 and 2 beyond the non-linear medium (the sum frequency mode 3 being filtered). In the case of the usual parametric amplification process $|\xi_1|, |\xi_3| \neq 0$, $|\xi_2| = 0$ and the bunching effect occurs $\langle(\Delta W)^2\rangle = 6g^2t^2|\xi_1|^2|\xi_3|^2 + 2g^2t^2|\xi_3|^2$.

Considering the signal mode alone, no ACE occurs because $\langle a_1^{\dagger 2}(t)a_1^2(t)\rangle - \langle a_1^\dagger(t)a_1(t)\rangle^2 = 2g^2t^2|\xi_1|^2|\xi_3|^2 \geqslant 0$ for both the solutions (3) and (6). Similarly for the idler mode. For the pumping mode 3 we obtain $\langle a_3^{\dagger 2}(t)a_3^2(t)\rangle - \langle a_3^\dagger(t)a_3(t)\rangle^2 = 0$, which means that this mode has a tendency to be coherent during the interaction [11].

Let us emphasize that the ACE does not occur when $\xi_1 = 0$ or $\xi_2 = 0$, i. e. the signal mode or the idler mode is in the vacuum state before the interaction. The ACE also does not occur if the field before the interaction is chaotic in at least one mode.

3. Degenerate case

The degenerate parametric amplification process (subharmonics generation) with classical pumping is described by the Hamiltonian

$$H = \hbar\omega a^\dagger a - \frac{\hbar\kappa}{2}\left[a^2 \exp(i2\omega t - i\phi) + a^{\dagger 2}\exp(-i2\omega t + i\phi)\right], \tag{8}$$

where a is the annihilation operator of the signal mode with the frequency ω. If the field is in the coherent state $|\xi\rangle$ ($\xi = |\xi| \exp i\phi_1$) at $t = 0$, then [2]

$$\langle(\Delta W)^2\rangle = \mathrm{sh}\,\kappa t\left[\frac{\mathrm{sh}\,3\kappa t - \mathrm{sh}\,\kappa t}{2} + 2\,\mathrm{sh}\,(3\kappa t)\,|\xi|^2 + 2\,\mathrm{ch}\,(3\kappa t)\,|\xi|^2 \sin(2\phi_1 - \phi)\right]. \tag{9}$$

One can see that if we put $\xi_1 = \xi_2 = \xi$ in (4), we obtain (9) multiplied by 2. This is comprehensible taking into account that the first two terms in (4) correspond to vacuum fluctuations in two modes and the last two terms correspond to two output or input modes ($|\xi|^2$ in (9) corresponds to $2|\xi|^2$ in (4)). If $\sin(2\phi_1 - \phi) < 0$, then the ACE occurs for some time interval after switching on the interaction again. This case has been also studied in greater detail from the point of view of the photocounting statistics including the numerical results in [7].

Considering quantum pumping, we can write the Hamiltonian in the form [12]

$$H = \sum_{j=1}^{2} \hbar\omega_j a_j^\dagger a_j - \hbar g(a_1{}^2 a_2^\dagger + a_1^{\dagger 2} a_2), \tag{10}$$

$\omega_2 = 2\omega_1$, a_1 being the annihilation operator of the signal (subharmonic) mode of the frequency ω_1 and a_2 being the annihilation operator of the pumping mode of the frequency ω_2. As the corresponding equations of motion for $a_1(t)$ and $a_2(t)$ cannot be solved in closed form again, we use the approximate solution taking into account terms up to the second order in t [13]

$$a_1(t) = \exp(-i\omega_1 t)\left[a_1 + 2igta_1^\dagger a_2 + g^2t^2(2a_1 a_2^\dagger a_2 - a_1^\dagger a_1{}^2)\right],$$

$$a_2(t) = \exp(-i\omega_2 t)\left[a_2 + igta_1{}^2 - g^2t^2(2a_1^\dagger a_1 a_2 + a_2)\right]. \tag{11}$$

Similarly as above we calculate the quantity

$$\langle(\Delta W)^2\rangle = \langle a_1^{\dagger 2}(t) a_1{}^2(t)\rangle - \langle a_1^\dagger(t) a_1(t)\rangle^2$$

$$= 4gt|\xi_1|^2|\xi_2|\sin(2\phi_1 - \phi_2) + 2g^2t^2(2|\xi_2|^2 + 12|\xi_1|^2|\xi_2|^2 - |\xi_1|^4). \tag{12}$$

Expanding (9) into a power series in t, keeping terms up to the second order in t and putting $\kappa = 2g\,|\xi_2|$, $|\xi| = |\xi_1|$, $\phi = \phi_2$, we obtain (12) except for the small term $-2g^2t^2\,|\xi_1|^4$, which is due to the quantum description of pumping in (10) and it is of higher order in the field intensity. Also no correlation occurs here in the pumping mode where $\langle a_2^{\dagger 2}(t)\, a_2^{\,2}(t)\rangle - \langle a_2^\dagger(t)\, a_2(t)\rangle^2 = 0$, which means that the pumping mode has a tendency to be coherent [13].

Similarly as in the non-degenerate case considering the second harmonic generation with $|\xi_2| = 0$ at $t = 0$ the pumping mode 1 detected beyond the medium will display the ACE with $\langle(\Delta W)^2\rangle = -2g^2t^2\,|\xi_1|^4$ (for initially chaotic light the bunching always occurs). For the usual process of the second subharmonic generation, $|\xi_1| = 0$, $|\xi_2| \neq 0$ and we have the bunching effect with $\langle(\Delta W)^2\rangle = 4g^2t^2\,|\xi_2|^2$.

We note that the same result for mode 1 has been obtained independently in [15, 16]. Moreover, it has been shown there that also mode 2 displays the anticorrelation effect starting $(gt)^6$. These authors have considered also the anticorrelation effect in the k-th harmonic generation process.

Related questions of the existence of the Glauber-Sudarshan quasi-distribution as well as consequences of the behaviour of various modes in the photocounting distribution will be discussed in a forthcoming paper on the basis of the generalized Fokker-Planck equation.

4. *Final remarks*

Detecting all three modes in the non-degenerate process, we obtain similarly as above

$$\langle \mathcal{N}(\sum_{j=1}^{3} a_j^\dagger(t)a_j(t))^2\rangle - \langle \sum_{j=1}^{3} a_j^\dagger(t)a_j(t)\rangle^2$$
$$= 4gt|\xi_1|\,|\xi_2|\,|\xi_3| \sin(\phi_1+\phi_2-\phi_3) + 2g^2t^2(2|\xi_2|^2|\xi_3|^2 + 2|\xi_1|^2|\xi_3|^2 + |\xi_3|^2 - |\xi_1|^2|\xi_2|^2), \tag{13}$$

which contains all terms of the same kind as (7) and thus the correlation of the pumping mode with the others does not have any effect on the above discussed possibilities of reaching the ACE. This is also reflected by

$$\langle \mathcal{N}(a_1^\dagger(t)a_1(t) + a_3^\dagger(t)a_3(t))^2\rangle - \langle a_1^\dagger(t)a_1(t) + a_3^\dagger(t)a_3(t)\rangle^2 = 0 \tag{14}$$

and the same relation for the idler and pumping modes. Similarly, in the degenerate process

$$\langle \mathcal{N}(a_1^\dagger(t)a_1(t) + a_2^\dagger(t)a_2(t))^2\rangle - \langle a_1^\dagger(t)a_1(t) + a_2^\dagger(t)a_2(t)\rangle^2$$
$$= 4gt|\xi_1|^2|\xi_2| \sin(2\phi_1-\phi_2) + 2g^2t^2(2|\xi_2|^2 + 8|\xi_1|^2|\xi_2|^2 - |\xi_1|^4). \tag{15}$$

However, detecting single modes separately by two photodetectors and correlating their outputs, we can observe the ACE (regardless of the initial phase condition) having

$$\langle a_1^\dagger(t)a_1(t)a_3^\dagger(t)a_3(t)\rangle - \langle a_1^\dagger(t)a_1(t)\rangle\,\langle a_3^\dagger(t)a_3(t)\rangle = -g^2t^2|\xi_1|^2|\xi_3|^2 \tag{16}$$

and the same relation for the idler and pumping modes, provided that the initial amplitudes are not zero. Similarly in the degenerate process

$$\langle a_1^\dagger(t)a_1(t)a_2^\dagger(t)a_2(t)\rangle - \langle a_1^\dagger(t)a_1(t)\rangle \langle a_2^\dagger(t)a_2(t)\rangle = -4g^2t^2|\xi_1|^2|\xi_2|^2. \tag{17}$$

Such a correlation experiment between the signal and idler modes also provides the above discussed possibilities of observing the ACE since

$$\langle a_1^\dagger(t)a_1(t)a_2^\dagger(t)a_2(t)\rangle - \langle a_1^\dagger(t)a_1(t)\rangle \langle a_2^\dagger(t)a_2(t)\rangle$$
$$= 2gt|\xi_1|\,|\xi_2|\,|\xi_3| \sin(\phi_1+\phi_2-\phi_3) + g^2t^2(2|\xi_1|^2|\xi_3|^2 + 2|\xi_2|^2|\xi_3|^2 + |\xi_3|^2 - |\xi_1|^2|\xi_2|^2). \tag{18}$$

An application of radiation exhibiting the ACE to optical communications has been outlined in [14].

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