DETERMINATION OF THE HEAT EMISSION COEFFICIENT AND COOLING CONSTANT OF SOLIDS BY ÅNGSTRÖM'S PERIODIC HEAT WAVE METHOD

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Applying Ångström's dynamical periodic heat wave method, equations are derived for the simultaneous determination of the cooling constant β, heat transmission coefficient α and heat loss coefficient G of solids. Measurements of these quantities are performed using an especially designed and constructed setup. A study is performed of the thermal diffusivity k and the three quantities β, α and G of methyl polymethacrylate versus temperature. Moreover, all four quantities are measured for methyl polymethracrylate in vacuum and various external gaseous media at room temperature.

1. *Introduction*

The temperature dependence of the thermal conductivity λ and temperature diffusivity k of dielectrics and semiconductors is a topic of interest to numerous authors, especially with regard to the temperature range close to the phase transition point. The fact that experimental studies are given, preference is probably related with the enormous difficulties besetting the mathematical treatment of thermal conductivity. In spite of these difficulties, the basic physical aspects of thermal conductivity are by now well understood.

It has been repeatedly established that, at the phase transition temperature, the change in structure of solids affects the value of their specific heat c_p ([1, 2] and others). This in turn affects the values of their thermal conductivity (ThC) and thermal diffusivity (ThD) coefficients [3]. Now the heat transmission coefficient (HTC) α and cooling constant β depend essentially on the specific heat, ThC and ThD. Information concerning the variations of these three properties is to be obtained from the temperature dependence of the ThC and ThD coefficients and cooling constant.

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Of especial interest to this author was the work of Pilawski [4, 5] who, applying the temperature method, determined the temperature dependence of the heat transmission coefficient α in ferroelectrics of the barium titanate group. In the phase transition region, be observed a marked maximum of α and confirmed experimentally the dependence of α on the specific heat and ThC coefficient.

Drabble and Goldsmid [6] as well as Kherlamov [7] lay stress on the lack of information concerning the loss of heat through the mantle of a cylindrical solid in the process of temperature investigation of its ThD. The present work deals with the problem of the simultaneous determination of the ThD, HTC, and cooling constant. Moreover, we make an attempt to evaluate experimentally the influence of heat losses through the mantle of a solid rod on the value obtained when measuring its ThD. The ratio of the cooling constant and ThD coefficient, obtained in simultaneous determinations of the two properties, is also a source of valuable information. This ratio, in fact, conveys information on the changes in specific heat, since it is directly proportional to the latter. The determination of the above ratio is of particular interest in the range of temperatures close to the phase transition point of the solid.

The simultaneous determination of the ThD coefficient, cooling constant, and heat transmission coefficient can be performed by the periodic heat wave (PHW) method of Ångström [8, 9]. Using this method, we derive equations for the cooling constant, the heat transmission constant, and all-over heat loss coefficient of solids. Furthermore, by applying an especially constructed measuring setup, we determine the temperature dependences of the three coefficients as well as of ThD for methyl polymethacrylate.

2. *Method for the determination of the heat transmission coefficient α and cooling constant β of solids*

The heat transmission coefficient and cooling constant of solids are accessible to determination by Ångström's PHW method [8, 9], applied and extended by the authors of Refs. [10–13] and many others. In all casse, a differential equation in partial derivatives has to be solved. The equation describes the ThD of the solid in the direction of the x-axis, which is that in which a non-steady state heat flow is made to propagate through the solid. It is of the following form [7]:

$$\frac{\partial T(x, t)}{\partial t} = k\frac{\partial^2 T(x, t)}{\partial x^2} - \beta T(x, t), \tag{1}$$

with k — the ThD coefficient, and β — the cooling constant. Eq. (1) takes into account the loss of heat through the external surface of the solid. The latter is a cylinder (rod), with diameter very small compared to its length. Indirectly, β describes the heat given off by the surface of the rod to the external medium.

Numerically, β is positive and has the same value at all points of the rod. According to Kondratiev's theory of the ordered state [14], the numerical value of β is independent of the initial temperature field in the solid, but depends on its ThD and ThC coefficients, its dimensions, and shape. It depends moreover on the external conditions of heat exchange. The value of β is a measure of the response of the solid to cooling or heating of the medium in which it is immersed.

The theory of the ordered state [14–17] shows that the cooling coefficient β and heat transmission coefficient α are related by the equation

$$\beta = a \cdot S \frac{\alpha}{C}, \tag{2}$$

where S is the external surface of the solid, C its total heat capacity ($C = Vc_p\varrho$), and a the number of inhomogeneities of the temperature field within it. If the temperature of the bulk of the solid is everywhere the same, we have $a = 1$, and Eq. (2) becomes

$$\beta = S \frac{\alpha}{C}. \tag{3}$$

For a solid in the shape of a cylinder of radius R, the constants β and α are related as follows:

$$\beta = \frac{2\alpha}{c_p \cdot \varrho \cdot R}. \tag{4}$$

The product of the specific heat c_p and density ϱ of a solid is referred to as its thermal accumulativity χ.

If the heat transmission coefficient α is known numerically, one is able to determine the influence of the surrounding medium on the body immersed in it. The value of α moreover defines the conditions of heat exchange between the surface of the solid and the surrounding medium (e. g. a gas, or liquid). Quite generally, the exchange of heat proceeds by conductivity, convection, and radiation. In the most general case, heat exchange can be expressed as follows:

$$\alpha = \alpha_k + \alpha_p, \tag{5}$$

where α_k accounts for convection and conductivity, and α_p accounts for the contribution of radiation.

In Ångström's PHW method, one assumes the following boundary conditions (of the first kind) when solving Eq. (1):

(a) for $x = 0$ i. e. at the front surface of the infinite rod

$$T(x, t) = \sum_{n=0}^{\infty} A_n \cos(n\omega t + \varphi_n), \tag{6}$$

(b) at $x \to \infty$

$$T(x, t) = 0, \tag{7}$$

where $T(x, t)$ is the difference in temperature of the body with respect to the temperature of the medium. In the method considered, the initial condition is of the form:

$$T(x, t) = 0, \quad t = 0 \tag{8}$$

After Ångström, the variation in temperature for the above stated boundary and initial conditions can be written in the form of the Fourier series

$$T'(x, t) = \sum_{n=0}^{\infty} (p_n \cos n\omega t + d_n \sin n\omega t), \tag{9}$$

with $p_n(x)$ and $d_n(x)$ — harmonic components, $\sqrt{p_n^2+d_n^2} = A_n$ — amplitude of the n-th harmonic of the periodic heat wave, and $\operatorname{arctg} \frac{p_n}{d_n} = \varphi_n$ — phase of the n-th harmonic.

The variations in temperature in the points $x_1 = L_0$ and $x_2 = L_0+l$ of the thin rod are, respectively

$$T_1(x, t) = \sum_{n=0}^{\infty} (p_{1n} \cos n\omega t + d_{1n} \sin n\omega t), \tag{10}$$

$$T_2(x, t) = \sum_{n=0}^{\infty} (p_{2n} \cos n\omega t + d_{2n} \sin n\omega t). \tag{11}$$

The amplitudes and phases of the n-th wave harmonic in the points x_1 and x_2 of the rod are calculated from the following relations:

$$A_{1n} = \sqrt{p_{1n}^2+d_{1n}^2}; \quad \varphi_{1n} = \operatorname{arc tg} \frac{p_{1n}}{d_{1n}}$$

$$A_{2n} = \sqrt{p_{2n}^2+d_{2n}^2}; \quad \varphi_{2n} = \operatorname{arc tg} \frac{p_{2n}}{d_{2n}}. \tag{12}$$

On introducing the notation

$$\Phi_n = |\varphi_{1n} - \varphi_{2n}|, \tag{13}$$

and

$$\ln D_n = \ln \frac{A_{1n}}{A_{2n}}, \tag{14}$$

one obtains in conformity with the PHW method [8, 9, 18] the following equations for the ThD coefficient k

$$k = \frac{\pi l^2}{\tau \Phi_n \ln D_n} = \frac{\omega l^2}{2\Phi_n \ln D_n} = \frac{v l^2}{2 \ln D_n}, \tag{15}$$

where Φ_n is the difference in phase of the n-th heat wave harmonic in the points x_1 and x_2, due to the finite propagation velocity v of the waves, τ is the period of the wave, ω its circular frequency, and D_n the ratio of amplitudes of the n-th heat wave harmonic in the points x_1 and x_2. In order to compute Φ_n and D_n numerically, one can apply the Fourier method of graphical harmonics analysis [19] and have recourse to computers.

In the case when sine waves are obtained at x_1 and x_2 the computational procedure simplifies considerably. The phase difference and amplitude ratio now become

$$\Phi = |\varphi_1 - \varphi_2|, \tag{16}$$

$$\ln D = \ln \frac{A_1}{A_2}. \tag{17}$$

Henceforth, we shall be concerned with the case when sine heat waves arise in the points x_1 and x_2 of a thin rod of radius $R \ll L$.

It has been shown [20] that the ratio of the cooling constant β and ThD coefficient k is equal to

$$\frac{\beta}{k} = \frac{\ln^2 D - \Phi^2}{l^2}, \tag{18}$$

where l is the distance from x_1 to x_2. By having recourse to Eqs. (15) and (18) we obtain β in the form

$$\beta = \frac{\pi(\ln^2 D - \Phi^2)}{\tau\Phi \ln D} = \frac{\omega(\ln^2 D - \Phi^2)}{2\Phi \ln D}. \tag{19}$$

The PHW method of Ångström moreover permits the determination of the heat transmission coefficient α. To this aim, we apply Eqs (4) and (15); this yields α in the following form:

$$\alpha = \frac{\pi\chi R(\ln^2 D - \Phi^2)}{2\tau\Phi \ln D} = \frac{\omega\chi R(\ln^2 D - \Phi^2)}{4\Phi \ln D}. \tag{20}$$

It is obvious from Eq. (20) that, in order to obtain α, we have to know — in addition to the quantities accessible by PHW investigation of ThD — the heat accumulativity χ and radius R of the rod.

The product $\alpha S = G$ is the over-all heat loss coefficient [16, 17], S being the surface area of the mantle of the cylindrical rod. If the diameter $2R$ of the latter is very small compared with its length, its external surface reduces approximately to its mantle. The heat loss coefficient G defines the heat received by the surrounding medium from the mantle of the rod per unit time per unit difference in temperature. From Eq. (20) for G, we derive the following expression:

$$G = \frac{\pi S\chi R(\ln^2 D - \Phi^2)}{2\tau\Phi \ln D} = \frac{\omega S\chi R(\ln^2 D - \Phi^2)}{4\Phi \ln D}. \tag{21}$$

The preceding considerations show that the sine PHW method, applied essentially in determinations of ThD, permits moreover the determination of the cooling constant, HTC, and heat loss coefficient. Once all three coefficients β, α and G, characterizing directly or indirectly the losses of heat through the mantle of the rod, are available, one is able to evaluate experimentally their influence on the numerical value of the ThD coefficient measured.

3. *Apparatus, and results of measurements*

When determining the numerical values of the heat transmission coefficient and cooling constant of solids, we had recourse to the setup show in Fig. 1. The supply circuit of the micro-heater (H) included a time-controlled switching system, synchronically switching on and off the current flowing in the circuit. The periodical changes in current gave rise to periodic heat waves in the heater. When studied in the latter, they were found to con-

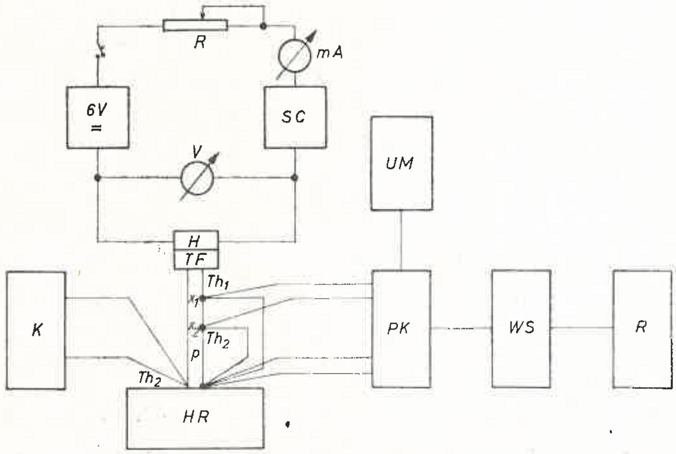

Fig. 1. Block diagram of measuring setup for the determination of the heat transmission coefficient, colling constant and thermal diffusivity coefficient of solids

tain higher harmonics. In our investigation, we applied sine waves. In order to obtain sine waves in the sample (P), the waves produced by the micro-heater were made to traverse a higher harmonics filter (TF) before attaining the sample. The filter consisted of platelets of appropriate thickness cut from a KCl single crystal [21]. A study of the waves in the sample showed that only sine wave propagated in the latter. The sample was glued with silver paste to a heat receiver (HR) of high heat capacity compared to that of the sample. The use of Degussa silver paste provided for permanent and close thermal contact between the micro-heater, higher harmonics filter, sample, and heat receiver. The heat waves propagating throughout the sample were studied by means of thermocouples (Th_1) and (Th_2) placed at the points x_1 and x_2 of the sample, respectively. The thermoelectric voltage signals from the thermocouples (Th_1) and (Th_2) proportional to the variations in temperature in the points x_1 and x_2 referred to the temperature of the surroundings, were transmitted to a contactron relay (PK) controlled by a non-stabilized multivibrator circuit (UM). In this way, the thermoelectric signals from the points x_1 and x_2 of the sample were fed to the d. c. input of an amplifier with a. c. transduction (WS). On amplification, the signals proceeded to the recorder (R). The static temperature of the sample was determined by the compensation method with a copper-constantan thermocouple and compensator (K).

Fig. 2 shows some heat waves, obtained with the measuring apparatus described above. The wave with the larger amplitude was recorded with the thermocouple (Th_1)

and that with the lower amplitude by (Th_2). The waves propagated in a sample of methyl polymethacrylate.

The following quantities were obtained numerically by measurement: the averaged amplitudes (A_1, A_2) of the heat waves, the averaged difference in phase angles (Φ), the circular frequencies of the waves (ω), the distance (l) from point x_1 to x_2 in the sample,

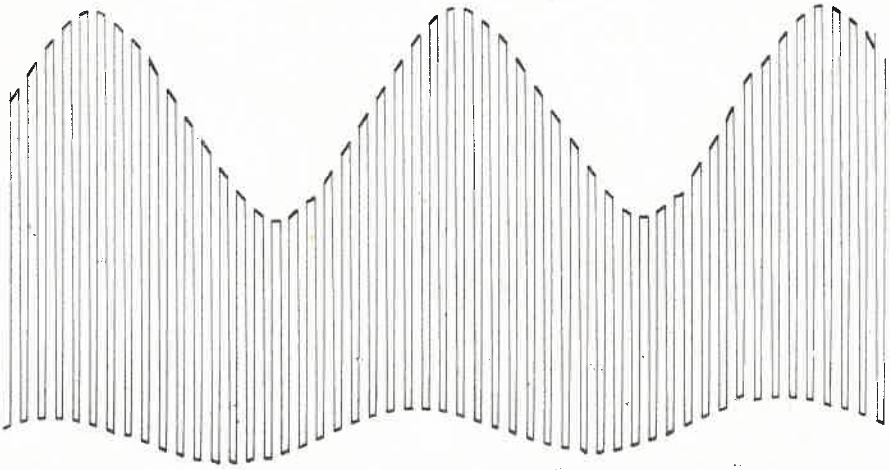

Fig. 2. Heat waves, recorded in the points $x_1 = L_0$ and $x_2 = L_0+l$ of the solid rod, respectively

as well as the radius (R) and length (L) of the latter. The ThD coefficient, cooling constant, HTC and heat loss coefficient were calculated from Eqs (15) and (19) — (21). respectively.

For investigation, we used methyl polymethacrylate samples in the shape of cylinders with a diameter of $2R = 0.4$ cm and length $L = 3.0$ cm. The circular frequency of the heat waves amounted to $\omega = 0.017\ s^{-1}$ and the distance between the points x_1 and x_2 of the rod to $l = 0.55$ cm. At these points, two holes were bored in the sample into which the copper-constantan thermocouples were introduced. The position of the holes with respect to the heated surface of the rod fulfilled the conditions specified in Ref. [22]. For the thermocouples, use was made of sufficiently long leads, 0.005 cm in diameter. The methyl polymethacrylate rod with micro-heater, thermocouples and heat receiver attached, was placed inside a vacuum holder, similar to the one proposed earlier by Krajewski [23]. The arrangement permitted the introduction of various gases at well defined pressures into the holder cavity. The temperature of the vacuum holder cavity was varied continuosly by means of a Hoeppler ultrathermostat.

Our investigation of the ThD coefficient, cooling constant, HTC, and heat loss coefficient was performed in vacuum, air, oxygen and hydrogen at room temperature. The three gases were at a pressure close to normal. The results obtained for the 4 quantities, measured in methyl polymethacrylate at 293 K, are given in Table I.

In the presence of the external gaseous media applied, the numerical ThD values measured refer to a solid-gas system. This is particularly important in the gas is hydrogen, since in this case the experimental ThD value is by one order of magnitude larger than

TABLE I

Numerical values of the thermal diffusivity coefficient k, cooling constant β, heat transmission coefficient α and heat loss coefficient G of methyl polymethacrylate at 293 K in vacuum and in various gaseous media

Solid investigated	k cm^2/s	β s^{-1}	α W/cm^2deg	G W/deg	External medium	Pressure N/m^2
Methyl polymethacrylate	2.1×10^{-3}	10^{-2}	1.24×10^{-4}	4.70×10^{-4}	vacuum	0.68
	2.2×10^{-3}	10^{-2}	1.27×10^{-4}	4.79×10^{-4}	oxygen	103360
	2.3×10^{-3}	10^{-2}	1.29×10^{-4}	4.86×10^{-4}	air (dry)	103360
	12.3×10^{-3}	2.5×10^{-2}	3.03×10^{-2}	11.50×10^{-4}	hydrogen	103360

in the case of vacuum. The heat losses through the mantle of the sample rod are now large compared to the losses in vacuum since the ThD of hydrogen at normal pressure is comparable to the ThD of silver. In agreement with theoretical predictions, the HTC, heat loss coefficient and cooling constant now become maximal.

The ThD of methyl polymethacrylate, measured in air and in oxygen, differs but slightly from the ThD measured in vacuum, the relative differences amounting to 10 and 5 per cent, respectively.

Methyl polymethacrylate is the polymer most commonly applied for technical purposes in the temperature range from about 293 to 353 K in air. We expect the polymer to

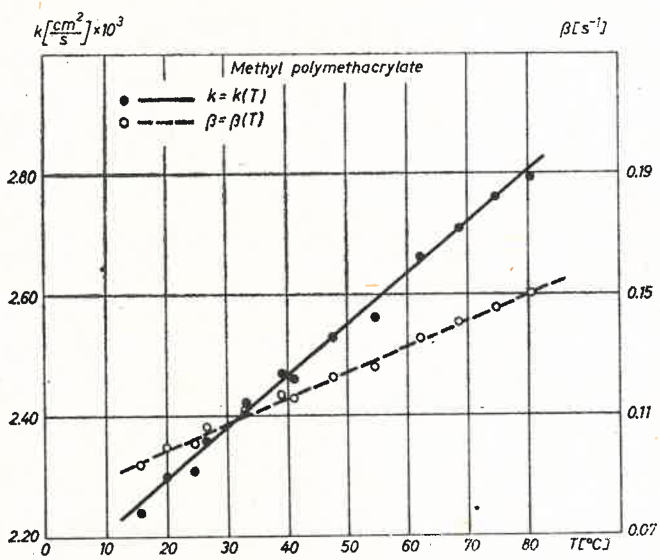

Fig. 3. Thermal diffusivity k and cooling constant β of methyl polymethacrylate vs temperature

be applied as a shield for desoxyribonucleic acid DNA in future studies of the thermal properties of the latter. This was one of the reasons which stimulated us to carry out additionally a study of the temperature-dependence of the ThD, cooling constant and HTC of methyl polymethacrylate in the above specified range of temperatures. It was absolutely necessary to determine all the thermal properties of this polymer, all the more so as such information was not available from the literature. Further on, we give the dependence of k, β, α and G vs temperature for methyl polymethacrylate.

The curves of Fig. 3 show the ThD coefficient k and cooling constant β of the polymer as functions of temperature (full circles denote experimental k-values, void circles — experimental β-values). One notes that k increases linearly from 2.24×10^{-3} cm²/s at 288.5 K to 2.80×10^{-3}/s at 353.3 K.

The temperature-dependence of the cooling constant β is also linear. For the above temperatures, β amounts to $0.094\ s^{-1}$ and $0.15\ s^{-1}$, respectively, and the ratio β/k to

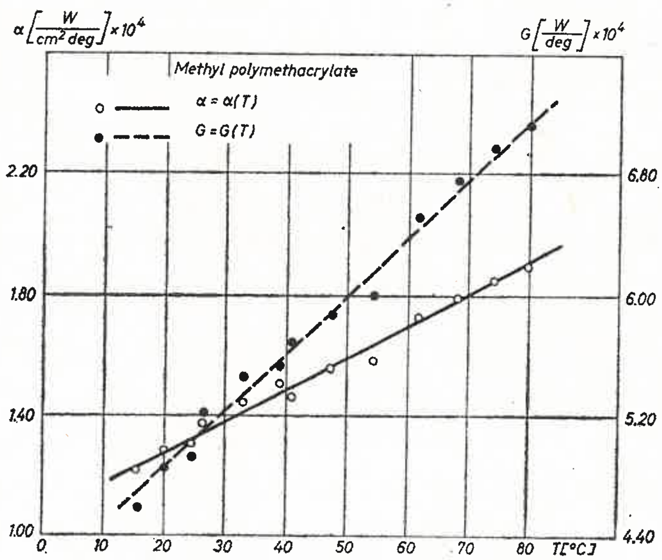

Fig. 4. Heat transmission coefficient α and heat loss coefficient G of methyl polymethacrylate vs temperature

$41.9\ cm^{-2}$ and $53.6\ cm^{-2}$. The increase in β points to larger and larger heat emission through the mantle of the rod into the surrounding medium.

The curves of Fig. 4 show the HTC α and heat loss coefficient G through the mantle of the methyl polymethacrylate rod as functions of temperature. Both α and G are linearly dependent on temperature in the range studied. For α we found 1.22×10^{-4} W/cm² deg at 288.5 K and 1.89×10^{-4} W/cm² deg at 353.3 K. For G, we obtained respectively 4.58×10^{-4} W/deg and 7.12×10^{-4} W/deg.

4. *Conclusion*

Investigation of the thermal diffusivity of solids by the dynamical periodic heat wave method of Ångström permits the simultaneous determination of their thermal diffusivity, cooling constant, heat transmission coefficient and heat loss coefficient, in cases of metals, semiconductors, as well as dielectrics. Once the cooling constant (thermal diffusivity ratio) is known, conclusions can be drawn concerning the variations of the specific heat and thermal conductivity of the solid.

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