MAGNETOROTATION IN THE NEIGHBOURHOOD OF AN OPTICAL RESONANCE LINE FOR THE TRANSITION IN NEON CORRESPONDING TO 6328 Å LINE

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The Faraday effect has been studied experimentally for neon line 6328 Å. In the experimental set-up crossed or nearly crossed polarizers were used. The comparison between experimental and theoretical diagrams for light intensity versus magnetic field enabled us to determine the value and the sign of the absorption coefficient for the $3s_2-2p_4$ transition in neon.

1. *Introduction*

During the twenties and thirties of our century the effect of magnetorotation in metalic vapours had been broadly used as a means for determining atomic constants, especially oscillator strength for the given optical transition [1]. The development of modern spectroscopic methods (radiofrequency double resonance, level crossing technique and others) caused interest in magnetorotation to decrease. The "old" notion of magnetorotation appeared natural in an article by Corney, Kible and Series [2]. The general

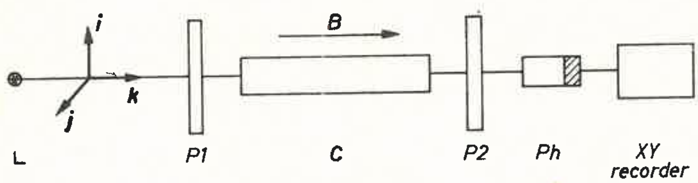

Fig. 1. Experimental arrangement

consideration presented by them were experimentally tested by several authors [3, 4, 5, 6]. This work is an experimental study of Corney, Kible, Series theoretical results for the $3s_2-2p_4$ transition (Paschen notation) in neon mixed with helium. Special attention is

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paid to the possibility of measuring the absorption coefficient κ_0 for a given transition in neon. The basic experimental arrangement is like that in figure 1.

Light, the frequency of which is resonant for the investigated atomic transition, emitted by source L and polarized by linear polarizer 1 propagates through the absorption tube C, situated in the magnetic field $\boldsymbol{B}$. The polarization axis of the polarizer 2 is rotated $\frac{\pi}{2} \pm \delta$ relative to that of polarizer 1. The light intensity detected by photomultiplier Ph is registered against the magnetic field $\boldsymbol{B}$. From comparison of theoretical with experimental plots we can get the absorption coefficient κ_0.

2. *Theoretical*

To describe the effect of magnetorotation we approach the problem semiclassically. The light propagating through the medium is treated according to Maxwell's equations whereas the same medium is characterized by a polarizability tensor which is to be obtained from quantum considerations. The calculations are done in a base of spherical vectors

$$\boldsymbol{e}_1 = -\frac{\boldsymbol{i}+i\boldsymbol{j}}{\sqrt{2}} \qquad \boldsymbol{e}_0 = \boldsymbol{k} \qquad \boldsymbol{e}_{-1} = \frac{\boldsymbol{i}-i\boldsymbol{j}}{\sqrt{2}}. \tag{1}$$

This base is particularity useful in our case, because the polarizability tensor is diagonal in it. Let us represent the polarizability tensor α as the sum of its real and imaginary parts: $\alpha = \alpha_r + i\alpha_i$. The substitution of tensor α into the wave equation yields, after standard calculation, the expression for electric vector $\boldsymbol{E}$ in a point z of our medium

$$\boldsymbol{E}(z, t) = \frac{A_0}{\sqrt{2}} e^{-i\left(\omega t - n^- \frac{\omega}{c}\right) - \kappa^- z} \boldsymbol{e}_1^*$$

$$+ \frac{A_0}{\sqrt{2}} e^{-i\left(\omega t - n^+ \frac{\omega}{c}\right) - \kappa^+ z} \boldsymbol{e}_{-1}^* \tag{2}$$

where

$$n^- = 1 + 2\pi\alpha_{11}^r \qquad n^+ = 1 + 2\pi\alpha_{-1-1}^r$$

$$\kappa^- = 2\pi\frac{\omega}{c}\alpha_{11}^i \qquad \kappa^+ = 2\pi\frac{\omega}{c}\alpha_{-1-1}^i$$

The foregoing formula is a well-known result of the classical theory: propagation of linearly polarized light through medium in magnetic field, parallel to the direction of propagation, can be described by decomposition of the electric vector $\boldsymbol{E} = A_0 e^{i\omega t}$ into two components in the base of spherical vectors $\boldsymbol{e}_1^*, \boldsymbol{e}_{-1}^*$. They represent two circularly polarized waves. Each wave propagate with the phase shift $n\frac{\omega}{c}$ and is absorbed. This absorption is described by absorption coefficient κ. Light intensity detected by a photomultiplier is given by

$$I = |\boldsymbol{E}\cdot\boldsymbol{e}|^2, \tag{3}$$

where $\boldsymbol{e}$ represents the position of the polarizer 2, rotated $\frac{\pi}{2}+\delta$ relative to polarizer 1

$$\boldsymbol{e} = \boldsymbol{i}\cos\delta+\boldsymbol{j}\sin\delta.$$

Substituting (2) into (3) and considering the spectral distribution of incident light intensity given by function $\varrho(\omega-\omega_0)$ we come to

$$I = \tfrac{1}{4}\int d\omega\varrho(\omega-\omega_0)\,[(e^{-\kappa^+ l}-e^{-\kappa^- l})^2+4\sin^2(\chi-\delta)e^{-(\kappa^++\kappa^-)l}] = I_A+I_B \tag{4}$$

where $\chi = \frac{\omega}{c}\,l\,\frac{n^+-n^-}{2}$ is an angle of magnetorotation. Equation (4) is a sum of two parts. First one connected with the absorption properties of a medium is called differential absorption. The second part connected with dispersional properties of the medium as well is called differential dispersion. To calculate the polarizability tensor α we use the same methods as Sargent, Lamb and Fork did in their article on the Zeeman laser published in 1968 [7]. The only correction we have to do is to replace the standing wave perturbation (Zeeman laser) by the progressive wave perturbation. Macroscopic polarization of 1 cubic unit of the medium is

$$\boldsymbol{P}(z,t) = \mathrm{Tr}\,(\varrho(z,t)\boldsymbol{p}),$$

where $\varrho(z,t)$ is the density matrix operator and $\boldsymbol{p}$ is the dipole operator for one atom. The trace is to be taken in the base of vectors $|\mu\rangle$, $|m\rangle$. Using the same methods of calculation as in [7] and under a similar assumption concerning the excitation function we come to the expression for $\boldsymbol{P}(z,t)$ which is proportional $\boldsymbol{E}$, when calculation are done in the first order of perturbation theory. From that we get immediately the polarizability tensor α

$$\alpha_{qq'} = \sum_{\mu}\left(-\frac{1}{2\hbar}\right)\left(\frac{N_b}{g_b}-\frac{N_a}{g_a}\right)\frac{1}{uk}\mathscr{Z}\left(\frac{\omega-\omega_{\mu,\mu+q}}{uk},\frac{\gamma}{uk}\right)$$

$$\times\mathscr{P}^2_{ba}\begin{pmatrix} j_b & 1 & j_a \\ -(q+\mu) & q & \mu\end{pmatrix}^2\delta_{qq'}, \tag{5}$$

where $\mathscr{P}_{ba}$ is reduced matrix element of the electric dipole operator, ω is the angular frequency of the incident light, $\mathscr{Z}(\eta,\varepsilon)$ is the plasma dispersion function, $\omega_{\mu,\mu+q}$ is the angular frequency of radiation emitted during the transition from the level $|\mu\rangle$ to $|m=\mu+q\rangle$, and

$$uk = \frac{2\pi\Delta\nu_D}{2\sqrt{\ln 2}} \qquad \gamma = \frac{\gamma_a+\gamma_b}{2}$$

Substituting in equation (5) for $\mathscr{P}_{ba}$ the expression containing the oscillator strength for the transition from level b to a we finally have

$$\alpha_{qq'} = \sum_{\mu} \frac{c}{2\pi\omega} \frac{\kappa_0}{\sqrt{\pi}} \mathscr{Z}\left(\frac{\omega-\omega_{\mu,\mu+q}}{uk}, \frac{\gamma}{uk}\right) \beta_{\mu,\mu+q}\delta_{qq'}, \tag{6}$$

where the coefficient of absorption κ_0 is defined by

$$\kappa_0 = 2\pi \sqrt{\frac{\ln 2}{\pi}} \frac{e^2}{mc} f_{ba} \frac{g_b}{\Delta\nu_D}\left(\frac{N_b}{g_b} - \frac{N_a}{g_a}\right),$$

and where $\beta_{\mu,\mu+q}$ are the relative intensities of the individual Zeeman components. During the numerical calculation we used the following approximations for real and imaginary parts of plasma dispersion function [1]

$$\mathscr{Z}^r(\eta, \varepsilon) = -2\left[F(\eta) - \frac{\varepsilon}{\sqrt{\pi}}(6\eta - 4\eta^3)e^{-\eta^3}\right]$$

$$\mathscr{Z}^i(\eta, \varepsilon) = \sqrt{\pi}\, e^{-\eta^2} - 2\varepsilon[1 - 2\eta F(\eta)]$$

$$F(\eta) = e^{-\eta^2} \int_0^{\eta} e^{t^2} \mathrm{dt}$$

To see what is the form of the function $I(B, \delta)$ (Eq. (4)) let us assume realistically simple conditions for functions under the integral in Eq. (4): $\varrho(\omega-\omega_0) = I_0\delta(\omega-\omega_0)$, $\varepsilon = 0$ and assume the normal Zeeman effect. In such a case we have

$$I(B, \delta) = I_0 \sin^2(\chi - \delta)\, e^{-2\kappa l} \tag{7}$$

where

$$\kappa = \kappa^+ = \kappa^- = \frac{\kappa_0}{2} e^{-b^2}, \quad b = \frac{\frac{\mu_0}{\hbar} B}{uk}, \quad \chi = \frac{\kappa_0 l}{4\sqrt{\pi}} F(b)$$

The discussion of the foregoing Eq. (7) will be made for different value of parameter $\kappa_0 l$.

(a) Let us allow $\kappa_0 l$ to be much less than 1. Eq. (7) may be rewritten as follows:

$$I(b, \delta) = I_0(\chi - b)^2 = I_0\left(\frac{\kappa_0 l}{4\sqrt{\pi}}\right)^2 (F(b) - a)^2, \tag{8}$$

where $a = \dfrac{\delta}{\kappa_0 l \cdot 4\sqrt{\pi}}$. Diagrams of the function $I(b, \delta)$ for various values of parameter a are shown in figure 2. To get different a parameters we can change the value of δ angle by turning the polarizer 2 — Fig. 1) or we can change $\kappa_0 l$. In the case $\delta = 0$ experimental dependence of the light intensity on magnetic field for different values of $\kappa_0 l$ permits us to find the relative absorption coefficients ratio between these coefficients

$$\left|\frac{\kappa_0}{\kappa_0'}\right| = \sqrt{\frac{A}{A'}},$$

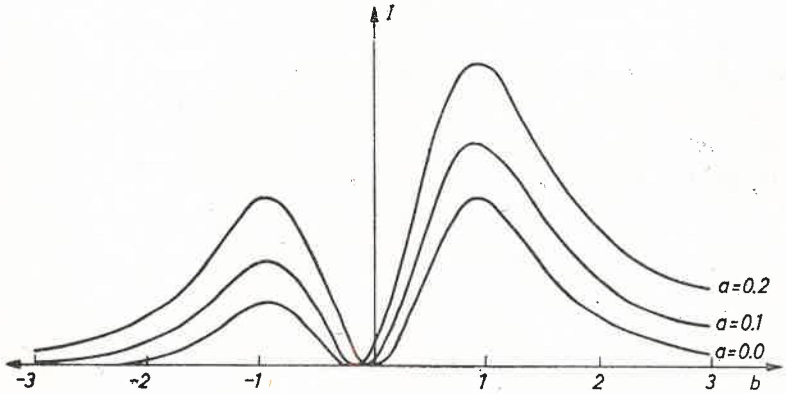

Fig. 2. Diagrams of the transmission curves given by formula 8 for different values of parameter a

where A, A' are indicated as in Fig. 5.

When $\delta \neq 0$, the fitting between experimental and theoretical diagrams of the function $I(b, \delta)$ leads directly to the value of the parameter a, which is independent of light intensity. Knowing it we are able to find $\kappa_0 l = \frac{\delta}{4\sqrt{\pi a}}$. That fact that $I(b, \delta) \neq I(-b, \delta)$ enables us to determine the sign of $\kappa_0 l$, which in the case of very small $\kappa_0 l$ is not trivial.

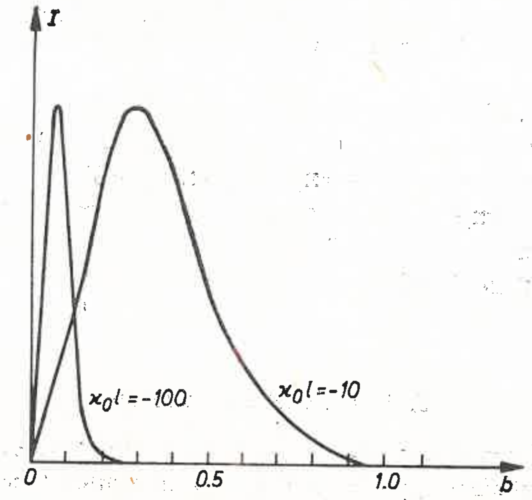

Fig. 3. Diagrams of the transmission curves (Eq. (8)) for two values of $\varkappa_0 l$. The heights of curves in maxima are normalized to the same value.

(b) When $\kappa_0 l$ is large enough, the sine of the magnetorotation angle can not be replaced by its argument. In this case one of the methods for $\kappa_0 l$ determination is to find the magnetic field B_0 for which $I(B, \delta)$ takes the maximum

$$\operatorname{tg}\left(\frac{\kappa_0 l F(b_0)}{4\sqrt{\pi}}\right) = -\frac{F'(b_0)}{4\sqrt{\pi}\, b_0} e^{b_0^2}.$$

Theoretical curves of the light intensity versus the parameter b (where b is proportional to the magnetic field) are shown in figure 3. It could seem that for a magnetorotation angle χ greater than π the function $I(b, \delta)$ should be undulatory because of a sine function (Eq. (7)). As however this sine function is multiplied by the strongly damping exponential factor, the second and successive maxima are too small to be observed when linear scale for

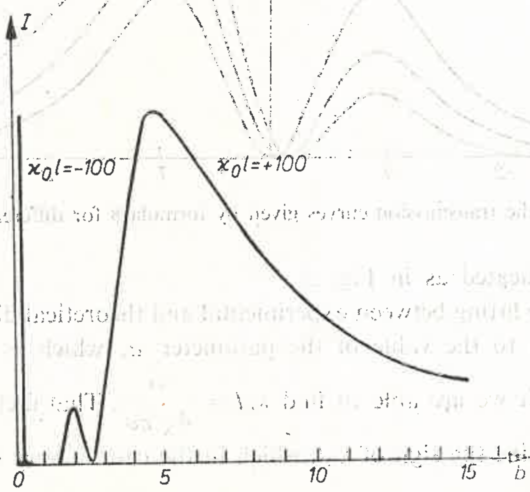

Fig. 4. Diagrams of the transmission curves for media with positive and negative absorption coefficients. The heights of the curves in maxima are normalized to the same value

light intensity is used. The comparison between results for medium with positive and negative absorption coefficients is presented in figure 4. The difference between these two plots comes from the exponential factor $\exp[-\kappa_0 l \exp(-b^2)]$ (Eq. (7)). When $\kappa_0 l$ is positive (absorbing medium) this exponential factor is negligible for small values of the parameter b. For negative $\kappa_0 l$ (laser medium) this factor is small when parameter b is large enough. The accurate shape of the function under consideration depends upon the definite value of $\kappa_0 l$.

3. *Experimental arrangement*

The basic experimental arrangement is shown in figure 1. Light from the laser L, polarized by linear polarizer 1, passes through the absorption tube C. The length of this tube is 100 cm, ID is 3 mm. The tube C is connected to vacuum system what allows changing the He–Ne mixture. The signal from the photomultiplier is amplified by a selective amplifier the output of which is connected to YY plates of the oscilloscope. The signal proportional to the magnetic field is connected to the XX plates. The spectral density function of the light emitted by the laser L was investigated by means of Fabry-Perot interferometer with mirrors distance 9.6 cm. Because the obtained picture of modes was unstable, one of the laser mirrors was fixed to the piezoelectric element. By applying alternative voltage, the mirror was caused to vibrate at 1 kHz. The source of light so obtained was, to a good approximation characterized by a gaussian spectral density function.

4. *Experimental results and discussion*

The experimental results for 6328 Å line in neon corresponding to $3s_2-2p_4$ transition (Paschen notation) are presented in figure 5. Polarizers were crossed ($\delta = 0$) and different curves were taken for various He–Ne mixtures. The comparison between theoret-

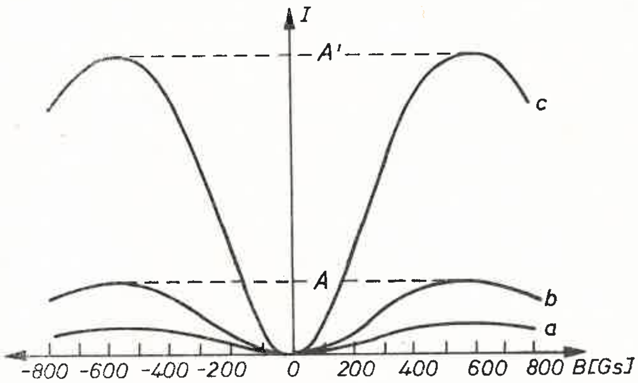

Fig. 5. Experimental transmission curves for various compositions of Ne—He mixture: *a* — 0.2 torr of Ne+0.2 torr of He, *b* — 0.2 torr of Ne+0.4 torr of He, *c* — 0.2 torr of Ne+0.6 torr of He

ical results is shown in the next figure. During theoretical calculations it was assumed that the spectral density function is gaussian with the width 1500 MHz. Because for the transition $3s_2-2p_4$ and the given length of the absorption tube, the value $\kappa_0 l$ is much less than 1, we approximated the absorption part of formula (4) by

$$I_A = \int d\omega \varrho(\omega-\omega_0)\,(\kappa^+ l-\kappa^- l)^2$$

For gaussian $\varrho(\omega-\omega_0)$ the above integral can be evaluated analytically.

Isotope shift of the $3s_2$, $2p_4$ levels was taken as -25 mK and -48 mK, respectively. The experimental results for a constant He–Ne mixture and various δ angles are shown in figure 7. Fitting between these experimental and theoretical diagrams leads to the value 4.10^{-3} for $\kappa_0 l$. It results from the theoretical analysis given in part 2, that for polarizers crossed by angle $\frac{\pi}{2}+\delta$ the value and sign of absorption coefficient can

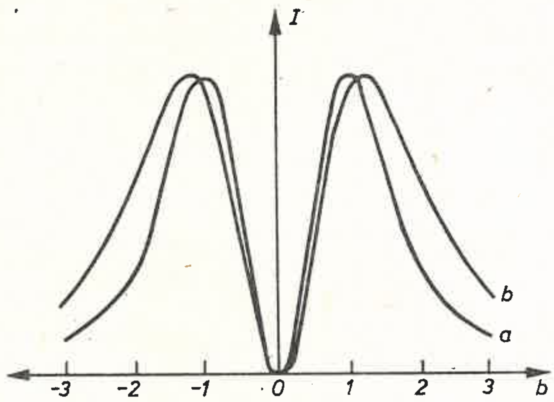

Fig. 6. Comparision between the theoretical curves in the case when the $\delta(\omega-\omega_0)$ shape for the spectral distribution function and no isotope shift were assumed curve *a* and the curve in the case when the spectral distribution function was assumed gaussian and isotope shift is present (curve *b*)

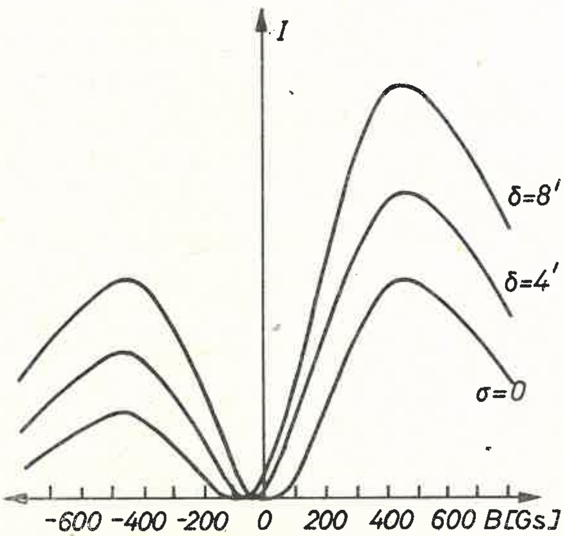

Fig. 7. Experimental transmission curves for various values of δ angle. The composition of Ne—He mixture was constant: 0.2 torr of Ne+0.5 torr of He

be found. That possibility has been used to demonstrate the change of magnetorotation angle when some helium is admitted into a neon medium. Diagram *a* (figure 8) represents the situation when the absorption tube is filled with neon only. Admittance of 0.2 torr of

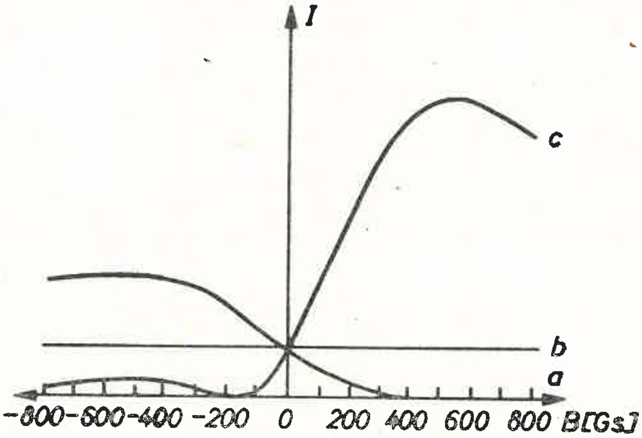

Fig. 8. Experimental transmission curves for constant δ angle and various composition of Ne—He mixture: *a* — 0.2 torr of Ne+0.0 torr of He, *b* — 0.2 torr of Ne+0.2 torr of He, *c* — 0.2 torr of Ne+0.3 torr of He

helium makes a medium transparent to incident light and no magnetorotation is observed (plot *b*). When helium at the pressure of 0.3 torr is present, the absorption coefficient is negative and higher maximum on curve *c* is shifted to the positive values of magnetic field *B*.

5. *Conclusion*

The experimental study of magnetorotation in neon has lead to the possibility of determining the absorption coefficient for the given atomic transition. Numerical calculations are much more simple when the source of light has a $\delta(\omega-\omega_0)$ type spectral density function. The presented method is particularly useful for small values of the absorption coefficient. That comes from the fact that the crossed polarizers method which has been used, allows the separation of the magnetorotational signal from strong primary radiation.

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