

TEMPERATURE-RENORMALIZED LONGITUDINAL MAGNETIC EXCITATIONS IN ANTIFERROMAGNETICALLY ORDERED Ce-GROUP-V COMPOUNDS*

BY P. SZWEYKOWSKI

Institute of Physics, A. Mickiewicz University, Poznań**

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Longitudinal magnetic excitations in antiferromagnetically ordered Ce-group-V compounds possessing an octahedral crystal field are analysed. For simplicity, a one-dimensional system with cyclic boundary conditions is considered. Then the temperature dependence of the longitudinal modes is derived within the framework of RPA (random-phase approximation) hybridized by MFA (molecular-field approximation). The results obtained concern the arbitrary range of the ratio of exchange and crystal fields, particularly the crossing range of the molecular-field levels arising from the lowest-lying crystal-field doublet T_7 .

1. Introduction

As known, the striking magnetic properties of light rare-earth compounds occur due to the competition between the crystal field and exchange effects. The role played by the crystal field is dominant over or at least, comparable with that of the exchange. Recently, there has been a great increase in interest in the magnetic properties of magnetically-ordered compounds consisting of the light-rare-earth ions with an even number of $4f$ -electrons [1-8].

Magnetic properties of the light rare earth compounds with an odd number of $4f$ -electrons, however, are also of great interest. In our investigation we study certain magnetic properties of the Ce-group-V compounds. The latter are all of the NaCl crystal structure. Moreover, as known they are antiferromagnetically ordered [9].

In further considerations, we assume colinear antiferromagnetic ordering, as verified by experiment, and consequently, resort to this in all calculations. In the present investigation we confine ourselves to longitudinal magnetic excitations i. e. excitations arising from the action of the \hat{J}^z component of the total angular momentum, the z -axis being the

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** Address: Instytut Fizyki, Uniwersytet im. A. Mickiewicza, Matejki 48/49, 60-769 Poznań, Poland.

direction of the mean magnetization. The longitudinal excitations consist of the single-ion transitions between the effective-field energy levels and propagate in the system due to the exchange couplings. Here, we assume that one can distinguish the longitudinal and transversal magnetic excitations both theoretically (due to the group-theoretical methods) and experimentally (in inelastic neutron diffraction). We start from the Hamiltonian including the single-ion crystal field and two-ion isotropic exchange operators. We consider both crystal-field energy-levels of a Ce^{+3} -ion in an octahedral environment: the Γ_7 -doublet and the excited Γ_8 -quartet. Within the framework of RPA, the temperature dependence of the four branches are expressed through the parameters of the Hamiltonian and by the statistical expectation values of the occupation-number operators of the single-ion effective-field energy levels. The latter are derived in MFA [10]. Only, at $T = 0\text{K}$ we arrive at the analytical expression for the energies of the collective modes. Whereas, at finite temperatures the mode energies are obtained numerically by solving an equation of the eighth order.

In order to reduce the number of the parameters and simultaneously maintaining the main physical aspects of the system considered, we assume a very simple model: a one-dimensional antiferromagnetically ordered chain in the form of a ring, consisting of the $2N$ Ce^{+3} ions affected by the cubic crystal field. The exchange interaction is limited to nearest neighbours only, reducing the number of the effective exchange isotropic parameters to one. The exchange parameter is estimated from the crystal-field-only susceptibility at $T = T_N$ following Bak and Lindgard [11].

2. The Hamiltonian

The antiferromagnetic collinear ordering is directed along the z -axis i. e. (001)-axis. While studying the longitudinal magnetic excitations, we can restrict the two-ion isotropic exchange interactions to their component parallel to the z -axis. In the approximation which considers only the nearest neighbours, the Hamiltonian, for the one-dimensional system with the cyclic boundary conditions, can be written as follows

$$\hat{\mathcal{H}} = \sum_{l=1}^{2N} \hat{\mathcal{H}}_{\text{cf}}(l) - \sum_{l=1}^{2N} F_{l,l+1} \hat{J}^z(l) \hat{J}^z(l+1), \quad (1)$$

where $F_{l,l+1}$ is the two-ion nearest-neighbour isotropic exchange parameter. The Hamiltonian (1) is projected onto the space L , this being the tensorial product of $2N$ six-dimensional spaces spanned over the eigen-states of the crystal-field single-ion Hamiltonian.

As known [9], the six-fold degenerated ground-state of the free Ce^{+3} ion under the action of the crystal-field operator with cubic symmetry splits into the Kramers doublet Γ_7 and the quartet Γ_8 which form the basis for our further calculations. The single-ion Hilbert space originating from the crystal-field wave-functions can be decomposed [12] into the direct sum of the three two-dimensional subspaces invariant with respect to $\hat{\mathcal{H}}_{\text{cf}}(l)$ and $\hat{J}^2(l)$

$$L(l) = L_1(l) \oplus L_2(l) \oplus L_3(l), \quad (2)$$

where $\hat{J}^z(l)$ takes the following form

$$\hat{J}^z(l) = \begin{vmatrix} \alpha & \beta & 0 & 0 & 0 & 0 \\ \beta & \gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & -\beta & 0 & 0 \\ 0 & 0 & -\beta & \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta & 0 \\ 0 & 0 & 0 & 0 & 0 & -\delta \end{vmatrix} \quad (2a)$$

$$\alpha = -5/6, \quad \beta = \frac{2\sqrt{5}}{3}, \quad \gamma = -11/6, \quad \delta = 1/2.$$

Since longitudinal magnetic excitations occur due to the single-ion transitions between the molecular-field energy levels originating from the Γ_7 -doublet and Γ_8 -quartet it is obvious that the space $L_3(l)$ does not play any important role and the molecular-field energy levels originating from the crystal-field states $|5/2, 1/2\rangle$, $|5/2, -1/2\rangle$ will be taken into account only indirectly through the partition function Z as they contribute to the statistical average of the total operator $\langle \hat{J}^2(l) \rangle$.

The Hamiltonian (1) can be now transformed by regrouping its terms into the single-ion Hamiltonian $\hat{\mathcal{H}}_{mf}$, including the crystal and molecular field Hamiltonians, and two-ion correction $\hat{\mathcal{H}}'$ being the difference between $\hat{\mathcal{H}}$ and $\hat{\mathcal{H}}_{mf}$ [12]. Diagonalization of the single-ion Hamiltonian $\hat{\mathcal{H}}_{mf}$ can be performed separately in each of the subspaces giving six molecular-field energy-levels:

$$E_{\Gamma_1(l)}, E_{\Gamma'_1(l)}, E_{\Gamma_2(l)}, E_{\Gamma'_2(l)}, E_{\Gamma_3(l)}, E_{\Gamma'_3(l)},$$

and the six corresponding eigen-functions $|\Gamma_1(l)\rangle$, $|\Gamma'_1(l)\rangle$ in the subspace $L_1(l)$; $|\Gamma_2(l)\rangle$ and $|\Gamma'_2(l)\rangle$ in the subspace $L_2(l)$ and $|\Gamma_3(l)\rangle$, $|\Gamma'_3(l)\rangle$ in the subspace $L_3(l)$.

The dependence of the six molecular-field energy levels of a Ce^{+3} ion in an octahedral environment upon the molecular-field strength is given in the paper of Cooper [9] or in that of Kowalewski et al. [13].

We consider the thermal populations of all the energy levels and thus our calculations concern the higher temperature region too. It should be noticed that in the description of longitudinal magnetic excitations, the thermal populations of the molecular-field energy levels of the subspaces $L_2(l)$ and $L_3(l)$ occur due to the transversal excitations i. e. the excitations arising from the single-ion transitions under the action of the operators $\hat{J}^+(l)$ and $\hat{J}^-(l)$.

Calculations of the longitudinal magnetic excitations require the consideration of the two subspaces $L_1(l)$ and $L_2(l)$ only, keeping in mind, however, that the molecular-field energy levels $|\Gamma_3(l)\rangle$ and $|\Gamma'_3(l)\rangle$ give their contribution to the excitation energy through the molecular field parameters. The reason for this simplification is that the transition from the molecular-field states of the subspaces $L_1(l)$ or $L_2(l)$ to $L_3(l)$ under the action of the operator \hat{J}^z is excluded. The two-ion correction $\hat{\mathcal{H}}'$ can be written in the following form

$$\hat{\mathcal{H}}' = - \sum_{l=1}^{2N} F_{l,l+1} \hat{J}^z(l) \hat{J}^z(l+1), \quad (3)$$

where $\hat{j}^z(I)$ are defined as follows (see for example [3]) $\hat{j}^z(I) = \hat{J}^z(I) - \langle \hat{J}^z(I) \rangle_T$, $\langle \hat{J}^z(I) \rangle_T$ being the statistical expectation value of the z-component of the total angular momentum operator. The operator $\hat{j}^z(I)$ can be expressed as the sum of the two two-dimensional "transition" operators $\hat{j}_1^z(I)$ and $\hat{j}_2^z(I)$ induced in the subspaces $L_1(I)$ and $L_2(I)$, respectively and the diagonal contribution from all three subspaces (see: Eq. (4a)).

3. The fermion representation and the diagonalization procedure

Before performing the diagonalization let us devote some attention to the antiferromagnetic ordering of the system. At each site of the chain there exist two subspaces of the molecular-field energy states i. e. $L_1(I)$ and $L_2(I)$, ($L_3(I)$ is not discussed here for the re-

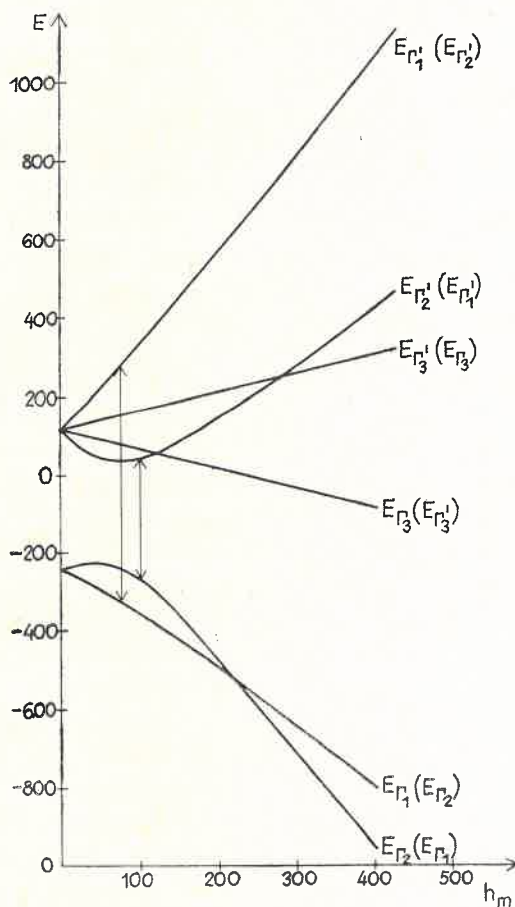


Fig. 1. The six molecular-field energy-levels of the Ce^{+3} ion in an octahedral environment versus the strength of the molecular field which is directed along the (001)-direction. The energy is expressed in units of the crystal-field parameter B_4 . The molecular field is given in dimensionless form, $h_m = g\mu_B H_{mf}/B_4$. The symbols in and outside the brackets label the molecular-field energy levels for an ion of the sublattice B and A , respectively. The permitted single-ion transitions are indicated by arrows

asons mentioned before). All the sites labelled by $l = 1, 3, \dots, 2N-1$ belong to the sublattice A , whereas all the sites $l = 2, 4, \dots, 2N$ belong to the sublattice B . The A and B ions differ in the ordering of their single-ion effective-field energy-levels: for the sublattice A the molecular-field ground-state is $|\Gamma_1(2l+1)\rangle$ whereas, for each B ion it is $|\Gamma_2(2l)\rangle$.

Such a description of the molecular-field energy-levels for the A and B ions results from that their magnetic moments are oppositely directed.

In order to diagonalize the Hamiltonian (3) we apply the Bogolyubov-type formalism [14]. On the basis of the effective-field eigen-states, we define the fermion operators [10] $\hat{\alpha}_{\Gamma_1}(l), \hat{\alpha}_{\Gamma_1'}(l), \hat{\alpha}_{\Gamma_1}^+(l), \hat{\alpha}_{\Gamma_1'}^+(l)$ in the subspaces $L_1(l)$ and analogously $\hat{\beta}_{\Gamma_2}(l), \hat{\beta}_{\Gamma_2'}(l), \hat{\beta}_{\Gamma_2}^+(l), \hat{\beta}_{\Gamma_2'}^+(l)$ in subspace $L_2(l)$. These operators annihilate and create, respectively, the eigen-states of the molecular-field Hamiltonian in the four-dimensional space $L(l) = L_1(l) \oplus L_2(l)$. Obviously, in space $L(l)$ all $\hat{\alpha}$'s commute with all $\hat{\beta}$'s. Evidently, the effective-field Hamiltonian $\hat{\mathcal{H}}_{\text{mf}}$ is diagonal in the fermion representation. The Hamiltonian (1) expressed in the fermion representation takes the following form:

$$\begin{aligned} \hat{\mathcal{H}} = & \text{const} + \sum_{\sigma} \sum_{l=1}^N E_{\sigma}(2l-1) \hat{\alpha}_{\sigma\sigma}^+(2l-1) \hat{\alpha}_{\sigma\sigma}(2l-1) \\ & + \sum_{\sigma} \sum_{l=1}^N E_{\sigma}(2l) \hat{\alpha}_{\sigma\sigma}^+(2l) \hat{\alpha}_{\sigma\sigma}(2l) - F \sum_{l=1}^{2N} \hat{j}^z(l) \hat{j}^z(l+1), \end{aligned} \quad (4)$$

where the operator $\hat{j}^z(l)$ in the fermion operators is

$$\begin{aligned} \hat{j}^z(l) = & \hat{J}_{\Gamma_1\Gamma_1'}^z(l) (\hat{\alpha}_{\Gamma_1}^+(l) \hat{\alpha}_{\Gamma_1}(l) + \hat{\alpha}_{\Gamma_1'}^+(l) \hat{\alpha}_{\Gamma_1'}(l) \\ & + \hat{J}_{\Gamma_2\Gamma_2'}^z(l) (\hat{\beta}_{\Gamma_2}^+(l) \hat{\beta}_{\Gamma_2}(l) + \hat{\beta}_{\Gamma_2'}^+(l) \hat{\beta}_{\Gamma_2'}(l) + \sum_{\sigma} \hat{J}_{\sigma\sigma}^z(l) (\hat{N}_{\sigma}(l) - \langle \hat{N}_{\sigma}(l) \rangle_T), \end{aligned} \quad (4a)$$

where $\hat{J}_{\Gamma_i\Gamma_i'}^z(l) = \langle \Gamma_i(l) | \hat{J}^z(l) | \Gamma_i'(l) \rangle$, ($i = 1, 2$); $\hat{J}_{\sigma\sigma}^z(l)$ are the appropriate diagonal matrix elements, and σ in the diagonal term of Eq. (4a) runs over all six molecular-field energy levels, $\hat{N}_{\sigma}(l)$ is the occupation-number operator ($\sigma = \Gamma_1, \Gamma_1', \Gamma_2, \Gamma_2', \Gamma_3, \Gamma_3'$; $l = 1, 2, \dots, \dots, 2N$). The statistical expectation values $\langle \hat{N}_{\sigma}(l) \rangle$ can be found from the usual molecular-field procedure as follows

$$\langle \hat{N}_{\sigma}(l) \rangle_T = \frac{1}{Z} \exp(-E_{\sigma}(l)/kT), \quad (5)$$

where Z is the partition function, k — Boltzman's constant.

As seen, the Hamiltonian $\hat{\mathcal{H}}'$ besides the diagonal terms contains also the fourth-order products of the fermion operators and has to be diagonalized.

Since the longitudinal magnetic excitations arise due to the single-ion transitions from the lower to excited molecular-field states, we introduce now the appropriate "transition" operators removing an ion from the lower excited level and back in the subspaces $L_1(l)$ and $L_2(l)$, respectively. The "transitions" operators are the pairs of the fermion operators as follows: $\hat{\alpha}_{\Gamma_1}^+(l) \hat{\alpha}_{\Gamma_1'}(l), \hat{\alpha}_{\Gamma_1'}^+(l) \hat{\alpha}_{\Gamma_1}(l)$ in the subspace $L_1(l)$, $\hat{\beta}_{\Gamma_2}^+(l) \hat{\beta}_{\Gamma_2'}(l), \hat{\beta}_{\Gamma_2'}^+(l) \hat{\beta}_{\Gamma_2}(l)$ in the subspace $L_2(l)$.

We assume that the diagonalization can be performed by having recourse to the new operators \hat{u}_λ^+ and \hat{v}_λ^+ (and their hermitian conjugates) which create the longitudinal magnetic excitations with the wave vector λ and which can be expressed as the appropriate linear combinations of all the $8N$ transition operators [10, 14]. The energies of the magnetic modes can be found from the usual procedure by applying the Bogolyubov-Tyablikov equation of motion for the new operators \hat{u}_λ^+ , \hat{v}_λ^+ and obviously their hermitian conjugates. Invoking an approximation of the RPA-type, we obtain the set of $8N$ equations for the transformation coefficients. In this case, RPA consists in replacing the occupation number operators of the single-ion energy levels $\hat{N}_\sigma(l)$; ($\sigma = \Gamma_1, \Gamma'_1, \Gamma_2, \Gamma'_2, \Gamma_3, \Gamma'_3$; $l = 1, 2, \dots, 2N$) by their expectation values.

For finite temperatures, the application of the before-mentioned procedure [10] yields the eight order characteristic equation for the energies of the longitudinal magnetic excitations. This equation was solved numerically providing us with the wave-vector and temperature-dependent four excitation energies. The occurrence of the four branches in the excitation spectrum is due to the following two reasons:

(i) two longitudinal collective magnetic excitations appear due the two single-ion transitions between the effective-field energy levels in both subspaces $L_1(l)$ and $L_2(l)$, respectively, transmitting along the chain via the exchange coupling;

(ii) each single-ion transition gives rise to the acoustical and optical branches because of the presence of the two inequivalent magnetic ions in the magnetic unit cell.

Below, we present in a more detail the discussion of the diagonalization at $T = 0\text{K}$, since it can be analytically performed. At $T = 0\text{K}$ only the lowest-lying molecular-field energy level is occupied for all the sites of the sublattice A i.e. $\langle \hat{N}_{\Gamma_1}(2l-1) \rangle = 1$ ($l = 1, 2, \dots, N$) similarly, at the sites of the sublattice B — $\langle \hat{N}_{\Gamma_2}(2l) \rangle = 1$ ($l = 1, 2, \dots, N$).

Resorting to the usual (described before) procedure we arrive at the fourth-order equation for the excitation energies, which yields the following two-branch dispersion relation

$$\varepsilon^{(1),(2)} = \Delta \sqrt{1 \pm A \cos \lambda} \quad (6)$$

with

$$\lambda = 0, \pm\pi/N, \dots, \pm\pi/N(2N-1),$$

where we have denoted

$$\Delta = E_{\Gamma'_1}(2l-1) - E_{\Gamma_1}(2l-1) = E_{\Gamma'_2}(2l) - E_{\Gamma_2}(2l)$$

and

$$A = \frac{4F\hat{J}_{\Gamma_1\Gamma_1}^z(2l-1)\hat{J}_{\Gamma_2\Gamma_2}^z(2l)}{\Delta}$$

The obtained result remains in agreement with that of the pseudo-boson low-temperature approach [1]. But due to the presence of two branches it differs from the zero-temperature dispersion relation calculated for the ferromagnetic ordered system [12].

4. The numerical results and their discussion

Our main purpose is to study the temperature dependence of the longitudinal magnetic modes in an antiferromagnetic system. Numerical calculations were performed for an antiferromagnetically ordered system of Ce^{+3} ions in octahedral environments. The crystal-field Hamiltonian for such systems can be expressed by means of only one crystal-field parameter B_4 , which is assumed to be positive. The discussion of the single-ion cubic anisotropy leads to the conclusion that for $B_4 > 0$ the preferred direction for an easy magnetic axis is one of the cubic three-fold axes e. g. (111) [16]. The appropriate distortion can change the easy magnetic direction for the (001)-axis if $B_4 > 0$. Discussing the excitation spectrum, however, we concentrate on the contribution due to the predominant cubic crystal field similarly like Wang and Cooper [9]. The ground crystal-field state is a Γ_7 -doublet which gives rise to the anomalous magnetic behaviour of the system [13].

We choose in our calculation the system orders antiferromagnetically below $T_N = 20$ K. Since the magnetic behaviour of light-rare-earth compounds are profoundly influenced by the ratio of the molecular and crystal fields, we consider several values of the energy gap between the ground Γ_7 -doublet and the excited Γ_8 -quartet putting $\Delta_{cf} = 20$ K, 30 K, 40 K and 60 K. Values of the nearest-neighbour exchange parameter F as well as those of the molecular-field constant are obtained from the crystal-field-only susceptibility χ_{cf}^0 at $T = T_N$ [11]. Thus calculated values are given in Table I, (obviously, the exchange parameters are assumed to take negative values).

TABLE I

$\Delta_{cf}[\text{K}]$	$F[\text{K}]$
20	3.592
30	3.804
40	4.093
60	4.819

The temperature variation of the sublattice molecular fields remain in perfect agreement with those given in Ref. [9, 13]. For Δ_{cf} comparable with T_N , the temperature-dependences of the molecular field exhibit anomalies which found their reflection in the temperature dependences of the considered magnetic modes. The anomalies are most pronounced for $\Delta_{cf} = 20$ K where occurs even a crossing of the lower-lying molecular-field energy levels [13].

The molecular-field calculations are the basis for obtaining the temperature-dependence of the longitudinal magnetic modes. The results are shown on Figs 2-5.

The energy spectrum consists of the four branches: each single-ion molecular-field energy-level transition gives rise via the exchange couplings, to a collective mode, which splits into acoustical and optical branches due to the presence of the two inequivalent magnetic ions in an antiferromagnetic unit cell. At $T \geq T_N$ the magnetic ordering vanishes

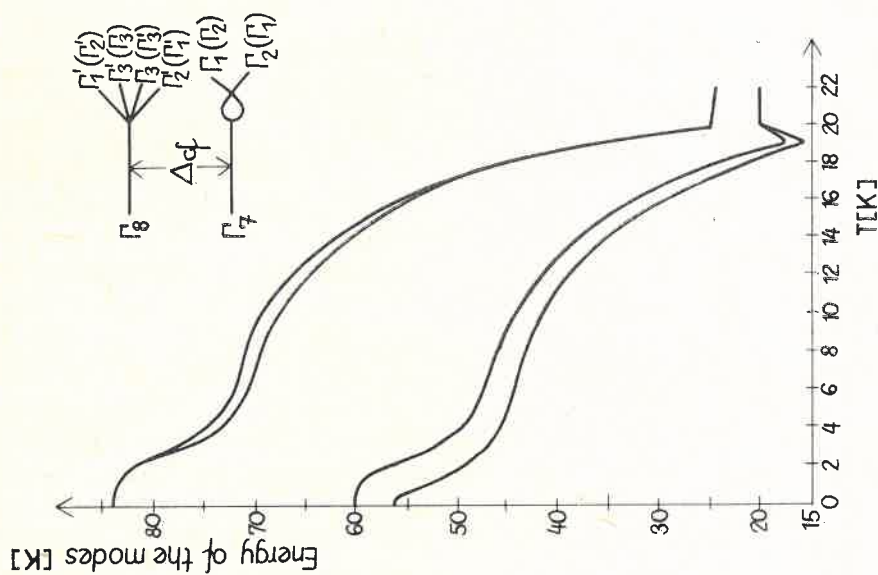


Fig. 2

Fig. 2. Temperature-variations of the longitudinal modes for $T_N = 20$ K, $\Delta_{cf} = 20$ K. The level scheme at the right-hand side of the diagram labels the modes (the crossing of the molecular field energy levels is indicated)

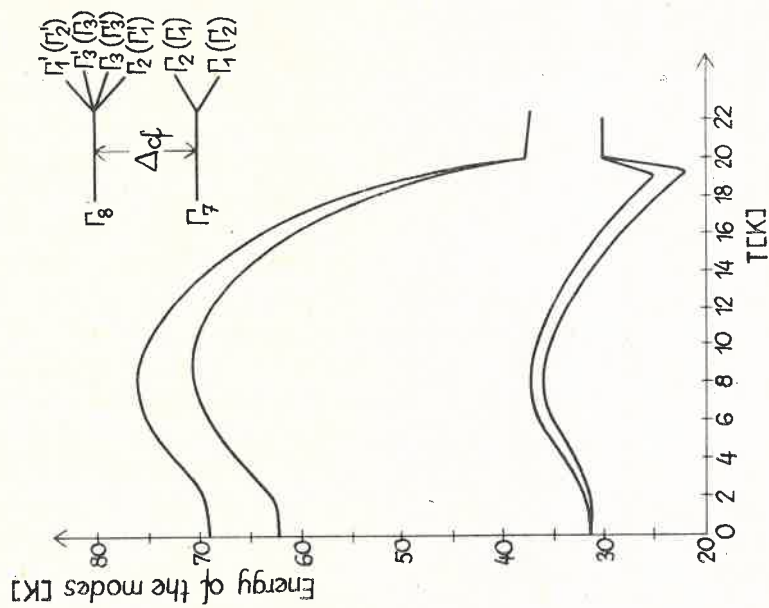


Fig. 3

Fig. 3. Temperature-variations of the longitudinal modes for $T_N = 20$ K, $\Delta_{cf} = 30$ K. The level scheme at the right-hand side of the diagram labels the modes

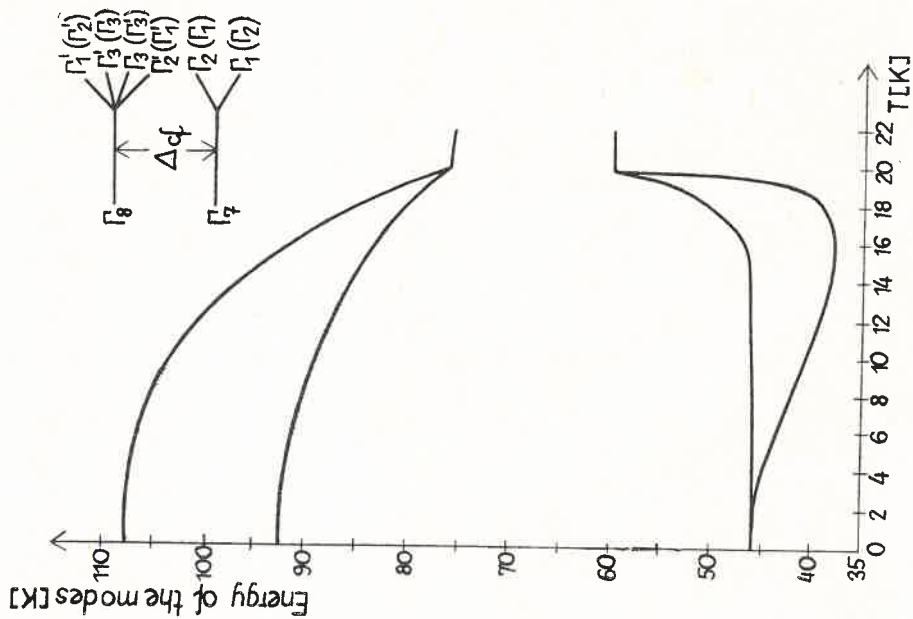


Fig. 4

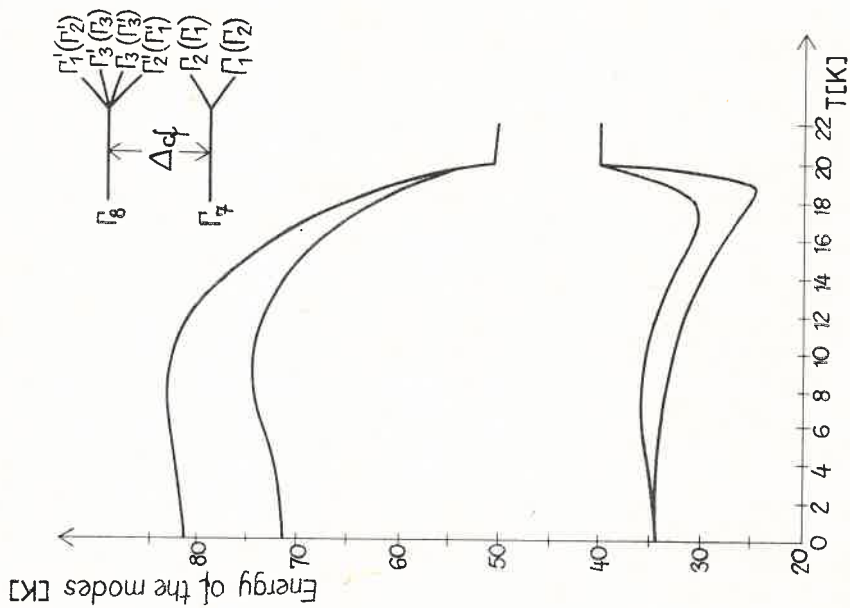


Fig. 5

Fig. 4. Temperature-variations of the longitudinal modes for $T_N = 20$ K, $\Delta_{ef} = 40$ K. The level scheme at the right-hand side of the diagram labels the modes

Fig. 5. Temperature-variations of the longitudinal modes for $T_N = 20$ K, $\Delta_{ef} = 60$ K. The level-scheme at the right-hand side of the diagram labels the modes

and consequently, only two of the four branches are preserved. The paramagnetic spectrum is like that obtained by Kowalewski et al. [12].

At $T = 0\text{K}$, only one molecular-field energy level is occupied (Γ_1 for the sublattice A , Γ_2 for $-B$) and therefore, the number of the dispersional spectrum branches is reduced to two, which are the acoustical and optical ones generated by the single-ion transition from the occupied molecular-field ground state. These modes are calculated analytically. Our result remains in good agreement with the low-temperature result of Grover [1] for an induced moment system; and moreover, it is also consistent with that calculated by Rao and Wang [15] for the antiferromagnetically-ordered induced moment system. One has to keep in mind, however, that the reduplication of the spectrum branches, in comparison with the result of Grover or that of Rao and Wang, is due to the presence of two possible intra-level transitions under the action of the operator \hat{J}^z at each ion of the system.

The temperature-dependence of the modes is the same as that of the appropriate ferromagnetic system. In the interpretation of the temperature behaviour of the excitations we consider that the modes are temperature-dependent by way of the occupation probabilities of the molecular-field energy levels and moreover, directly via the quantities which are molecular-field functions: the energy values E_σ ; ($\sigma = \Gamma_1, \Gamma'_1, \Gamma_2, \Gamma'_2$) and the matrix elements $\hat{J}_{\Gamma_1\Gamma'_1}^z, \hat{J}_{\Gamma_2\Gamma'_2}^z$. It is then obvious that all the molecular-field anomalies are reflected in the temperature behaviour of the modes.

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REFERENCES

- [1] B. Grover, *Phys. Rev.* **A140**, 1944 (1965).
- [2] D. A. Pink, *J. Phys.* **C1**, 1246 (1968).
- [3] Y. L. Wang, B. R. Cooper, *Phys. Rev.* **185**, 696 (1969).
- [4] Y. Y. Hsieh, M. Blume, *Phys. Rev.* **B6**, 2684 (1972).
- [5] S. R. P. Smith, *J. Phys.* **C5L**, 157 (1972).
- [6] M. E. Lines, *J. Phys.* **C7**, 287 (1974).
- [7] W. J. L. Buyers, T. M. Holden, Proc. Int. Conf. on Crystalline Electric Field Effects in Metals and Alloys, Montreal 1974.
- [8] W. L. J. Buyers, T. M. Holden, A. Perreault, *Phys. Rev.* **B11**, 266 (1975).
- [9] Y. L. Wang, B. R. Cooper, *Phys. Rev.* **2**, 2607 (1970).
- [10] L. Kowalewski, A. Lehmann-Szweykowska, P. Szweykowski, *Acta Phys. Pol.* **A50**, 55 (1976).
- [11] P. Bak, P. A. Lindgard, 1974 Proc. Int. Conf. on Magnetism, Nauka, Moscow 1973.
- [12] L. Kowalewski, A. Lehmann-Szweykowska, P. Szweykowski, *Acta Phys. Pol.* **A49**, 197 (1976).
- [13] L. Kowalewski, A. Lehmann-Szweykowska, P. Szweykowski, *J. Phys. C*, 1976 (in print).
- [14] S. Krompiewski, H. Puzkarski, *Acta Phys. Pol.* **A40**, 709 (1971).
- [15] B. N. Rao, Y. L. Wang, *J. Phys. Chem. Solids* **37**, 129 (1976); **37**, 137 (1976).
- [16] Cz. Rudowicz, L. Kowalewski, *Physica* **80B**, 517 (1975).