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THE ELECTROMAGNETIC RADIATION CONDITIONS FOR A MEDIUM WITH UNIAXIAL ELECTRIC ANISOTROPY

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On the basis of electromagnetic Huygens principle for a medium with uniaxial electric anisotropy the electromagnetic radiation conditions were obtained. These radiation conditions, formulated by us, can be regarded as conditions of the Rubinowicz type. It is sufficient to use radiation conditions to exclude the existence of radiation sources at infinity.

1. Introduction

In order to apply the electromagnetic Huygens principle to diffraction problems one must first formulate the so called radiation conditions. The integration surface in the electromagnetic Huygens principle possesses then always such a fragment which reaches infinity (Fig. 1). Observation point P will then receive the waves originating — from the

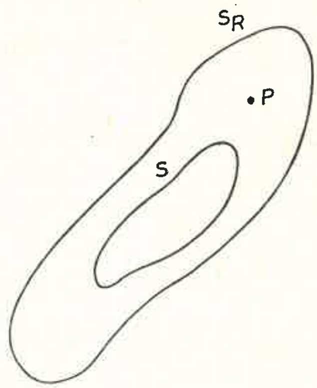

Fig. 1. The integration surface in the Huygens principle is composed of two closed surface S and S_R. In order to apply the Huygens principle to diffraction problems the surface S_R is moved to infinity. P is the observation point at which the electromagnetic field is calculated

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integration surface, which is located at a finite distance, as well as from points at infinity. In dealing with physical problems we must exclude, however, waves originating at infinity because of the lack of electromagnetic field sources there. The radiation conditions reduce themselves to the requirement that the energy flux long distances from the source and on diffracting bodies is directed outward and that infinity does not contribute to the field at the observation point.

The electromagnetic radiation principles formulated for the first time for the case of an isotropic medium by Claus Müller were of the form:

$$\lim_{R\to\infty} R\left(E_i + \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\varepsilon_{ijk} n_j H_k\right) = 0 \tag{1.1}$$

$$\lim_{R\to\infty} R\left(H_i - \sqrt{\frac{\varepsilon_0}{\mu_0}}\,\varepsilon_{ijk} n_j E_k\right) = 0 \tag{1.2}$$

$$\lim_{R\to\infty} RE_i < \infty \tag{1.3}$$

$$\lim_{R\to\infty} RH_i < \infty, \tag{1.4}$$

E_i, H_i are here the electric and magnetic field strength vector components, n_j — components of unit vector in the direction of vector $\overrightarrow{PQ}$ connecting the observation point P with point Q at infinty, where the electromagnetic field from formulas (1.1)–(1.4) is localized. The conditions (1.1)–(1.4) were derived by Claus Müller from the electromagnetic Huygens principle formulated by Tedone [10]. Contributions originating from different area elements in this formulae do not fulfill the Maxwell equations and cannot be treated as secondary wavelets, which must be implied in genuine Huygens principle.

Rubinowicz [7] who considered the exact form of the Huygens–Lorentz principle obtained weaker electromagnetic radiation conditions, than those given by formulae (1.1)–(1.4). They are as follows

$$\lim_{R\to\infty} R\left(E_i^{\perp} + \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\varepsilon_{ijk} n_j H_k\right) = 0 \tag{1.5}$$

$$\lim_{R\to\infty} R\left(H_i^{\perp} - \sqrt{\frac{\varepsilon_0}{\mu_0}}\,\varepsilon_{ijk} n_j E_k\right) = 0, \tag{1.6}$$

where

$$E_i^{\perp} = E_i - n_i(n_j E_j), \qquad H_i^{\perp} = H_i - n_i(n_j H_j).$$

Conditions (1.5) and (1.6) contain only the tangential field components, whereas the Claus Müller conditions contain total field vectors.

In this paper the electromagnetic radiation conditions for the medium with uniaxial electric anisotropy will be considered. It will be based on the electromagnetic Huygens principle formulated for this medium by Wünsche [11].

2. The electromagnetic Huygens principle for a medium with uniaxial anisotropy

Let the space limited by closed surface S, area V, contain a nonmagnetic medium with uniaxial electric anisotropy. Electric properties of this medium are described by the dielectric and the magnetic permeability tensors of the form:

$$\varepsilon_{ij} = \varepsilon^0(\delta_{ij} - c_i c_j) + \varepsilon^e c_i c_j \tag{2.1}$$

$$\mu_{ij} = \delta_{ij}, \tag{2.2}$$

where ε^e is the dielectric permeability if the medium in the direction parallel to the optic axis, ε^0 — the dielectric permeability in the direction perpendicular to optic axis, $\vec{c}$ — the unit vector along the direction of optic axis.

The electromagnetic field at an arbitrary point inside this area is given by integrals over the limiting surface S.

$$E_i(P) = \frac{1}{4\pi} \oint_S d^2x'_Q \{[\vec{N} \times \vec{E}(Q)]_k \varepsilon_{ilm} \nabla'_l \mu^{-1}_{mk} G^{(\vec{E})}_{ij}(\vec{r} - \vec{r}') - i\omega\mu_0 G^{(\vec{E})}_{ij}(\vec{r} - \vec{r}')\,[\vec{N} \times \vec{H}(Q)]_j\} \tag{2.3}$$

$$H_i(P) = \frac{1}{4\pi} \oint_S d^2x'_Q \{[\vec{N} \times \vec{H}(Q)]_k \varepsilon_{jlm} \nabla'_l \varepsilon^{-1}_{mk} G^{(\vec{H})}_{ij}(\vec{r} - \vec{r}') + i\omega\varepsilon_0 G^{(\vec{H})}_{ij}(\vec{r} - \vec{r}')\,[\vec{N} \times \vec{E}(Q)]_j\}, \tag{2.4}$$

where

$$\vec{r} = \{x_1, x_2, x_3\}, \quad \vec{r}' = \{x'_1, x'_2, x'_3\}$$

Green's tensors $G^{(\vec{E})}_{ij}$, $G^{(\vec{H})}_{ij}$ were found by Wünsche and their form is follows:

$$\begin{aligned} G^{(\vec{E})}_{ij}(\vec{r} - \vec{r}') = \frac{1}{\sqrt{\varepsilon^0}} \Bigg\{ & (k_0^{-2} \nabla_i \nabla_j - \varepsilon^0 \varepsilon^e \varepsilon^{-1}_{ij}) \frac{e^{ik_0 R^e}}{R^e} \\ & - \left(\varepsilon^e \frac{e^{ik_0 R^e}}{R^e} - \varepsilon^0 \frac{e^{ik_0 R^0}}{R^0} \right) \frac{[(\vec{r} - \vec{r}') \times \vec{c}]_i [(\vec{r} - \vec{r}') \times \vec{c}]_j}{\varrho^2} \\ & - \frac{e^{ik_0 R^e} - e^{ik_0 R^0}}{ik_0 \varrho^2} \left(\delta_{ij} - c_i c_j - 2 \frac{[(\vec{r} - \vec{r}') \times \vec{c}]_i [(\vec{r} - \vec{r}') \times \vec{c}]_j}{\varrho^2} \right) \Bigg\} \end{aligned} \tag{2.5}$$

$$\begin{aligned} G^{(\vec{H})}_{ij}(\vec{r} - \vec{r}') = \sqrt{\varepsilon^0} \Bigg\{ & (k_0^2 \nabla_i \nabla_j - \varepsilon^0 \delta_{ij}) \frac{e^{ik_0 R^0}}{R^0} \\ & + \left(\varepsilon^e \frac{e^{ik_0 R^e}}{R^e} - \varepsilon^0 \frac{e^{ik_0 R^0}}{R^0} \right) \frac{[(\vec{r} - \vec{r}') \times \vec{c}]_i [(\vec{r} - \vec{r}') \times \vec{c}]_j}{\varrho^2} \\ & + \frac{e^{ik_0 R^e} - e^{ik_0 R^0}}{ik_0 \varrho^2} \left(\delta_{ij} - c_i c_j - 2 \frac{[(\vec{r} - \vec{r}') \times \vec{c}]_i [(\vec{r} - \vec{r}') \times \vec{c}]_j}{\varrho^2} \right) \Bigg\}, \end{aligned} \tag{2.6}$$

where

$$R^0 = \sqrt{\varepsilon^0 (z^2 + \varrho^2)} \qquad R^e = \sqrt{\varepsilon^0 z^2 + \varepsilon^e \varrho^2},$$

z, ϱ are the cylindrical components related to the optical axis of the medium.

If we choose the optical axis so that it is one of the main dielectric permeability tensor axes, and coincides with axis ox_3 of the cartesian frame coordinates x_1, x_2, x_3 we have $\vec{c} = \{0, 0, 1\}$. Introducting further, unit vector $\vec{n}$ in the direction of vector $\vec{R}$ connecting point Q on surface S with point P where the electromagnetic field is calculated

$$\vec{n} = \frac{\vec{R}}{R}, \qquad \vec{n} = \{n_1, n_2, n_3\}$$

$$\vec{R} = \{x_1 - x_1', x_2 - x_2', x_3 - x_3'\} \tag{2.7}$$

after some transformations of (2.1)–(2.6) we get:

$$\begin{aligned}
E_j(P) = \frac{i}{4\pi\sqrt{\varepsilon^0}} \oint_s d^2x_Q' \Big\{ & \frac{k_0\varepsilon^e\varepsilon^0}{\alpha^2} \varepsilon_{jkl}\varepsilon_{kpq}N_pE_qn_l \\
& + \frac{\omega\mu_0\varepsilon^e}{\alpha^3} \kappa_{jk}n_kn_l\varepsilon_{lpq}N_pH_q\left(1 - \frac{3}{ik_0\alpha R} - \frac{3}{k_0^2\alpha^2R^2}\right) \\
& + \frac{\omega\mu_0(\varepsilon^0 - \varepsilon^e)}{\alpha^3} \kappa_{jk}n_k\varepsilon_{3pq}N_pH_q\left(1 - \frac{3}{ik_0\alpha R} - \frac{3}{k_0^2\alpha^2R^2}\right) - \frac{\omega\mu_0}{\alpha}\kappa_{jk}\varepsilon_{kpq}N_pH_q\Big\}\frac{e^{ik_0\alpha R}}{R} \\
& + \frac{ik_0\sqrt{\varepsilon^0}}{4\pi}\frac{\varepsilon_{j3k}n_k}{(1-n_3^2)}\oint_s d^2x_Q'\{n_3n_l\varepsilon_{ipq}N_pE_q - \varepsilon_{3pq}N_pE_q\}\left(\frac{\varepsilon^e}{\alpha^2}\frac{e^{ik_0\alpha R}}{R} - \frac{e^{ik^0R}}{R}\right) \\
& - \frac{i\omega\mu_0}{4\pi\sqrt{\varepsilon^0}}\frac{\varepsilon_{j3k}n_k}{(1-n_3^2)}\oint_s d^2x_Q'\{\varepsilon_{3lm}\varepsilon_{lpq}N_pH_qn_m\}\left(\frac{\varepsilon^e}{\alpha}\frac{e^{ik_0\alpha R}}{R} - \sqrt{\varepsilon^0}\frac{e^{ik^0R}}{R}\right) \\
& + \frac{1}{4\pi\sqrt{\varepsilon^0}}\oint_s d^2x_Q'\left\{\frac{\varepsilon^e\varepsilon^0}{\alpha^3}\varepsilon_{jkl}\varepsilon_{lpq}n_kN_pE_q + \sqrt{\frac{\mu_0}{\varepsilon_0}}\alpha^{-2}\kappa_{jk}\varepsilon_{kpq}N_pH_q\right\}\frac{e^{ik_0\alpha R}}{R^2} \\
& - \frac{\sqrt{\varepsilon^0}\,\varepsilon_{j3k}n_k}{4\pi(1-n_3^2)}\oint_s d^2x_Q'\{n_3n_l\varepsilon_{lpq}N_pE_q - \varepsilon_{3pq}N_pE_q\}\left(\frac{\varepsilon^e}{\alpha^3}\frac{e^{ik_0\alpha R}}{R^2} - \frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right) \\
& - \frac{\sqrt{\varepsilon^0}\,\varepsilon_{j3k}n_kn_3}{4\pi(1-n_3^2)^2}\oint_s d^2x_Q'\{n_l\varepsilon_{lpq}N_pE_q - n_3\varepsilon_{3pq}N_pE_q\}\left(\frac{1}{\alpha}\frac{e^{ik_0\alpha R}}{R^2} - \frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right) \\
& + \frac{\sqrt{\varepsilon^0}\,n_jn_3}{4\pi(1-n_3^2)^2}\oint_s d^2x_Q'\varepsilon_{3kl}\varepsilon_{kpq}N_pE_qn_l\left(\frac{1}{\alpha}\frac{e^{ik_0\alpha R}}{R^2} - \frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right)_{j=1,2} \\
& + \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon^0}}\frac{n_j}{4\pi(1-n_3^2)^2}\oint_s d^2x_Q'\{n_l\varepsilon_{lpq}N_pH_q - n_3\varepsilon_{3pq}N_pH_q\}\left(\frac{e^{ik_0\alpha R}}{R^2} - \frac{e^{ik^0R}}{R^2}\right)_{j=1,2} \\
& + \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\varepsilon_{jk3}n_k}{4\pi\sqrt{\varepsilon^0}(1-n_3^2)^2}\oint_s \varepsilon_{3ml}\varepsilon_{mpq}N_pH_qn_l\left(\frac{e^{ik_0\alpha R}}{R^2} - \frac{e^{ik^0R}}{R^2}\right)d^2x_Q' \\
& + \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{i}{4\pi\alpha^3k_0\sqrt{\varepsilon^0}}\oint_s \kappa_{jk}\varepsilon_{kpq}N_pH_q\frac{e^{ik_0\alpha R}}{R^3}d^2x_Q',
\end{aligned} \tag{2.8}$$

and

$$H_j(P) = \frac{i\sqrt{\varepsilon^0}}{4\pi} \oint_s d^2x'_Q \Big\{ k_0 \varepsilon_{jkl}\varepsilon_{kpq} N_p H_q n_l$$

$$- \omega\varepsilon_0 \sqrt{\varepsilon^0}\, n_j n_k \varepsilon_{kpq} N_p E_q \left(1 - \frac{3}{ik_0\sqrt{\varepsilon^0}\,R} - \frac{3}{k_0^2 \varepsilon^0 R^2}\right) + \omega\varepsilon_0 \sqrt{\varepsilon^0}\, \varepsilon_{jpq} N_p E_q \Big\} \frac{e^{ik^0R}}{R}$$

$$- \frac{ik_0\sqrt{\varepsilon^0}}{4\pi} \frac{\varepsilon_{j3k}n_k}{(1-n_3^2)} \oint_s d^2x'_Q \{n_3 n_l \varepsilon_{lpq} N_p H_q - \varepsilon_{3pq} N_p H_q\} \left(\frac{\varepsilon^e}{\alpha^2} \frac{e^{ik_0\alpha R}}{R} - \frac{e^{ik^0R}}{R}\right)$$

$$- \frac{i\omega\varepsilon_0\sqrt{\varepsilon^0}}{4\pi} \frac{\varepsilon_{j3k}n_k}{(1-n_3^2)} \oint_s d^2x'_Q \varepsilon_{3lm}\varepsilon_{lpq} N_p E_q n_m \left(\frac{\varepsilon^e}{\alpha} \frac{e^{ik_0\alpha R}}{R} - \sqrt{\varepsilon^0}\frac{e^{ik^0R}}{R}\right)$$

$$+ \frac{1}{4\pi} \oint_s d^2x'_Q \left\{ \varepsilon_{jkl}\varepsilon_{lpq} n_k N_p H_q - \sqrt{\frac{\varepsilon_0\varepsilon^0}{\mu_0}}\, \varepsilon_{jpq} N_p H_q \right\} \frac{e^{ik^0R}}{R^2}$$

$$+ \frac{\sqrt{\varepsilon^0}}{4\pi} \frac{\varepsilon_{j3k}n_k}{(1-n_3^2)} \oint_s d^2x'_Q \{n_3 n_l \varepsilon_{lpq} N_p H_q - \varepsilon_{3pq} N_p H_q\} \left(\frac{\varepsilon^e}{\alpha^3} \frac{e^{ik_0\alpha R}}{R^2} - \frac{1}{\sqrt{\varepsilon^0}} \frac{e^{ik^0R}}{R^2}\right)$$

$$+ \frac{\sqrt{\varepsilon^0}\, n_3 \varepsilon_{j3k} n_k}{4\pi(1-n_3^2)^2} \oint_s d^2x'_Q \{n_l \varepsilon_{lpq} N_p H_q - n_3 \varepsilon_{3pq} N_p H_q\} \left(\frac{1}{\alpha} \frac{e^{ik_0\alpha R}}{R^2} - \frac{1}{\sqrt{\varepsilon^0}} \frac{e^{ik^0R}}{R^2}\right)$$

$$- \frac{\sqrt{\varepsilon^0}\, n_3 n_j}{4\pi(1-n_3^2)^2} \oint_s d^2x'_Q \varepsilon_{3kl}\varepsilon_{kpq} N_p H_q n_l \left(\frac{1}{\alpha} \frac{e^{ik_0\alpha R}}{R^2} - \frac{1}{\sqrt{\varepsilon^0}} \frac{e^{ik^0R}}{R^2}\right)_{j=1,2}$$

$$+ \frac{1}{4\pi}\sqrt{\frac{\varepsilon_0\varepsilon^0}{\mu_0}} \frac{n_j}{(1-n_3^2)^2} \oint_s d^2x'_Q \{n_k \varepsilon_{kpq} N_p E_q - n_3 \varepsilon_{3pq} N_p E_q\} \left(\frac{e^{ik_0\alpha R}}{R^2} - \frac{e^{ik^0R}}{R^2}\right)_{j=1,2}$$

$$+ \frac{1}{4\pi}\sqrt{\frac{\varepsilon_0\varepsilon^0}{\mu_0}} \frac{\varepsilon_{j3k}n_k}{(1-n_3^2)^2} \oint_s d^2x'_Q \varepsilon_{3lm}\varepsilon_{lpq} N_p E_q n_m \left(\frac{e^{ik_0\alpha R}}{R^2} - \frac{e^{ik^0R}}{R^2}\right)$$

$$- \frac{i}{4\pi k_0}\sqrt{\frac{\varepsilon_0}{\mu_0}} \oint_s \varepsilon_{jpq} N_p E_q \frac{e^{ik^0R}}{R^3}\, d^2x'_Q, \tag{2.9}$$

where the symbols mean:

$$\alpha = (\varepsilon^e n_1^2 + \varepsilon^e n_2^2 + \varepsilon^0 n_3^2)^{1/2}$$

$$k_0 = \omega\sqrt{\varepsilon_0\mu_0}, \qquad k^0 = \sqrt{\varepsilon^0}\, k_0$$

$$\kappa_{ij} = \varepsilon^e(\delta_{ij} - c_i c_j) + \varepsilon^0 c_i c_j, \qquad \vec{c} = \{0, 0, 1\}.$$

The j index in expresions (2.8) and (2.9) runs through 1, 2, 3 and 1, 2 only in those terms for which it is clearly indicated.

3. The formulation of the electromagnetic radiation conditions for a medium with uniaxial electric anisotropy

The integration surface in (2.8), (2.9) is composed of two closed surface, the first one contains field sources and difracting bodies, the second one surrounds the first one (Fig. 1) and is a spherical surface with radius extending to infinity. This surface is denoted further by S_∞.

If S_∞ is a spherical surface with its centre at observation point P, so the unit vectors $\vec{N}$ and $\vec{n}$ from (2.8)–(2.9) are antiparallel, and the area element is equal to $d^2x'_Q = R^2 d\gamma$, where $d\gamma$ is the solid angle under which the area element from the observation point is seen.

Because

$$\varepsilon_{jkl}\varepsilon_{kpq} = \delta_{jq}\delta_{lp} - \delta_{jp}\delta_{lq},$$

integrating (2.8) over the infinite surface of the sphere we obtain the expression

$$-\frac{ik_0}{4\pi\alpha\sqrt{\varepsilon^0}} \int\limits_{S_\infty} \frac{e^{ik_0\alpha R}}{R}\left(\frac{\varepsilon^e\varepsilon^0}{\alpha} E_j^{\perp} + \sqrt{\frac{\mu_0}{\varepsilon_0}}\, \kappa_{jk}\varepsilon_{kpq}N_pH_q\right) R^2 d\gamma$$

$$-\frac{i\omega\mu_0}{4\pi\sqrt{\varepsilon^0}} \frac{\varepsilon^0-\varepsilon^e}{\alpha^3} \kappa_{jk}N_k \int\limits_{S_\infty} \varepsilon_{3pq}N_pH_qR^2 d\gamma$$

$$+\frac{i\omega\mu_0}{4\pi\sqrt{\varepsilon^0}} \frac{\varepsilon_{jk3}N_k}{1-N_3^2} \int\limits_{S_\infty} H_3^{\perp}\left(\frac{\varepsilon^e}{\alpha}\frac{e^{ik_0\alpha R}}{R} - \sqrt{\varepsilon^0}\frac{e^{ik^0R}}{R}\right) R^2 d\gamma$$

$$+\frac{ik_0\sqrt{\varepsilon^0}}{4\pi} \frac{\varepsilon_{j3k}N_k}{1-N_3^2} \int\limits_{S_\infty} \varepsilon_{3pq}N_pE_q\left(\frac{\varepsilon^e}{\alpha^2}\frac{e^{ik_0\alpha R}}{R} - \frac{e^{ik^0R}}{R}\right) R^2 d\gamma$$

$$+\frac{1}{4\pi\alpha^2\sqrt{\varepsilon^0}} \int\limits_{S_\infty} \frac{e^{ik_0\alpha R}}{R^2}\left(\frac{\varepsilon^e\varepsilon^0}{\alpha} E_j^{\perp} + \frac{\mu_0}{\varepsilon_0} \kappa_{jk}\varepsilon_{kpq}N_pH_q\right) R^2 d\gamma$$

$$+\frac{\sqrt{\varepsilon^0}}{4\pi} \frac{\varepsilon_{j3k}N_k}{1-N_3^2} \int\limits_{S_\infty} \varepsilon_{3pq}N_pE_q\left(\frac{\varepsilon^e}{\alpha^3}\frac{e^{ik_0\alpha R}}{R^2} - \frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right) R^2 d\gamma$$

$$-\frac{\sqrt{\varepsilon^0}\,N_3^2}{4\pi} \frac{\varepsilon_{j3k}N_k}{(1-N_3^2)^2} \int\limits_{S_\infty} \varepsilon_{3pq}N_pE_q\left(\frac{1}{\alpha}\frac{e^{ik_0\alpha R}}{R^2} - \frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right) R^2 d\gamma$$

$$-\frac{\sqrt{\varepsilon^0}\,N_3}{4\pi} \frac{N_j}{(1-N_3^2)^2} \int\limits_{S_\infty} E_3^{\perp}\left(\frac{1}{\alpha}\frac{e^{ik_0\alpha R}}{R^2} - \frac{1}{\sqrt{\varepsilon^0}}\frac{e^{ik^0R}}{R^2}\right)_{j=1,2} R^2 d\gamma$$

$$+\frac{3}{4\pi}\sqrt{\frac{\mu_0}{\varepsilon^0\varepsilon_0}}\,\frac{\varepsilon^0-\varepsilon^e}{\alpha^4}\,\kappa_{jk}N_k\int\limits_{S_\infty}\frac{e^{ik_0\alpha R}}{R^2}\,\varepsilon_{3pq}N_pH_qR^2d\gamma$$

$$-\frac{N_3}{4\pi}\sqrt{\frac{\mu_0}{\varepsilon^0\varepsilon_0}}\,\frac{N_j}{(1-N_3^2)^2}\int\limits_{S_\infty}\varepsilon_{3pq}N_pH_q\left(\frac{e^{ik_0\alpha R}}{R^2}-\frac{e^{ik^0R}}{R^2}\right)_{j=1,2}R^2d\gamma$$

$$+\frac{1}{4\pi}\sqrt{\frac{\mu_0}{\varepsilon^0\varepsilon_0}}\,\frac{\varepsilon_{j3k}N_k}{(1-N_3^2)^2}\int\limits_{S_\infty}H_3^{\perp}\left(\frac{e^{ik_0\alpha R}}{R^2}-\frac{e^{ik^0R}}{R^2}\right)R^2d\gamma$$

$$+\frac{i\kappa_{jk}N_k}{4\pi k_0}\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon^0}}\,\frac{\varepsilon^0-\varepsilon^e}{\alpha^5}\int\limits_{S_\infty}\frac{e^{ik_0\alpha R}}{R^3}\,\varepsilon_{3pq}N_pH_qR^2d\gamma$$

$$+\frac{i}{4\pi k_0\alpha^3}\sqrt{\frac{\mu_0}{\varepsilon^0\varepsilon_0}}\int\limits_{S_\infty}\frac{e^{ik_0\alpha R}}{R^3}\,\kappa_{jk}\varepsilon_{kpq}N_pH_qR^2d\gamma \tag{3.1}$$

and similarly, integrating (2.9) over S_∞ we obtain

$$-\frac{ik_0\sqrt{\varepsilon^0}}{4\pi}\int\limits_{S_\infty}\frac{e^{ik^0R}}{R}\left(H_j^{\perp}-\sqrt{\frac{\varepsilon^0\varepsilon_0}{\mu_0}}\,\varepsilon_{jpq}N_pE_q\right)R^2d\gamma$$

$$-\frac{ik_0\sqrt{\varepsilon^0}}{4\pi}\,\frac{\varepsilon_{j3k}N_k}{1-N_3^2}\int\limits_{S_\infty}\varepsilon_{3pq}N_pH_q\left(\frac{\varepsilon^e}{\alpha^2}\,\frac{e^{ik_0\alpha R}}{R}-\frac{e^{ik^0R}}{R}\right)R^2d\gamma$$

$$-\frac{i\omega\varepsilon_0\sqrt{\varepsilon^0}}{4\pi}\,\frac{\varepsilon_{j3k}N_k}{1-N_3^2}\int\limits_{S_\infty}E_3^{\perp}\left(\frac{\varepsilon^e}{\alpha}\,\frac{e^{ik_0\alpha R}}{R}-\sqrt{\varepsilon^0}\,\frac{e^{ik^0R}}{R}\right)R^2d\gamma$$

$$+\frac{\sqrt{\varepsilon^0}}{4\pi}\int\limits_{S_\infty}\frac{e^{ik^0R}}{R^2}\left(H_j^{\perp}-\sqrt{\frac{\varepsilon^0\varepsilon_0}{\mu_0}}\,\varepsilon_{jpq}N_pE_q\right)R^2d\gamma$$

$$-\frac{\sqrt{\varepsilon^0}}{4\pi}\,\frac{\varepsilon_{j3k}N_k}{1-N_3^2}\int\limits_{S_\infty}\varepsilon_{3pq}N_pH_q\left(\frac{\varepsilon^e}{\alpha^3}\,\frac{e^{ik_0\alpha R}}{R^2}-\frac{1}{\sqrt{\varepsilon^0}}\,\frac{e^{ik^0R}}{R^2}\right)R^2d\gamma$$

$$+\frac{\sqrt{\varepsilon^0}\,N_3^2}{4\pi}\,\frac{\varepsilon_{j3k}N_k}{(1-N_3^2)^2}\int\limits_{S_\infty}\varepsilon_{3pq}N_pH_q\left(\frac{1}{\alpha}\,\frac{e^{ik_0\alpha R}}{R^2}-\frac{1}{\sqrt{\varepsilon^0}}\,\frac{e^{ik^0R}}{R^2}\right)R^2d\gamma$$

$$-\frac{\sqrt{\varepsilon^0}\,N_3}{4\pi}\,\frac{N_j}{(1-N_3^2)^2}\int\limits_{S_\infty}H_3^{\perp}\left(\frac{1}{\alpha}\,\frac{e^{ik_0\alpha R}}{R^2}-\frac{1}{\sqrt{\varepsilon^0}}\,\frac{e^{ik^0R}}{R^2}\right)_{j=1,2}R^2d\gamma$$

$$-\frac{N_3}{4\pi}\sqrt{\frac{\varepsilon^0\varepsilon_0}{\mu_0}}\frac{N_j}{(1-N_3^2)^2}\int\limits_{S\infty}\varepsilon_{3pq}N_pE_q\left(\frac{e^{ik_0\alpha E}}{R^2}-\frac{e^{ik^0R}}{R^2}\right)_{j=1,2}R^2d\gamma$$

$$+\frac{1}{4\pi}\sqrt{\frac{\varepsilon^0\varepsilon_0}{\mu_0}}\frac{\varepsilon_{j3k}N_k}{1-N_3^2}\int\limits_{S_\infty}E_3^\perp\left(\frac{e^{ik_0\alpha R}}{R^2}-\frac{e^{ik^0R}}{R^2}\right)R^2d\gamma$$

$$-\frac{i}{4\pi k_0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\int\limits_{S_\infty}\frac{e^{ik^0R}}{R^3}\varepsilon_{jpq}N_pE_qR^2d\gamma, \tag{3.2}$$

where E_j, H_j, denote the tangential components of the electric and magnetic field strength at the surface S_∞.

$$E_j^\perp = E_j-N_j(N_kE_k), \quad H_j^\perp = H_j-N_j(N_kH_k).$$

The integration surface in (3.1) and (3.2) is proportional to R^2 whereas the terms in these equations contain R in different powers. Finally, expressions (3.1), (3.2) contain integrals of three types. The integrals of the first type, in which R is found in the first power, the second type of integrals are independent of R and the third type contain R^{-1}.

Because infinity must not contribute to the field at the observation point the expressions (3.1), (3.2) must be equal to zero. This is possible only when integrals of all three types vanish.

In the limiting case $R\to\infty$ integrals of the third type vanish if the absolute value of the vector product $\varepsilon_{jpq}N_pH_q$, $\varepsilon_{jpq}N_pE_q$ has a finite value.

Let us further notice that one should postulate:

$$\lim_{R\to\infty} R\left(\frac{\varepsilon^e\varepsilon^0}{\alpha}E_j^\perp+\sqrt{\frac{\mu_0}{\varepsilon_0}}\kappa_{jk}\varepsilon_{kpq}N_pH_q^\perp\right)=0, \tag{3.3}$$

$$\lim_{R\to\infty} RH_3^\perp=0, \tag{3.4}$$

$$\lim_{R\to\infty} R\varepsilon_{3pq}N_pE_q^\perp=0, \tag{3.5}$$

$$\lim_{R\to\infty} R\varepsilon_{3pq}N_pH_q^\perp=0 \tag{3.6}$$

in order that the first type of integrals vanish.

In order to demand the vanishing of the second type of integrals in this expression we postulate

$$\lim_{R\to\infty}\left(\frac{\varepsilon^e\varepsilon^0}{\alpha}E_j^\perp+\sqrt{\frac{\mu_0}{\varepsilon_0}}\kappa_{jk}\varepsilon_{kpq}N_pH_q^\perp\right)=0, \tag{3.7}$$

$$\lim_{R\to\infty} E_3^\perp=0, \tag{3.8}$$

$$\lim_{R\to\infty} H_3^\perp=0, \tag{3.9}$$

$$\lim_{R\to\infty} \varepsilon_{3pq} N_p E_q^{\perp} = 0, \tag{3.10}$$

$$\lim_{R\to\infty} \varepsilon_{3pq} N_p H_q^{\perp} = 0 \tag{3.11}$$

similarly, considering expression (3.2), we have the conditions

$$\lim_{R\to\infty} R\left(H_j^{\perp} - \sqrt{\frac{\varepsilon^0 \varepsilon_0}{\mu_0}}\, \varepsilon_{jpq} N_p E_q^{\perp}\right) = 0 \tag{3.12}$$

$$\lim_{R\to\infty} R E_3^{\perp} = 0 \tag{3.13}$$

$$\lim_{R\to\infty} R \varepsilon_{3pq} N_p H_q^{\perp} = 0 \tag{3.14}$$

and for the second type of integrals, respectively

$$\lim_{R\to\infty} \left(H_j^{\perp} - \sqrt{\frac{\varepsilon^0 \varepsilon_0}{\mu_0}}\, \varepsilon_{jpq} N_p E_q^{\perp}\right) = 0 \tag{3.15}$$

$$\lim_{R\to\infty} E_3^{\perp} = 0 \tag{3.16}$$

$$\lim_{R\to\infty} H_3^{\perp} = 0 \tag{3.17}$$

$$\lim_{R\to\infty} \varepsilon_{3pq} N_p H_q^{\perp} = 0, \tag{3.18}$$

$$\lim_{R\to\infty} \varepsilon_{3pq} N_p E_q^{\perp} = 0. \tag{3.19}$$

In the expression (3.3)–(3.19) we used the selfevident identity

$$\varepsilon_{jpq} N_p E_q = \varepsilon_{jpq} N_p E_q^{\perp}, \quad \varepsilon_{jpq} N_p H_q = \varepsilon_{jpq} N_p H_q^{\perp}.$$

As it is easily seen, the conditions (3.4)–(3.6) are stronger than (3.7), (3.9) — (3.11) and these are automatically fulfilled if the conditions (3.4)–(3.6) are fulfilled. We can say the same about conditions (3.12)–(3.14) and (3.15), (3.16), (3.18). If we notice moreover that condition (3.13) is stronger than (3.8), (3.4) and (3.5) and they in turn are stronger than (3.17) and (3.19) respectively, we formulate for the anisotropic medium under consideration the following radiation conditions:

$$\lim_{R\to\infty} R\left(\frac{\varepsilon^e \varepsilon^0}{\alpha} E_j^{\perp} + \sqrt{\frac{\mu_0}{\varepsilon_0}}\, \kappa_{jk} \varepsilon_{kpq} N_p H_q^{\perp}\right) = 0, \tag{3.20}$$

$$\lim_{R\to\infty} R\left(H_j^{\perp} - \sqrt{\frac{\varepsilon^0 \varepsilon_0}{\mu_0}}\, \varepsilon_{jpq} N_p E_q^{\perp}\right) = 0, \tag{3.21}$$

$$\lim_{R\to\infty} R E_3^{\perp} = 0, \tag{3.22}$$

$$\lim_{R\to\infty} RH_3^{\perp} = 0, \tag{3.23}$$

$$\lim_{R\to\infty} R\varepsilon_{3pq}N_pE_q^{\perp} = 0, \tag{3.24}$$

$$\lim_{R\to\infty} R\varepsilon_{3pq}N_pH_q^{\perp} = 0. \tag{3.25}$$

4. Conclusions

Electromagnetic radiation conditions obtained by us contain only tangential components not total field vectors and on account of this they can be regarded as conditions of the Rubinowicz type (1.5) and (1.6).

The characteristic of the medium with uniaxial anisotropy is its symmetry axis. This characteristic has an influence on the electromagnetic radiation conditions. For those reasons these conditions can be divided into two groups.

In the first group we include

$$\lim_{R\to\infty} R\left(\frac{\varepsilon^e\varepsilon^0}{\alpha}E_j^{\perp} + \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\kappa_{jk}\varepsilon_{kpq}N_pH_q^{\perp}\right) = 0, \tag{4.1}$$

$$\lim_{R\to\infty} RH_3^{\perp} = 0, \tag{4.2}$$

$$\lim_{R\to\infty} \varepsilon_{3pq}RN_pE_q^{\perp} = 0 \tag{4.3}$$

in the second one we include

$$\lim_{R\to\infty} R\left(H_j^{\perp} - \sqrt{\frac{\varepsilon^0\varepsilon_0}{\mu_0}}\,\varepsilon_{jpq}N_pE_q^{\perp}\right) = 0, \tag{4.4}$$

$$\lim_{R\to\infty} RE_3^{\perp} = 0, \tag{4.5}$$

$$\lim_{R\to\infty} \varepsilon_{3pq}RN_pH_q^{\perp} = 0. \tag{4.6}$$

If conditions (4.1)–(4.6) are fulfilled we can characterize the behaviour of the electromagnetic field at infinity by stating that it behaves similar to the superposition of plane waves of the TM and TE types moving in the N direction.

The following conditions are then fulfilled:

$$\frac{\varepsilon^0\varepsilon^e}{\alpha}E_j^{\perp} + \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\kappa_{jk}\varepsilon_{kpq}N_pH_q^{\perp} = 0, \tag{4.7}$$

$$H_3^{\perp} = 0, \tag{4.8}$$

$$\varepsilon_{3pq}N_pE_q^{\perp} = 0, \tag{4.9}$$

and

$$H_j^{\perp} - \sqrt{\frac{\varepsilon^0 \varepsilon_0}{\mu_0}}\, \varepsilon_{jpq} N_p E_q^{\perp} = 0, \tag{4.10}$$

$$E_3^{\perp} = 0, \tag{4.11}$$

$$\varepsilon_{3pq} N_p H_q^{\perp} = 0. \tag{4.12}$$

Conditions (4.1)–(4.3) correspond to the TM field, and conditions (4.4)–(4.6) to the TE field.

The electromagnetic radiation conditions obtained above are sufficient to exclude the possibility of radiation from infinity.

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