TEMPERATURE DEPENDENCE OF SPIN WAVE RESONANCE IN FERROMAGNETICS

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The temperature dependence of standing spin wave resonance in ferromagnetics, in particular resonance peak separations and intensities, are calculated. The temperature dependence of the threshold conditions for surface mode is derived. The calculations are carried out for a ferromagnetic system with symmetric boundary conditions and inhomogeneities localized at the boundary and next-to-boundary nodes.

1. Introduction

The experimental study of ferromagnetic resonance (FMR) is a source of information on such basic properties of ferromagnetics as magnetization, magnetocrystalline anisotropy energy, relaxation time, splitting factor, etc. Additional information can be gained from the fine structure analysis of the FMR spectrum (spin wave resonance, SWR). The resonance peak separation of the SWR spectrum is large only in the case of thin films.

Kittel [1] first proved that a uniform microwave field generates standing spin waves if appropriate surface spin pinning conditions are fulfilled. Seavey and Tannenwald [2] first obtained SWR experimentally in permalloy thin film. To date, numerous theoretical and experimental papers on spin wave resonance have been published.

Whereas most experimental studies on SWR are performed at room temperature, theoretical papers are concerned with the low temperature region. The experimental results for the temperature dependence of SWR are currently interpreted on the basis of theoretical results for the temperature dependence of propagating spin waves [3–6], although in SWR standing spin waves are excited. Since no theory of temperature-dependent

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SWR is as yet available, we undertook the present theoretical study in order to bridge the gap.

There are many sides to the problem of the temperature dependence of SWR. Highly essential is the variation in resonance peak separation with increasing temperature as a method for the direct determination of the temperature dependence of the exchange coupling parameter. Highly important is the analysis of the temperature dependence of the threshold conditions for surface modes. The investigation of the existence conditions for surface mode provides information on the surface paramaters and thus yields the characteristics of the surface of magnetic crystals.

2. *The assumptions*

The analysis of a finite system is concerned with the problem of its boundary conditions.

We assume symmetric boundary conditions and, in accordance with the surface inhomogeneity (SI) model, inhomogeneities localized at the film surfaces. At finite temperatures, however, the inhomogeneities cannot be restricted to the film surfaces only ($l = 0$ and $N-1$), but have to extend to adjacent lattice layers parallel to the film surface ($l = 1, 2, \ldots$ and $N-3, N-2$) [7, 8]. Therefore we specify boundary conditions with inhomogeneities localized in the boundary layers and the lattice layers adjacent on either surface.

The effective field acting on an individual spin belonging to the layer l is

$$\vec{H}_l^{\mathrm{eff}} = \vec{H}_{\mathrm{bulk}}^{\mathrm{eff}} + \vec{H}_{\mathrm{surf}}^{\mathrm{eff}}[(\delta_{l,0}+\delta_{l,N-1}) + p(\delta_{l,1}+\delta_{l,N-2})], \tag{1}$$

where p is the ratio of the surface anisotropy field in the second lattice layer and the surface anisotropy field $\vec{H}_{\mathrm{surf}}^{\mathrm{eff}}$ in the boundary layer.

With increasing temperature, the number of perturbed layers near the surface, with magnetization differing from the bulk magnetization, increases [7–10]. However, we obtain the characteristic features of the influence of magnetization inhomogeneity on SWR by assuming that the inhomogeneity is localized entirely on the film surface, i. e.

$$\langle n \rangle = \begin{cases} \langle \hat{n} \rangle & \text{for } l = 1, 2, \ldots, N\text{-}2 \\ \langle \hat{n} \rangle_s & \text{for } l = 0 \text{ and } N\text{-}1. \end{cases} \tag{2}$$

We restrict the exchange interactions to the nearest neighbours, and in addition assume that

$$J = \begin{cases} J & \text{for } l,\ l' = 1, 2, \ldots N\text{-}2 \\ J' & \text{for } l \text{ or } l' = 0 \text{ and } N\text{-}1. \end{cases} \tag{3}$$

In accordance with Valenta's model of a magnetic thin film [11], one can reduce the secular problem to that of a finite linear chain perpendicular to the film surface, e. g. [12–13]. Therefore for simplicity we consider a finite linear chain, consisting of N parallel spin S with lattice constant a. The sites are labelled $l = 0, 1, \ldots N$-1. In this way we obtain correct results because, in spin wave resonance, only spin waves with a wave vector $\vec{k}$

normal to the film plane are important. In the present paper, we consider SWR with the d. c. magnetic field $\vec{H}_0$ perpendicular to the film plane.

By Heisenberg's theory, the Hamiltonian of a ferromagnetic finite linear chain is of the form:

$$\hat{\mathscr{H}} = -\tfrac{1}{2}\sum_{l,l'} J_{ll'}\hat{\vec{S}}_l \cdot \hat{\vec{S}}_{l'} - \mu g \sum_{l} \vec{H}_l^{\text{eff}} \cdot \hat{\vec{S}}_l, \tag{4}$$

where $\hat{\vec{S}}_l$ is the spin operator of the l-th site in $\hbar$ units, g — the Landé factor, and μ — Bohr's magneton.

We consider the temperature renormalization of magnons resulting from the interactions between spin waves. We transform the Hamiltonian (4) to the oscillator form by means of Oguchi's transformation [14], which preserves the predominant part of the dynamical interactions between spin wawes and excludes all kinematical interactions, which are unimportant in the phenomenon of spin wave resonance. By Oguchi's transformation and considering that the effective field is directed along the chain axis ($H_l^{\text{eff}\pm} = 0$) and the equilibrium conditions of the system are $\hat{\mathscr{H}}_1 = \hat{\mathscr{H}}_3 = 0$, we obtain the Hamiltonian to biquadratic order in the form:

$$\hat{\mathscr{H}} = E_0 + \hat{\mathscr{H}}_2 + \hat{\mathscr{H}}_4 \tag{5}$$

with:

$$E_0 = -\tfrac{1}{2}\sum_{l,l'} J_{ll'}S_lS_{l'} - \mu g \sum_{l} H_l^{\text{eff}\,z} S_l, \tag{6a}$$

$$\hat{\mathscr{H}}_2 = \sum_{l,l'} A_{ll'}\hat{a}_l^+ \hat{a}_{l'}, \tag{6b}$$

$$\hat{\mathscr{H}}_4 = -\tfrac{1}{8}\sum_{l,l'} J_{ll'}(\hat{a}_l^+ \hat{n}_{l'}\hat{a}_{l'} + \hat{a}_l^+ \hat{n}_l\hat{a}_{l'} + \hat{a}_l\hat{a}_{l'}^+ \hat{n}_{l'} + \hat{n}_l\hat{a}_l\hat{a}_{l'}^+ - 4\hat{n}_l\hat{n}_{l'}), \tag{6c}$$

where

$$A_{ll'} = -SJ_{ll'} + \delta_{ll'}\Big(\sum_{l''} SJ_{ll''} + \mu g H_l^{\text{eff}\,z}\Big). \tag{7}$$

3. *Temperature renormalization of spin wave energy in a ferromagnetic thin film*

The Hamiltonian (5) can be diagonalized only in approximation by applying suitable methods which simplify the interaction part of the Hamiltonian $\hat{\mathscr{H}}_4$, e. g., the random-phase approximation (RPA) [15, 16], the first random-phase approximation of Keffer and Loudon (RPA-I) [17–19], or the magnon renormalization approximation (MRA) proposed by Bloch [20, 21].

In agreement with RPA-I, on taking into account all symmetrical decouplings of $\hat{\mathscr{H}}_4$, we obtain the renormalized Hamiltonian

$$\hat{\tilde{\mathscr{H}}} = E_0 + \sum_{l,l'} \tilde{A}_{ll'}\hat{a}_l^+ \hat{a}_{l'} \tag{8}$$

with

$$\tilde{A}_{ll'} = -SJ_{ll'}\left[1-\frac{1}{2S}(\langle\hat{n}_l\rangle+\langle\hat{n}_{l'}\rangle)\right]+\delta_{ll'}\left[\sum_{l''} SJ_{ll''}\left(1-\frac{\langle n_{l''}\rangle}{S}\right)+\mu g H_l^{\text{eff}}\right]. \tag{9}$$

Due to RPA-I, performed in the real crystal lattice, the local variation in spin deviation at the film surface essential to the phenomenon of SWR can be taken into account.

In the case considered, the secular problem can be written in matrix form as follows:

$$\begin{bmatrix} \tilde{A}_0+a-E_{\vec{k}}, \ \tilde{A}_1+c, & & & \\ \tilde{A}_1+d, \quad \tilde{A}_0+b-E_{\vec{k}}, \ \tilde{A}_1, & & & \\ \cdots & & & \\ & \tilde{A}_1, \ \tilde{A}_0-E_{\vec{k}}, \ \tilde{A}_1, & & \\ \cdots & & & \\ & & \tilde{A}_1, \ \tilde{A}_0+b-E_{\vec{k}}, \ \tilde{A}_1+d & \\ & & \tilde{A}_1+c, \quad \tilde{A}_0+a-E_{\vec{k}} & \end{bmatrix}_{N\times N} \times \begin{bmatrix} u_0(\vec{k}) \\ u_1(\vec{k}) \\ \cdots \\ u_l(\vec{k}) \\ \cdots \\ u_{N-2}(\vec{k}) \\ u_{N-1}(\vec{k}) \end{bmatrix} = 0 \tag{10}$$

where $u_l(\vec{k})$ denote the canonical transformation functions, $E_{\vec{k}}$ the spin wave energy with wave vector $\vec{k}$, $\tilde{A}_0$ and A_1 are temperature renormalized parameters for the bulk of the sample, whereas a, b, c, d are surface parameters. Hitherto, in the simple SI model with symmetric surface inhomogeneities, only one surface parameter has been taken into consideration. In the RPA–1 approximation

$$\tilde{A}_0 = 2JS\alpha(T)+\mu g|\vec{H}_{\text{bulk}}^{\text{eff}}|, \tag{11a}$$

$$\tilde{A}_1 = -JS\alpha(T), \tag{11b}$$

$$a = \mu g\vec{H}_{\text{surf}}^{\text{eff}}(T)-(2J-J')S\alpha(T), \tag{11c}$$

$$b = p\mu g\vec{H}_{\text{surf}}^{\text{eff}}(T)-(J-J')S\alpha(T)-\tfrac{1}{4}J'(\langle\hat{n}\rangle_s-\langle\hat{n}\rangle), \tag{11d}$$

$$c = d = (J-J')S\alpha(T)+\tfrac{1}{2}J'(\langle\hat{n}\rangle_s-\langle\hat{n}\rangle), \tag{11e}$$

where $\alpha(T)$ denotes the temperature renormalization coefficient, equal to the relative magnetization σ in RPA approximation.

The solution of the secular problem leads to the temperature-dependent spin wave energy (shown in Fig. 2)

$$E_{\vec{k}}(T) = 2JS(1-\cos k)\alpha(T)+\mu g H_{\text{bulk}}^{\text{eff}}(T) \tag{12}$$

and, simultaneously, to a set of boundary equations which can be reduced to a characteristic equation, whence the allowed temperature-dependent wave vector values k are obtained.

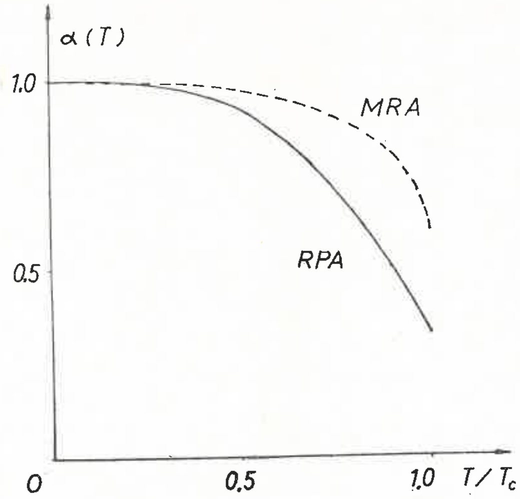

Fig. 1. Comparison of temperature renormalization coefficients in **RPA—I** and **MRA** approximations

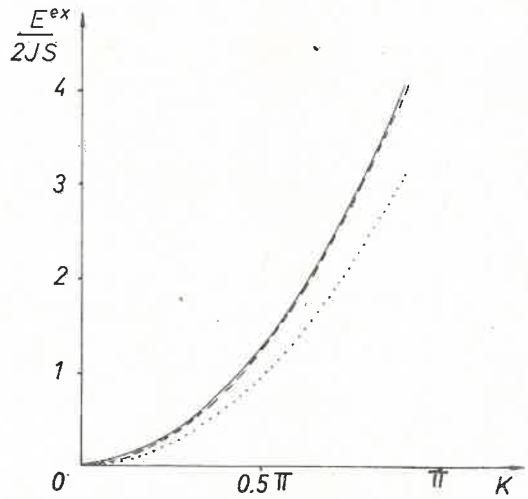

Fig. 2. The dispersion relation in **RPA—I** approximation at reduced temperatures equal to a) 0.0 (solid curve), b) 0.3 (dashed curve), c) 0.7. Data used in calculations: $N = 11$, $S = 1.0$, $J' = 0.5$ J, $\sigma_s = 0.5\sigma$, $p = 0.5$

In the case considered, the modified characteristic equation has the form

$$A(T)\left[\exp\left(ik\,\frac{N-1}{2}\right)\pm\exp\left(-ik\,\frac{N-1}{2}\right)\right]=\left[\exp\left(ik\,\frac{N+1}{2}\right)\pm\exp\left(-ik\,\frac{N+1}{2}\right)\right]$$
$$+B(T)\left[\exp\left(ik\,\frac{N-3}{2}\right)\pm\exp\left(-ik\,\frac{N-3}{2}\right)\right]+C(T)\left[\exp\left(ik\,\frac{N-5}{2}\right)\pm\exp\left[-ik\,\frac{N-5}{2}\right)\right] \tag{13}$$

involving three temperature-dependent surface parameters

$$A(T) = \frac{a+b}{A_1}, \tag{14a}$$

$$B(T) = \frac{ab-cd}{A_1^2} - \frac{c+d}{A_1}, \tag{14b}$$

$$C(T) = -\frac{b}{A_1}. \tag{14c}$$

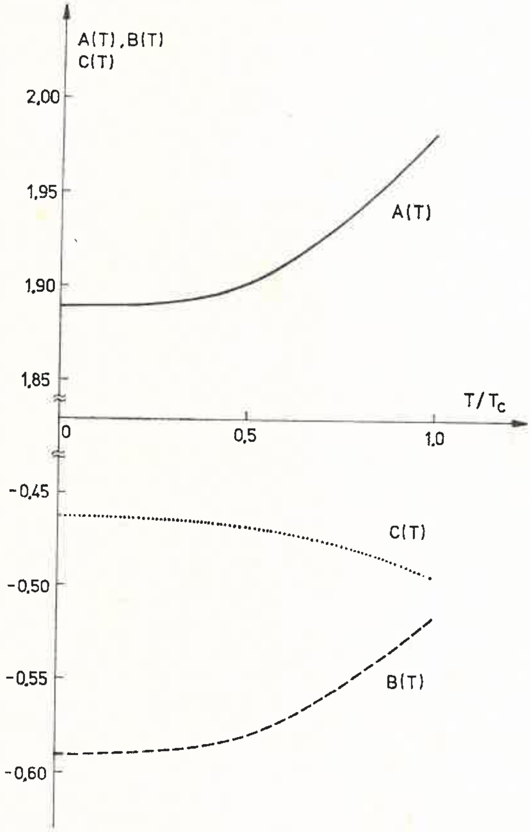

Fig. 3. Temperature dependence of the surface parameters $A(T)$, $B(T)$ and $C(T)$ for a ferromagnetic finite linear chain in RPA—I. Data used in calculations: $N = 11$, $S = 1.0$, $J = J' = 2.5\times 10^{-14}$ erg, $H_{surf}(0) = 5.0\times 10^5$ Oe, $\sigma_s = 0.5\sigma$, $p = 0.5$

The characteristic equation (13) is a transcendental one and can be solved in approximation numerically or graphically.

In a ferromagnetic finite linear chain, the following types of modes can be excited: space (bulk) modes with real k, acoustic surface modes with imaginary $k(k = ik')$, and optical surface modes with complex $k(k = \pi+ik')$. In a symmetric system only even modes or odd modes can be excited, whereas even and odd modes can be excited simultaneously in a thin film if its boundary conditions are asymmetric.

The characteristic equation for symmetric space modes takes the form

$$\cos\left(\frac{N+1}{2}k\right)+B(T)\cos\left(\frac{N-3}{2}k\right)+C(T)\cos\left(\frac{N-5}{2}k\right)=A(T)\cos\left(\frac{N-1}{2}k\right). \quad (15)$$

Our boundary conditions with three parameters $A(T)$, $B(T)$ and $C(T)$ provide a better description of the real situation at the boundary of a magnetic system than the conditions applied hitherto which took into account the perturbation at the boundary only.

The temperature renormalization of the standing spin wave energy in a magnetic thin film is more complicated than in a bulk crystal because not only the temperature renormalization coefficient $\alpha(T)$ occurs in the dispersion relation but moreover the allowed wave vector values depend on temperature, and this dependence is different for different regions of the Brillouin zone.

The analysis of the threshold conditions describing the emergence of surface modes is principally important. Characteristic equations with one temperature independent surface parameter A give its threshold values defined by the number of lattice layers N only:

$$A_s \geqslant 1 \quad \text{and} \quad A_a \geqslant \frac{N+1}{N-1}, \quad (16)$$

where "s" refers to symmetric modes and "a" — to antisymmetric ones.

It is obvious that the conditions at the surfaces of a thin film change as the temperature rises; this implies the possibility of surface mode excitation by a change in temperature [22]. The theory of the temperature dependence for standing spin waves leads to new threshold conditions for the existence of surface modes in the form:

$$A_s(T) \geqslant 1+B(T)+C(T) \quad (17a)$$

and

$$A_a(T) \geqslant \frac{N+1}{N-1}+\frac{(N-3)B(T)+(N-5)C(T)}{N-1}. \quad (17b)$$

The threshold conditions (17a, b) are more general than the conditions (16) because they depend on temperature and include parameters specific to the magnetic thin film.

4. *The temperature dependence of standing spin wave resonance*

We now proceed to calculate the probability of resonance transitions from an initial state at temperature T to the excited state induced by a linearly polarized microwave field $\vec{h}$ perpendicular to the d. c. magnetic field $\vec{H}_0$.

The perturbation Hamiltonian of the spin system in the microwave field $\vec{h}$ has the form

$$\mathscr{H}_{\rm rf} = -\mu g\vec{h}\cdot\sum_{l}\hat{\vec{S}}_{l}. \tag{18}$$

The circular component $\vec{h}^{+}$ is associated with a creation operator of spin deviation $\hat{a}_{l}^{+}$ localized in the lattice point $\vec{l}$ and causing excitation of a spin wave with wave vector $\vec{k}$. We thus consider only one part of the perturbation Hamiltonian (18), including the inhomogeneity of spin deviations at the ends of the chain

$$\hat{\mathscr{H}}_{\rm rf} = -\sqrt{\frac{S}{2}}\,\mu g h_0 \exp(i\omega t)\Bigg[\hat{a}_0^{+}\left(1-\frac{\langle\hat{n}\rangle_s}{4S}\right)$$

$$+\sum_{l=1}^{N-2}\hat{a}_l^{+}\left(1-\frac{\langle\hat{n}\rangle}{4S}\right)+\hat{a}_{N-1}^{+}\left(1-\frac{\langle\hat{n}\rangle_s}{4S}\right)\Bigg]. \tag{19}$$

Above, we took into account the most essential consequences of our assumptions regarding magnetization inhomogeneity (2) for the surface parameters and, indirectly, the set of allowed wave vectors $\vec{k}$. We now rewrite (19) in the following form:

$$\hat{\mathscr{H}}_{\rm rf} = -\frac{\sqrt{2S}}{2}\,\mu g h_0 \exp(i\omega t)\beta(T)\sum_{l}\sum_{\vec{k}}u_l(\vec{k})\hat{b}_{\vec{k}}^{+}, \tag{20}$$

where the coefficient

$$\beta(T) = \tfrac{1}{4}(3+\sigma) \tag{21}$$

describes the temperature dependence of the perturbation operator.

The probability of a transition of the system from a state $|m\rangle$ with energy E_m to a state $|n\rangle$ with energy E_n during time τ is

$$W_{nm}(\tau) = \frac{1}{\hbar^2}\left|\int_0^{\tau}\langle n|\mathscr{H}_{zab}(t)\,|m\rangle \exp\left[it(E_n-E_m)/\hbar\right]dt\right|^2. \tag{22}$$

The microwave field $\vec{h}$ generates an additional spin wave with energy dependent on temperature. The probability of the transition induced by the microwave field can be written in the form

$$P_{|0\rangle_T\to|\vec{k}\rangle_T} = \frac{2\pi}{\hbar}\left|{}_T\langle\vec{k}|\hat{\mathscr{H}}_{\rm rf}|0\rangle_T\right|^2\delta(E_{\vec{k}}(T)-\hbar\omega), \tag{23}$$

where $E_{\vec{k}}(T)$ denotes the energy required to excite the additional spin wave with wave vector $\vec{k}$ at the temperature T. Taking into account formula (20), we obtain

$$P_{|0\rangle_T \to |\vec{k}\rangle_T} = \frac{\pi S}{\hbar} (\mu g h_0)^2 \beta^2(T) \gamma(\vec{k}) \delta(E_{\vec{k}}(T) - \hbar\omega), \tag{24}$$

where

$$\gamma(\vec{k}) = |\sum_{l=0}^{N-1} u_l(\vec{k})|^2 = \begin{cases} \dfrac{2\sin^2 \dfrac{Nk}{2}}{\sin^2 \dfrac{k}{2}\left(N + \dfrac{\sin Nk}{\sin k}\right)} & \text{for symmetric space solutions,} \\ 0 & \text{for antisymmetric space solutions,} \\ \dfrac{2\,\mathrm{sh}^2 \dfrac{Nk}{2}}{\mathrm{sh}^2 \dfrac{k}{2}\left(N + \dfrac{\mathrm{sh}\, Nk}{\mathrm{sh}\, k}\right)} & \text{for symmetric acoustic surface solutions,} \\ 0 & \text{for antisymmetric acoustic surface solutions.} \end{cases} \tag{25}$$

With regard to the symmetric boundary conditions, only symmetric modes are excited. The generation probability of antisymmetric modes is zero. The allowed wave vector values k, obtained from the characteristic equation (13) (with sign "+" for the case of a symmetrical thin film), depend moreover on temperature.

It results from formula (24) that resonance absorption of the microwave field radiation occurs when the resonance condition

$$E_{\vec{k}}(T) = \hbar\omega \tag{26}$$

is fulfilled.

Above, the spin wave energy is

$$E_{\vec{k}}(T) = 2JS(1 - \cos k)\alpha(T) + \mu g[H_0 + H_{an}(0)\sigma^9 - 4\pi M_s(0)\sigma]. \tag{27}$$

The value of the temperature renormalization coefficient $\alpha(T)$ depends on the approximation applied.

With increasing temperature, the resonance peak intensities decrease as follows:

$$P_n(T) = \frac{2\pi}{\hbar} S(\mu g h_0)^2 \beta^2(T) \frac{\sin^2 \dfrac{Nk_n}{2}}{\sin^2 \dfrac{k_n}{2}\left(N + \dfrac{\sin Nk_n}{\sin k_n}\right)}, \tag{28}$$

where n labels the mode corresponding to the wave vector k_n.

In a first approximation, the intensities of all resonance peaks (irrespective of the mode number n) decrease in the same way when the temperature rises, namely proportionally to $\beta^2(T)$; this dependence is shown in Fig. 4.

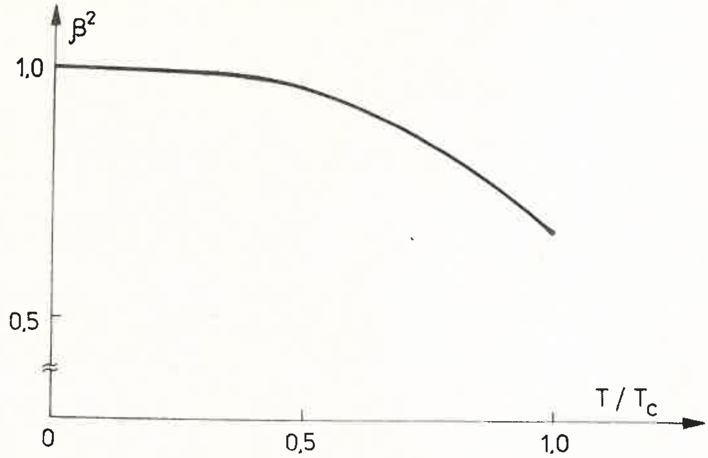

Fig. 4. Approximate dependence of the resonance line intensity on reduced temperature

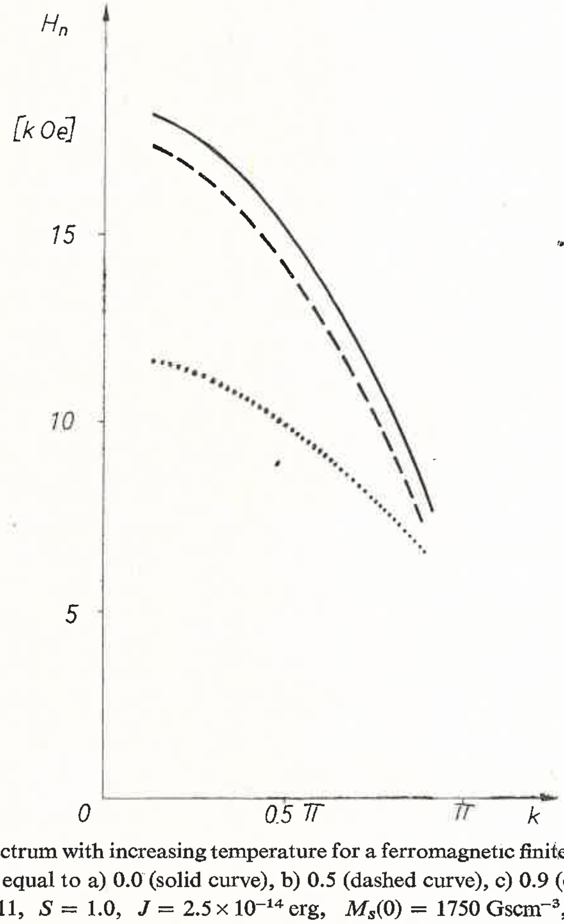

Fig. 5. Shift in SWR spectrum with increasing temperature for a ferromagnetic finite linear chain in RPA—I at reduced temperature equal to a) 0.0 (solid curve), b) 0.5 (dashed curve), c) 0.9 (dotted curve). Data used in calculations: $N = 11$, $S = 1.0$, $J = 2.5 \times 10^{-14}$ erg, $M_s(0) = 1750$ Gscm^{-3}, $H_{an}(0) = 5.0 \times 10^2$ Oe, $H_{surf}(0) = 5.0 \times 10^5$ Oe, $\sigma_s = 0.5\sigma$, $J' = 0.5$ J, $p = 0.5$, $h = 10$ Oe, $\nu = 9.4$ GHz

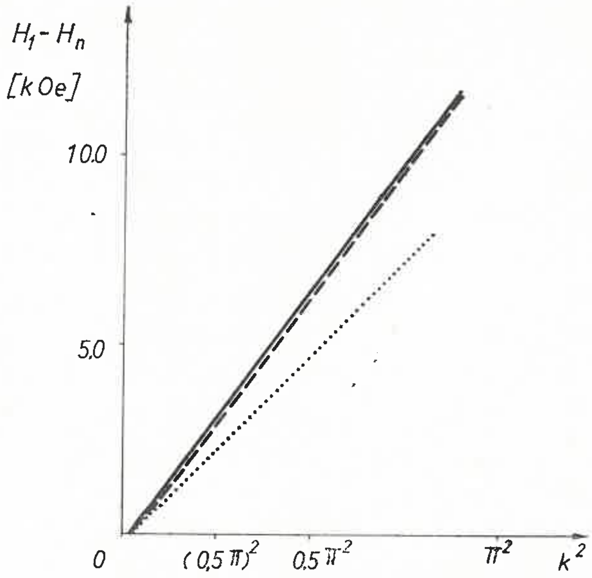

Fig. 6. Separations between the main res onance peak and other peaks vs k^2 at reduced temperatures equal to a) 0.0 (solid curve), b) 0.3 (dashed curve), c) 0.7 (dotted curve). Data as in Fig. 5

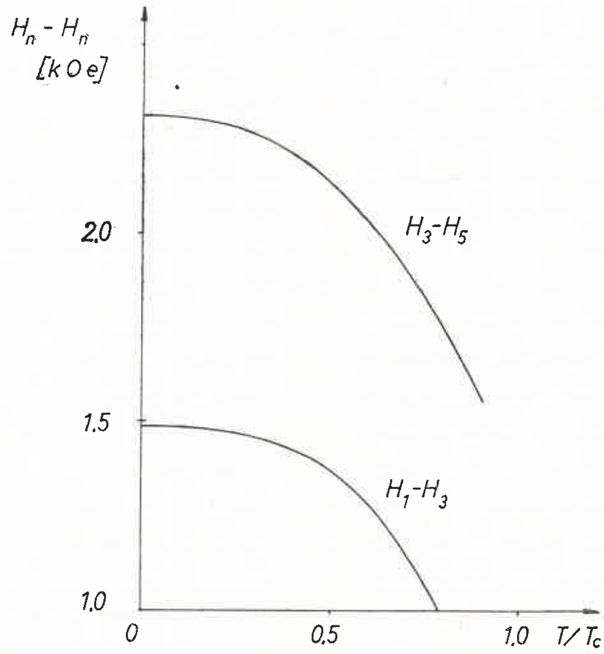

Fig. 7. Temperature dependence of two successive resonance peaks. Data as in Fig. 5

The positions of the individual resonance peaks (in d. c. magnetic field scale) depend on temperature and are given as:

$$H_n(T) = \frac{\hbar\omega}{\mu g} - H_{an}(0)\sigma^9 + 4\pi M_s(0)\sigma - \frac{2JS}{\mu g}\alpha(T)(1-\cos k_n). \tag{29}$$

We assume as the main peak the resonance line corresponding to the space mode with the smallest wave vector. The separation between the main resonance peak and the other peaks decreases with increasing temperature as follows

$$H_1(T) - H_n(T) = \frac{2JS}{\mu g}\alpha(T)(\cos k_1 - \cos k_n) \tag{30}$$

or

$$H_1(T) - H_n(T) = \frac{JS}{\mu g}\alpha(T)[k_n^2(T) - k_1^2(t)]. \tag{31}$$

This dependence is shown in Fig. 6.

Likewise, we obtain the peak separations

$$H_n(T) - H_{n'}(T) = \frac{JS}{\mu g}\alpha(T)[k_{n'}^2(T) - k_n^2(T)]. \tag{32}$$

Fig. 7. shows the temperature dependence of successive resonance peaks in two different regions of the SWR spectrum.

5. *Conclusions*

As the temperature increases the spectrum of spin wave resonance becomes more and more compact. In experiment, the lines coalesce into one as T approaches the Curie point. The peak separations decrease proportionally to the temperature renormalization coefficient $\alpha(T)$ of spin wave energy. This dependence is modified by the temperature dependence of the allowed wave vector values, which is different for different regions of the Brillouin zone. The line intensities decrease proportionally to $\beta^2(T)$.

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