

## ZEEMAN EFFECT IN THE MULTIPOLE LINES OF ANTIMONY

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Relative intensities of the  $\Delta M = \pm 1$  and  $\Delta M = 0, \pm 2$  transitions of the hyperfine components of  $\lambda = 541.5$  nm and  $\lambda = 609.8$  nm antimony lines are calculated as functions of the external magnetic field for the longitudinal  $L$  and transverse  $\sigma, \pi$  directions of observation. The comparison of the computed and experimental intensities is considered to be satisfactory.

## 1. Introduction

Multipole lines permitted for second order radiation can be magnetic dipole (M1), electric quadrupole (E2) or mixed type *i.e.* having the character of both M1 and E2 lines. The Zeeman effect of mixed multipole lines was investigated theoretically by Milańczuk [1], Gerjuoy [2], Shortley *et al.* [3] and by Dembiński *et al.* [4]. Milańczuk [1] and independently Shortley *et al.* [3] have predicted the existence of an interference phenomenon between the M1 and E2 radiations in  $\Delta M = \pm 1$  transitions. This effect was observed experimentally by Jenkins and Mrozowski [5], by Hults [6] and by Heldt and Mrozowski [7] in the mixed multipole lines of lead PbI, PbII and bismuth BiI. Recently Heldt and Hults [8] performed corresponding experiments on the  $\lambda = 541.5$  nm ( $^2P_{3/2}^0 \rightarrow ^4S_{3/2}^0$ ) and  $\lambda = 609.8$  nm ( $^2P_{1/2}^0 \rightarrow ^4S_{3/2}^0$ ) lines of antimony. These lines resulted from transitions between levels of  $5s^2 5p^3$  electron configuration are of mixed type. The contribution of E2 radiation, calculated by Garstang [10], is  $1.5 \times 10^{-3}\%$  and  $1.43\%$  in the  $\lambda = 541.5$  nm and  $609.8$  nm lines respectively.

The purpose of this work was to perform calculations of relative intensities of  $\Delta M = \pm 1$  and  $\Delta M = 0, \pm 2$  Zeeman components for two directions of observation (transverse  $\sigma, \pi$  and longitudinal  $L$ ) using the value of the electric quadrupole admixture as estimated by Garstang [10]. For numerical computations the formulae for the relative intensities for the transitions in question were adapted from [4].

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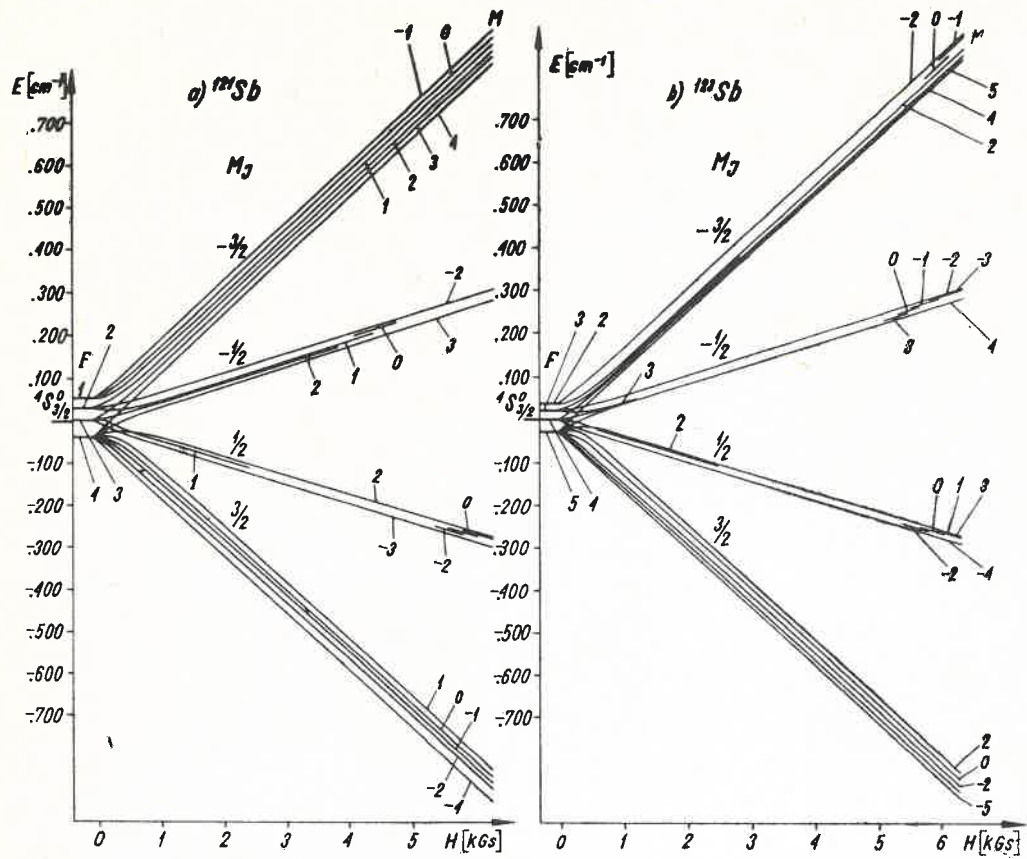


Fig. 1a

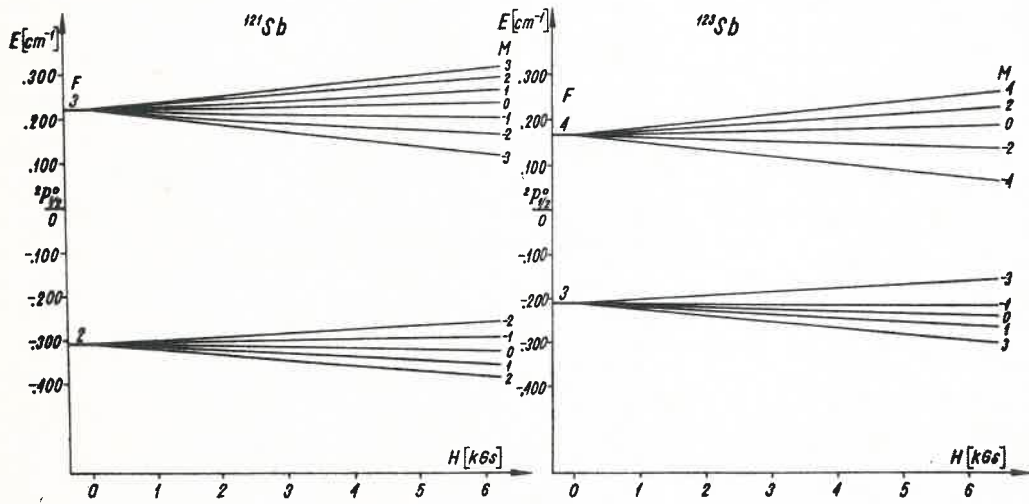


Fig. 1b

## 2. Method of calculation

For a line of mixed type the transition probability amplitude is given by

$$\langle a | \mathcal{H}^{\text{int}} | b \rangle = (2\pi\hbar)^{1/2} \{ \omega^{1/2} \langle a | e^j \hat{Q} | b \rangle + i c \omega^{-1/2} \langle a | \mathbf{M}(\mathbf{k} \times \mathbf{e}^j) | b \rangle \} \quad (1)$$

where  $a, b$  are states belonging to the same electron configuration,  $\mathbf{k}$  is the wave vector and  $\mathbf{M} = e(\mathbf{L} + 2\mathbf{S})/2mc$ ,  $\hat{Q} = e\mathbf{r} \otimes \mathbf{r}$  are the operators of the magnetic dipole and electric quadrupole moments of the atom, respectively. Other symbols have their standard meanings. The spontaneous emission transition probability which equals the square of the amplitude is given by

$$|\langle a | \mathcal{H}^{\text{int}} | b \rangle|^2 = 2\pi\hbar \sum_j \left\{ \frac{\omega}{4} |(e^j \hat{Q} \mathbf{k})_{ab}|^2 + c^2 \omega^{-1} |(\mathbf{M}(\mathbf{k} \times \mathbf{e}^j))_{ab}|^2 + 2 \operatorname{Im} \frac{c}{2} (\mathbf{M}(\mathbf{k} \times \mathbf{e}^j))_{ab} (e^j \hat{Q} \mathbf{k})_{ab} \right\}. \quad (2)$$

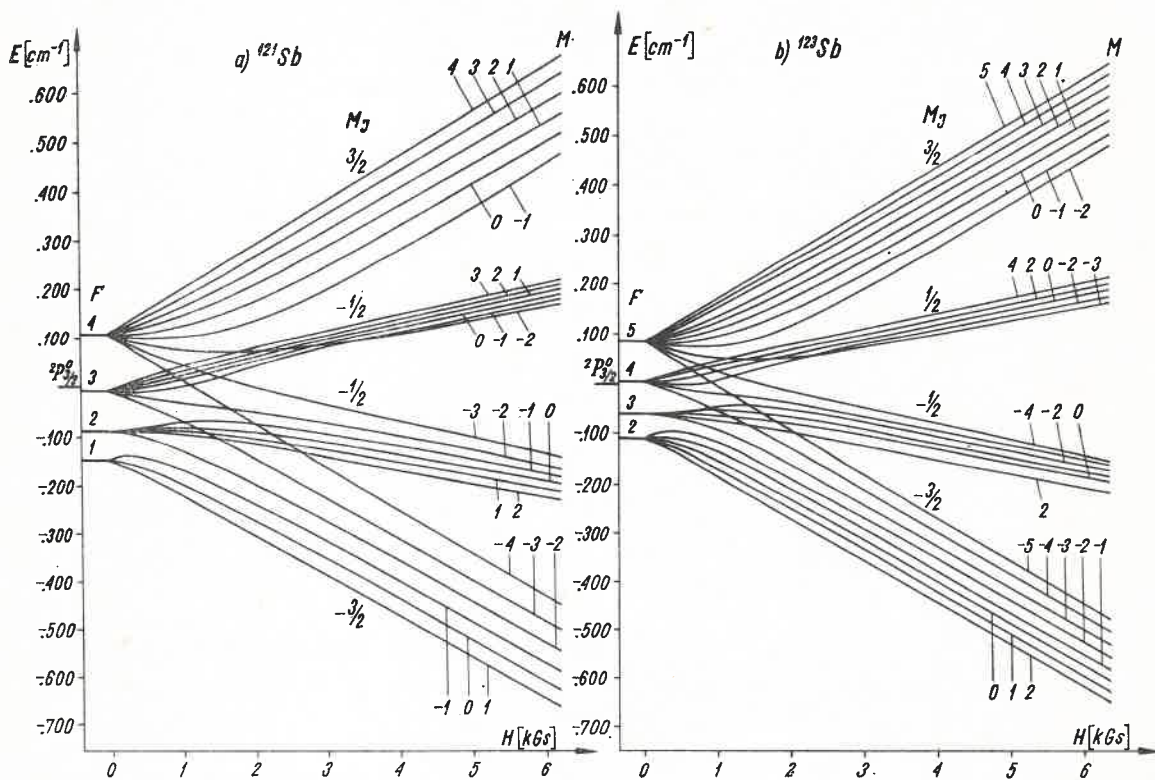


Fig. 1c

Fig. 1. The Breit-Rabi diagram of a)  $4S_{3/2}^0$ , b)  $2P_{1/2}^0$ , c)  $2P_{3/2}^0$  electronic states of  $^{121}\text{Sb}$  and  $^{123}\text{Sb}$ . In order to avoid overcrowding some Zeeman sublevels are omitted in the figures

The expression in Eq. (2) is summed over two independent polarization vectors  $e^j$  of the emitted photon. In the presence of an external magnetic field the resultant Hamiltonian has the form

$$\mathcal{H} = \mathcal{H}_{\text{hfs}} + \mathcal{H}_{\text{mag}} = \mathcal{H}_{\text{hfs}} - \mu_B H - \mu_I H. \quad (3)$$

The nuclear contribution to the Hamiltonian (3) has been omitted as being negligibly small. In the magnetic field only  $M, I, J$  remain good quantum numbers. For this reason

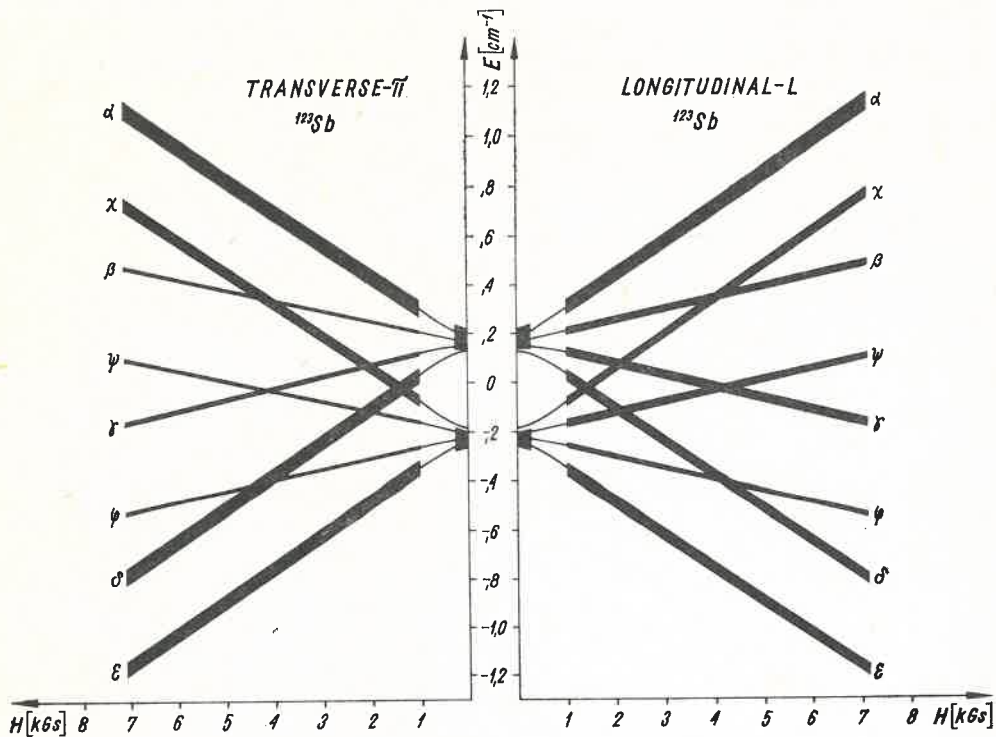


Fig. 2. Calculated relative intensities of the Zeeman patterns  $\Delta M = \pm 1$  for the 609.8 nm line of  $^{123}\text{Sb}$  in two directions: longitudinal  $L$  and transverse  $\pi$ , as a function of the magnetic field from 1.0 to 7.0 kGs

the calculations have been carried out in a representation with atomic eigenstates expressed as a linear combination of the form

$$|nM\rangle = \sum_F c_{MF}^n |\gamma I J F M\rangle. \quad (4)$$

The coefficients  $c_{MF}^n$  and the energies of Zeeman hfs levels  $E_M^n$  are solutions of the set of linear equations

$$\sum_F c_{MF}^n (\mathcal{H}_{F'M,FM} - \delta_{FF'} E_M^n) = 0. \quad (5)$$

The absolute intensity of a line corresponding to the transition  $b \rightarrow a$  for a given direction

$r$  can be represented as (see Dembiński *et al.* [4])

$$J_{ab}^r = R_1^2 I_{Qab}^r + R_2^2 I_{Dab}^r + R_1 R_2 I_{QDab}^r \quad (6)$$

where  $I_{Qab}^r$ ,  $I_{Dab}^r$ ,  $I_{QDab}^r$  are combinations of the Clebsch-Gordan, Racah and  $c_{MF}^n$  coefficients.  $R_1$  and  $R_2$  depend only on the quantum numbers  $\gamma$ ,  $J$ ,  $J'$  and energy difference  $\omega$ .

The relative intensity of the line corresponding to a transition  $nM \rightarrow n'M'$  is given by

$$J_{\text{rel. } ab}^r = \frac{J_{ab}^r}{\sum_{nM, n'M'} J_{ab}^r} \quad (7)$$

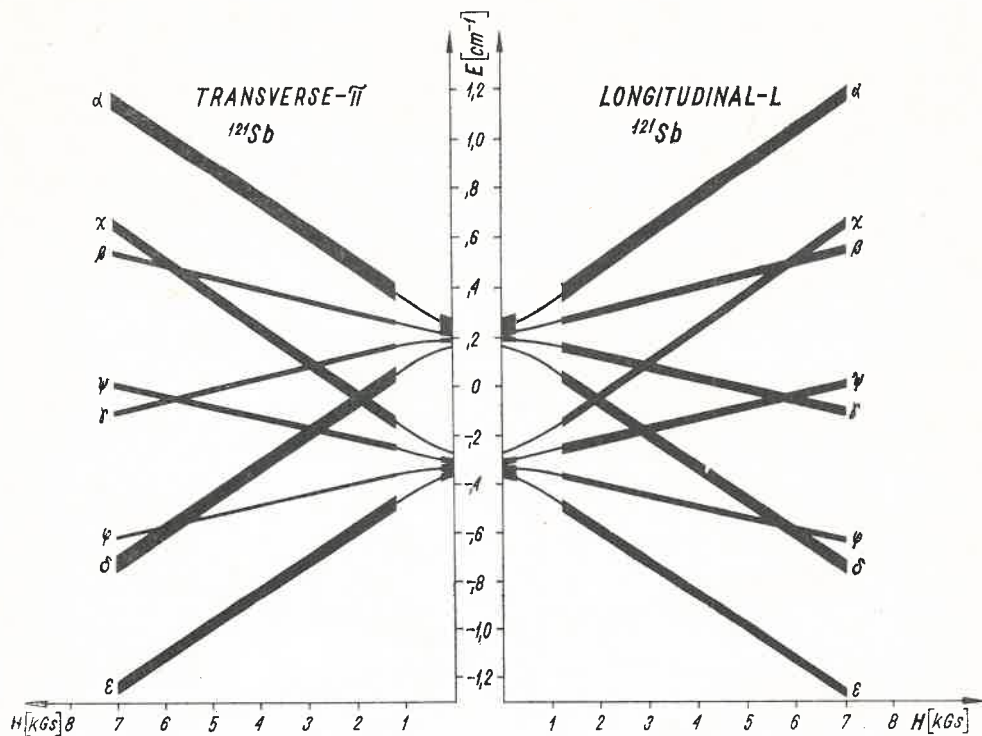


Fig. 3. Calculated relative intensities of the Zeeman patterns  $\Delta M = \pm 1$  for the 609.8 nm line of  $^{121}\text{Sb}$  in two directions: longitudinal  $L$  and transverse  $\pi$ , as a function of the magnetic field from 1.2 to 7.0 kGs

The relative electric quadrupole admixture, expressed in percents, in a transition  $nM \rightarrow n'M'$  is

$$Y = 100\% \frac{R^2 \sum_{nM, n'M'} I_{Qab}^r}{\sum_{nM, n'M'} J_{ab}^r} \quad (8)$$

Eq. (8) can be used for two purposes:

- for calculating the  $E2$  admixture knowing the relative intensity of the lines
- for calculating relative intensities of a line knowing the  $Y$  value.

TABLE I

Interaction constants for the  $4S_{3/2}^0$ ,  $2P_{1/2}^0$ ,  $2P_{3/2}^0$  states of  $5p^3$  configuration of  $^{121}\text{Sb}$  and  $^{123}\text{Sb}$ 

		$4S_{3/2}^0$	$2P_{1/2}^0$	$2P_{3/2}^0$
$^{121}\text{Sb}$	$A \times 10^{-3}\text{cm}^{-1}$	-9.974	163.0	28.06
	$B \times 10^{-3}\text{cm}^{-1}$	-0.123		
	$g_J$	1.971	0.688	1.277
$^{123}\text{Sb}$	$A \times 10^{-3}\text{cm}^{-1}$	-5.418	95.0	16.16
	$B \times 10^{-3}\text{cm}^{-1}$	0.155		
	$g_J$	1.971	0.688	1.277

The constants appearing in the hfs Hamiltonian, Eq. (3), determined from data concerning atomic beam resonance [11] and optical hfs measurements [12a, b] for the two isotopes of antimony, are collected in Table I. The calculated eigenvalues of the Hamiltonian, Eq. (5) as a function of the magnetic field for the  $4S_{3/2}^0$ ,  $2P_{1/2}^0$ ,  $2P_{3/2}^0$  states of two isotopes of SbI are shown in Fig. 1a, b, c. The Zeeman splitting of the hfs levels of the  $2P_{1/2}^0$  state is much smaller than that in the  $4S_{3/2}^0$  and  $2P_{3/2}^0$  states. This is due to the

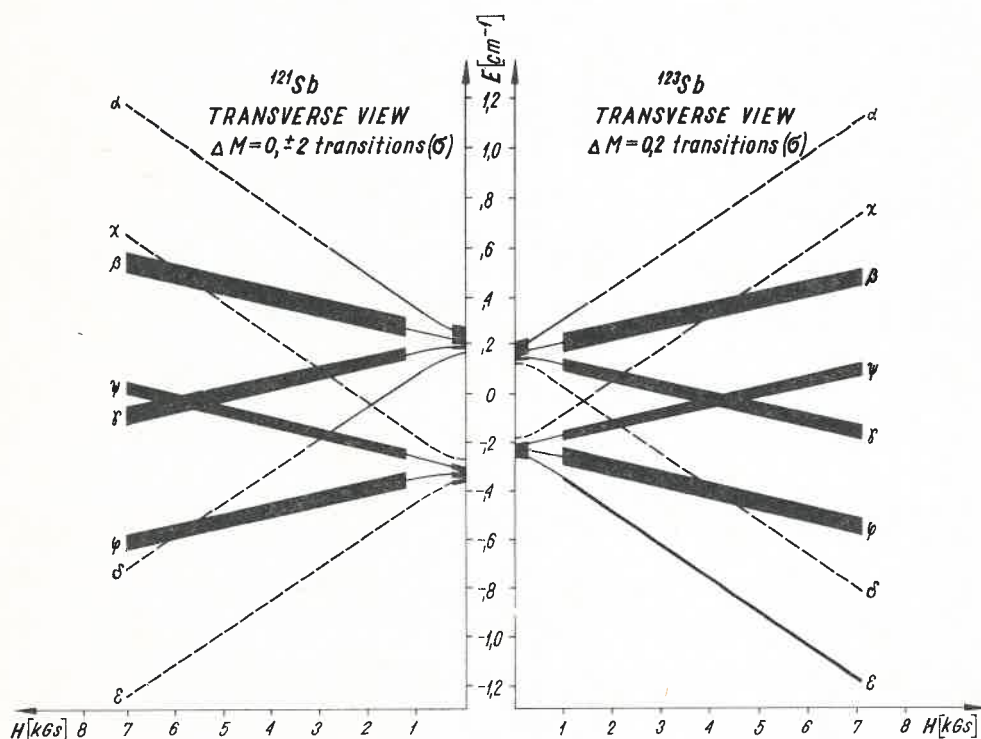


Fig. 4. Calculated relative intensities of the Zeeman patterns  $\Delta M = 0, \pm 2$  for the 609.8 nm line of a)  $^{121}\text{Sb}$ , b)  $^{123}\text{Sb}$  as a function of the magnetic field. Intensities smaller than 1.0% are marked by broken lines

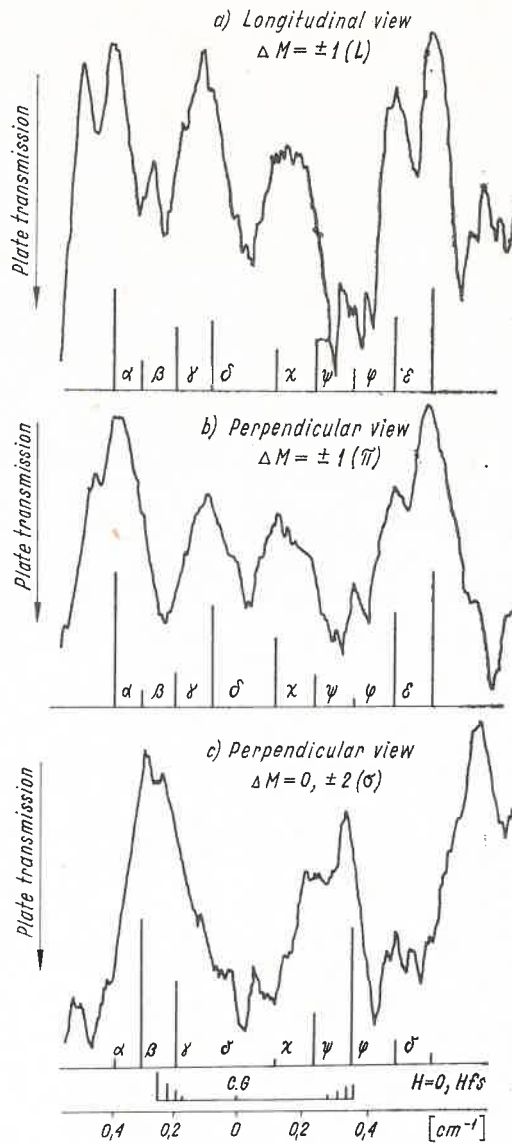


Fig. 5. Microphotometer traces showing the Zeeman patterns of the 609.8 nm line of  $^{121}\text{Sb}$  at magnetic field 1.2 kGs in a) longitudinal  $L$ , b) transverse  $\pi$  and c) transverse  $\sigma$  directions for a  $t = 5.17$  nm etalon separator. Theoretical relative intensities are indicated below. The zero-field hfs is given in the lower parts of the figures

large hyperfine splitting of the  $^2P_{1/2}^0$  state and to the difference in  $g_J$  factors. Since the transitions were investigated for magnetic field up to 7 kGs, one can ignore the interaction of the hfs levels of  $^2P_{1/2}^0$  state and corresponding coefficients can be put as  $c_{MF}^n = \delta_{nF}$ . In the  $^4S_{3/2}^0$  and  $^2P_{3/2}^0$  states, however, interaction of levels corresponding to different values  $F$  is strong and the coefficients  $c_{MF}^n$  must be obtained numerically from Eq. (5).

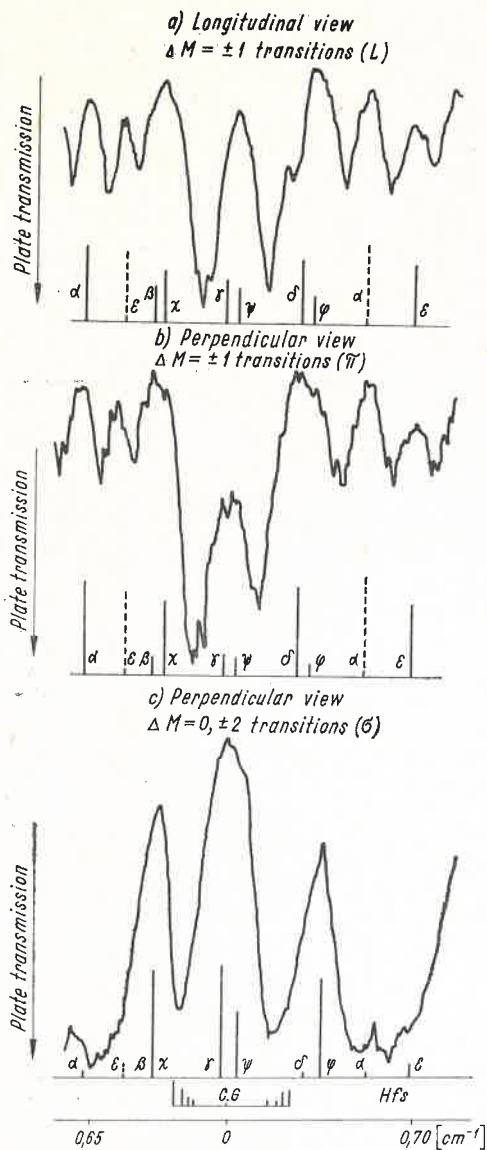


Fig. 6. Microphotometer traces showing the Zeeman patterns of the 609.8 nm line of  $^{123}\text{Sb}$  at magnetic field 3.5 kGs in a) longitudinal  $L$ , b) transverse  $\pi$  and c) transverse  $\sigma$  directions for a  $t = 4.25$  mm etalon separator. Theoretical relative intensities are indicated below. The zero-field hfs is given in the lower parts of the figures

A. The line 609.8 nm ( $^2P_{1/2}^0 \rightarrow ^4S_{3/2}^0$ )

The  $\lambda = 609.8$  nm line is composed of two narrow groups each of four hfs components separated by a large distance which corresponds to the hyperfine splitting of the upper level  $^2P_{1/2}^0$ . In the range above the Goudsmit-Back effect for  $^4S_{3/2}^0$  state, *i. e.* for fields higher than







1 kGs, the transitions between Zeeman sublevels should be grouped into eight components. The resolving power of a Steinheil GS glass spectrograph and silver coated Fabry-Perot etalon was not sufficient to resolve individual transitions. Therefore we computed only the relative intensities of the eight groups of transitions. The grouped transitions, called by us components, are designated by Greek letters. The letter designation is the same as in [4] and [13] where a complete level diagram is given for transitions of the type

TABLE IV

Calculated relative intensities of the  $\Delta M = 0$  transitions of the line 541.5 nm of  $^{123}\text{Sb}$  at various magnetic fields

$H$ [kGs]	1.00	1.20	1.50	1.75	2.00	2.50	3.00	3.50	3.75	4.00
$E[\text{cm}^{-1}]$										
-1.0									0.2	1.8
-0.9								1.9	1.6	0.0
-0.8							2.1	0.0	0.0	0.0
-0.7						2.3	0.0	8.0	8.3	8.8
-0.6					1.2	0.0	7.5	0.6	1.1	0.9
-0.5			1.6	2.8	1.3	7.3	1.0	0.8	1.2	6.1
-0.4	1.4	3.7	1.4	6.1	6.6	0.6	3.0	8.6	12.4	11.8
-0.3	6.6	4.9	5.8	0.3	2.9	16.4	10.2	15.8	10.0	6.7
-0.2	8.9	15.8	23.9	25.8	23.3	10.8	4.7	3.3	3.5	4.0
-0.1	45.1	33.7	23.5	21.0	19.5	12.7	8.8	8.3	7.9	7.6
0.1	15.8	16.8	13.6	12.2	11.7	14.6	14.8	12.7	12.3	11.7
0.2	16.0	16.9	12.8	11.0	8.5	10.9	2.5	3.8	3.8	3.6
0.3	6.2	6.7	14.8	18.9	10.5	5.6	10.7	11.3	11.5	5.2
0.4			2.0	1.2	12.0	7.5	13.3	5.8	0.5	7.0
0.5				0.3	0.8	10.3	10.4	8.1	14.2	14.8
0.6						0.6	0.0	9.2	0.9	1.1
0.7							0.5	0.0	8.4	8.1
0.8								0.5	0.3	0.0
0.9									0.2	0.4
1.0										0.1

$J = 3/2 \rightarrow J = 1/2$ . The relative intensities of each component observed in longitudinal  $L$  and transverse  $\pi$  and  $\sigma$  directions were calculated for  $Y = 1.43\%$  and for various magnetic fields with the step of 250 Gs using an Odra 1204 computer. The results are presented in Table II for  $^{121}\text{Sb}$  and in Table III for  $^{123}\text{Sb}$ , and graphically in Figs 2 and 3. The hyperfine splitting in the absence of a magnetic field is given in centre of the figures according to [11] and [12]. The width of a band represents the intensity of components  $\alpha \dots \epsilon$ . Relative intensities of transitions between Zeeman sublevels were not computed for fields where crossing of the energy occurs (0-1 kGs). The asymmetry in Figs 2 and 3 confirm the presence of the interference phenomenon in a mixed multipole transition  $^2P_{1/2}^0 \rightarrow ^4S_{3/2}^0$  of  $^{121}\text{Sb}$  and  $^{123}\text{Sb}$ . In Figs 5 and 6 we give the microphotometer traces of the observed structure of Zeeman patterns of the line obtained by Heldt and Hults for magnetic fields 1.2 and

3.5 kGs. In order to compare the observed structure of these patterns with theory, we give theoretical relative intensities and calculated positions of the components in the lower part of the figures. The agreement of the calculated and observed patterns seems to be satisfactory. It shows that a 1.43% content of E2 radiation for the line 609.8 nm of SbI, as calculated by Garstang, gives a good coincidence.

TABLE V  
Calculated relative intensities of the  $\Delta M = \pm 1$  transitions of the line 541.5 nm of  $^{123}\text{Sb}$  at various magnetic fields

$E[\text{cm}^{-1}] \backslash H[\text{kGs}]$	1.00	1.20	1.50	1.75	2.00	2.50	3.00	3.50	3.75	4.0
-1.1										0.5
-1.0									3.0	2.6
-0.9								3.0	0.1	0.0
-0.8							3.0	0.0	0.0	2.5
-0.7						3.0	0.0	6.4	6.3	9.6
-0.6					3.0	0.0	6.7	10.0	11.1	6.6
-0.5			3.0	3.0	0.0	12.9	11.1	5.1	4.3	5.1
-0.4	3.1	3.1	0.1	13.6	17.4	7.0	5.5	6.4	7.4	6.7
-0.3	15.5	17.5	20.1	7.8	4.7	9.9	10.4	6.3	5.7	6.2
-0.2	11.3	11.4	15.4	14.9	15.3	8.9	7.5	9.9	11.3	11.4
-0.1	22.8	19.4	14.4	15.9	15.5	15.0	12.9	10.4	7.9	7.5
0.1	25.6	25.2	15.8	12.1	11.4	8.2	7.2	6.5	6.2	6.1
0.2	20.0	20.4	20.6	20.7	19.0	15.8	5.1	4.1	3.7	3.3
0.3	3.1	4.5	7.6	7.7	6.1	8.9	15.3	11.8	12.9	6.2
0.4		0.3	3.0	3.2	3.8	3.1	7.2	6.5	5.3	8.9
0.5				0.9	3.5	2.8	2.6	5.0	5.9	9.2
0.6						3.5	0.1	2.6	0.2	0.6
0.7							3.9	0.0	2.4	2.4
0.8							0.3	3.4	1.1	0.0
0.9								0.8	3.0	3.3
1.0										0.9

### B. The line 541.5 nm $^2P_{3/2}^0 \rightarrow ^4S_{3/2}^0$

The second multipole line of the SbI spectrum, the line 541.5 nm, according to Garstang's calculations [10] has a very small quadrupole admixture  $1.5 \times 10^{-3}\%$ . Since there is no difference of relative intensities of the  $\Delta M = \pm 1$  hyperfine Zeeman patterns in longitudinal and transverse directions, we have neglected the E2 admixture in our calculations. We have computed the intensities of all permitted transitions between Zeeman compounds of hfs levels and put them in order with regard to transition energy. Then we have divided the whole region into parts of width  $\Delta E = 0.1 \text{ cm}^{-1}$  and summed all the intensities of transitions within these parts. Such a procedure gives the general outline of Zeeman patterns as a block diagram in which the height of a rectangle is equal to the sum of intensities of transitions within the range  $E, E + \Delta E$ . Relative intensi-

ties of  $\Delta M = 0, \pm 1$  transitions of  $^{123}\text{Sb}$  for various magnetic fields are collected in Table IV and V.

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