THE IONIZATION EFFECTS IN PLASMA

By B. Grabowski

Department of Experimental Physics, Higher Pedagogical School, Opole*

(Received October 10, 1972, Revised paper received May 3, 1973)

In this paper two effects are discussed: 1) line merging at the series limit, and 2) lowering of the ionization energy in plasma. The first problem is considered on the basis of the Mozer and Baranger (1960) field-strength distribution functions. We obtained (in the Inglis-Teller model) a relation useful for determining electron density. When passing to the Holtsmark distribution, our relation reduces to the Griem (1964) one. The second problem is considered from the standpoint of the tunnel effect. Comparing a) spontaneous and b) tunnel depopulation of atomic levels we obtained the relation between external electric field-strength (or electron density) and the principal quantum number of the last quantized level of the hydrogen atom.

1. Introduction

The spectroscopic diagnostics of an optically thin layer can be performed on the basis of the connections between the emission (or absorption) coefficient in the lines and variables describing the state of plasma. Especially, the electron density can be determined in a simple way, without exact knowledge of the temperature and without the necessity of accepting any assumption regarding LTE. The hydrogen lines, the half widths $\Delta\lambda_S$ of which strongly depend upon the electron density N_e (as a consequence of the Stark effect), can be treated as "barometric lines" of plasma. The quantitative dependence can be written in the form (Griem 1964)

$$N_e = C(N_e, T)\Delta\lambda_S^{3/2}, \tag{1}$$

where $C(N_e, T)$ is a coefficient very weakly dependent on N_e and T. This relation, according to the considered line, holds with high accuracy (within 5 to 15 per cent) in a wide range $10^{14} < N_e < 10^{17}$ cm^{-3} and almost without any influence of temperature T (in the 5000 to 40000 K range) (see also: Wiese 1965 and Lochte-Holtgreven 1968).

In optically thick layers the linear dependence between the number of atoms producing the given line and the ordinate of its profile breaks down. This dependence fails first of all

* Address: Katedra Fizyki Doświadczalnej, Wyższa Szkoła Pedagogiczna, Oleska 48, 45-052 Opole, Poland.

in the core of the line. Owing to the variability of the number of participating atoms in the region within the profile of the line, there is no relation between the measured line width and $\Delta\lambda_S$.

The great optical depth and inhomogeneity of stellar atmospheres are the main difficulties encountered when analyzing stellar spectra. Furthermore, there are some difficulties in applying the Saha law to the determination of the mean electron density in sources of low temperatures (of the order of 5000 K). In these cases an error of the order of 10 per cent in the determination of the temperature of optically thick layers causes an error of the factor of 10 in the obtained electron density (*cf.* Grabowski 1973). Unsöld's method (Unsöld 1955), in its original version, is based on Holtsmark's absorption coefficient; in a more general version (Grabowski 1969) the wings of the lines are described by means of the Kolb-Griem approximation. Unsöld's method is not, however, a "self-sufficient" one, because it requires knowledge of the number of atoms excited to the second quantum level "above 1 cm^2 of the stellar atmosphere". This last quantity can be established with the assumption — especially controversial in the case of stellar spectra with very broadened hydrogen lines — of small optical thickness of the considered layer. Only the theories of the disappearance of spectral lines at the series limit can be transferred to the optically thick layers without introducing any significant ambiguity.

In the range of energy levels corresponding to the spectral region at the series limit, processes take place which are decisive for two effects:

1) the spectrally observed shift of the series limit (mainly due to the Stark effect), and

2) the depression of the ionization energy.

(This last effect, important *e. g.*, when calculating the partition functions, is not easily "readable" from the observations.)

2. The line merging at the series limit

Before the series limit of the hydrogen-like atoms is reached, a quasi-continuum is caused by the overlapping wings of the neighbouring lines. The highest principal quantum number n^*, above which the quasi-continuum begins, can be approximately determined in two ways:

a) by comparing the distance between the neighbouring lines and the line-broadening caused by fluctuating intermolecular electric microfields (Pannekoek's model);

b) by comparing the separation (in terms of energy) between two successive levels and the static splitting of the given level in a homogeneous electric field (Inglis-Teller model).

2.1. Pannekoek's model

The quantitative dependence between the number of the last visible line in a series and the electron density was obtained for the first time by Pannekoek (1938) by means of the superposition of the profiles calculated (in Pannekoek's approximation, Pannekoek 1930)

for different electron densities. After some simplifications this relation can be written in the form (see also: Ivanov-Kholodny *et al.* 1961):

$$\log N_e \approx 22.2\text{–}6 \log n^*. \tag{2}$$

In several papers by Kurochka the influence of both the Stark and Doppler effects on the line merging is similarly considered. (Kurochka makes use of Griem's approximation for the line-profile, Griem 1960.) The essential result obtained by him is the following (Kurochka 1967):

$$\log N_e \approx 22\text{–}7 \log n^*. \tag{3}$$

Vidal (1966) employs for the line-profile the quasi-static approximation. On the basis of the Mozer and Baranger (1960) distribution of microfields (which takes into account both the screening of the field and the ion-ion correlation effects), Vidal introduces the envelope curves of the intensity distribution in the emission lines at the series limit as the datum more useful for determining electron density. The measurements of the ratio of the envelope curves can be precisely performed. However, this quantity is suitable for the electron density determination only in the case when the assumption of the optical thinness of the considered layer is rigorously satisfied. If this assumption is not fulfilled, the saturation in the center of the line (where the ratio of the envelope curves is measured) gives observational effects similar to that due to an increase of the electron density. Owing to the restrictions placed on the optical thickness of the layers and the emission spectra, Vidal's method is not useful for astrophysical purposes. Furthermore, this method — even for the optically thin layers — has not been sufficiently verified by experiments.

2.2. The Inglis-Teller model

The highest quantum number n^*, relating to the last visible spectral line, can be approximately obtained when the Stark splitting of the level is compared with the energy separation between two neighbouring levels of different principal quantum numbers:

$$\frac{3a_0 n^{*2}\langle F\rangle}{2Z} = \frac{Ze^2}{2a_0 n^{*3}}, \tag{4}$$

where Z is the effective ion charge (for a neutral atom $Z = 1$), and $\langle F\rangle$ represents the field averaged over the space fluctuations in the plasma.

Figure 1 shows the field-strength distribution functions $W_r(\beta)$ (Mozer and Baranger 1960) in the undimensional scale $\beta = F/F_0$, where $F_0 = e/\bar{\varrho}^2 = 1.25\times 10^{-9} N_e^{2/3}$, $\bar{\varrho}$ being the mean distance between the ions. The parameter of the represented curves is the shielding and the ion-ion correlation parameter

$$r = \bar{\varrho}/\varrho_D = 9.0\times 10^{-2} N_e^{1/6} T^{-1/2}. \tag{5}$$

Here, ϱ_D denotes the Debye radius.

The mean field $\langle F \rangle$ is determined from $\langle F \rangle = F_0 \langle \beta \rangle$, where it is most natural to define the mean value $\langle \beta \rangle$ as

$$\langle \beta \rangle_r = \int \beta W_r(\beta) d\beta / \int W_r(\beta) d\beta.$$

Inglis and Teller (1939), accepting *ad hoc* 1.4 as the Holtsmark mean value of β, obtained the known relation,

$$\log N_e = 22.96 - 7.5 \log n^* + 4.5 \log Z. \tag{6}$$

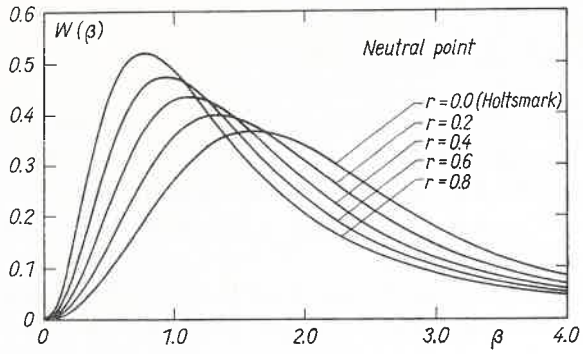

Fig. 1. The Mozer and Baranger (1960) field-strength distribution functions $W_r(\beta)$. The curve $W_{r=0}(\beta)$ covers with the Holtsmark distribution. (The drawing after Cowley 1970)

1 the basis of the Mozer and Baranger distribution functions $W_r(\beta)$ we obtain the approximate relations[1]:

$$\log \langle \beta \rangle_r = \underset{\pm .001}{0.41} - \underset{\pm .001}{0.19r} \quad (\text{for } Z = 1)$$

$$\log \langle \beta \rangle_r = \underset{\pm .005}{0.41} - \underset{\pm .10}{0.27r} \quad (\text{for } Z = 2)$$

for neutral and single ionized atoms, respectively, with errors that can be neglected. Substituting these mean values into (4) and taking into account (5) yields:

$$\log N_e = 22.57 - 7.5 \log n^* + 2.6 \times 10^{-2} N_e^{1/6} T^{-1/2}, \tag{7}$$

$$\log N_e = 23.92 - 7.5 \log n^* + 3.6 \times 10^{-2} N_e^{1/6} T^{-1/2}, \tag{8}$$

again for neutral and singly ionized atoms, respectively. The last terms, the values of which under typical conditions are of the order of 0.10 to 0.30, represent both the screening and ion-ion correlation effects.

By comparing the distance (in the energy scale) between successive high levels and the line width (in the Holtsmark approximation of Underhill and Waddel 1959) Griem (1964) obtained the following result:

$$\log N_e = 22.57 - 7.5 \log n^* + 4.5 \log Z. \tag{9}$$

[1] These results are found by numerical integration and the least square method.

For $Z = 1$ and $Z = 2$ this result is equivalent to the formulae (7) and (8), respectively, expect for the terms dependent on N_e and T.

The equations (7), (8) and (9) were received with different physical models (Griem's calculations rather correspond to Pannekoek's model). The agreement of these results seems to indicate that Eqs (7) to (9) are more physically justified than the earlier approximations. A similar conclusion arises from Fig. 2, in which the curves of log n^* *versus* log N_e are compared. This figure also contains points relating to Mohler's (1939) experimental

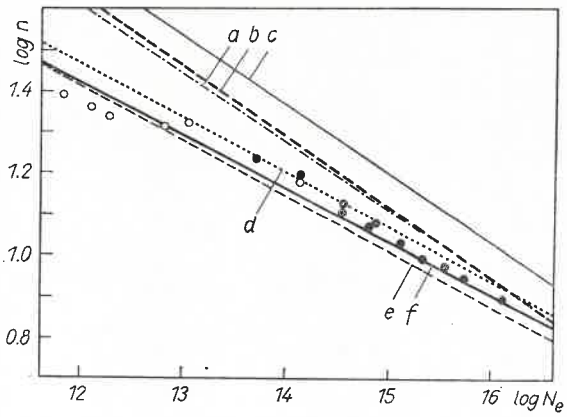

Fig. 2. The dependences of log n^* *versus* log N_e. Designations: c — the Pannekoek Eq. (2); d —Inglis-Teller Eq. (6); e — Griem Eq. (9); f — Eq. (7) being the result obtained in this paper. The Mohler's measurements are designated by circles. The curves b and a represent the tunnel effect: b — Eq. (22), c — Eq. (14)

measurements of the alkali spectra, made for a wide range of electron densities. The filled circles correspond to the measurements made under conditions of over 50 per cent ionization, whereas the open circles represent the measurements with low currents and low ionization. The measure of the degree of the importance of the last term in Eq. (7) (in the 3000 to 5000 K range, approximately corresponding to that of Mohler's experiments) is the discrepancy between the e (Eq. (9)) and f (Eq. (7)) curves.

The formulae (6) to (9) are obtained in quasi-static approximation for both ions and electrons (for comments see Griem's book, p. 126). They are applicable to lines originating from higher levels of the majority of atoms and ions (because in this case the Stark broadening always becomes hydrogen-like).

3. Depopulation of energy levels by the tunnel effect

The reduction of the ionization energy in an external electric field was first theoretically discussed by Robertson and Dewey (1928). They considered this problem from the standpoint of the classical quantization of the orbits in a homogeneous field. Oppenheimer (1928) gave a rough quantum-mechanical treatment of this problem. More exact calculations were made by Lanczos (1931). He calculated, in WKB approximation, the critical fields in which the extreme components of some lines of the Balmer series disappears

owing to the tunnel effect. This problem was recently considered by Ivanov-Kholodny *et al.* (1960).

Unsöld (1948), with the help of a simple nearest neighbour approximation, employed the classical quantized orbits method to plasma conditions. In this approximation the effective potential energy function $U(r)$ varies along the r-axis according to the formula:

$$U(r) = -e^2\left(\frac{1}{r} + \frac{1}{r_0 - r} - \frac{1}{r_0}\right). \tag{10}$$

(This axis is attached to the atomic nucleus and directed towards the perturbing proton placed at a distance of r_0; Unsöld assumes r_0 is equal to the mean distance between the ions.)

Between the atom and the perturber arises a potential barrier, the maximum height of which is given by $U(r_0/2)$. If the intra-atomic electron is in the n-th state, the total energy E_n of which is higher than $U(r_0/2)$, then it passes classically to the potential well of the perturber. From the condition $E_n^* = U(r_0/2)$ follows Unsöld's relation,

$$\log N_e = 21.88 - 6 \log n^*, \tag{11}$$

(n^* is the last quantized energy level) as well as an equivalent formula for the reduction of the ionization energy (in eV):

$$\Delta\chi = 6.96 \times 10^{-7} N_e^{1/3}. \tag{12}$$

The geometry of the field (10) is appropriate for one-dimensional motion. In Unsöld's method — relating to three-dimensional quantum system — this geometry was used for defining the maximum height of the potential barrier and, consequently, for performing the classical separation of periodic and of aperiodic orbits. Ivanov-Kholodny *et al.* (1960) employ this geometry in their considerations of the tunnel effect in higher states, *i.e.* the quantum effects, the realization of which occur in the time interval corresponding to 10^6 to 10^8 oscillations of the atomic electron.

In simple cases the consideration of the tunnel effect can be reduced to the one-dimensional Schrödinger equation; the necessary-condition is here the separability of the potential energy function relating to the space variables. We introduce the co-ordinate system attached to the atomic core with the z-axis directed towards the perturber. In these co-ordinates the potential energy function of the atom-perturber system has the simplest form:

$$U(x, y, z, X, Y, Z) = e^2[Z^{-1} - (x^2+y^2+z^2)^{-1/2} - \{x^2+y^2+(Z-z)^2\}^{-1/2}], \tag{13}$$

where the co-ordinate of the perturber are denoted by capital letters, and the co-ordinates of electron by small letters. This function is not separable in any orthogonal co-ordinate system[2]. Separation can be performed only when $x = y = 0$, *i.e.* in the particular case

[2] Exceptionally, the separation is possible in the spheroidal-prolated (eli psoidal-degenerated) co-ordinate system. However, this result is not valid along the z-axis (J. Halenka 1972, private information).

of one-dimensional motion along the z-axis. Then, the Eq. (13), after putting $z \to r$ and $Z \to r_0$, reduces to Eq. (10). Concluding, we state that acceptance of the field geometry (10) as suitable (from the point of view of the tunnel effect) for a three-dimensional quantum system is convincing rather by intuition than by the mathematical formalism.

Using the relation (10), Ivanov-Kholodny *et al.* (1960) obtained

$$\log N_e = 21.65 - 6 \log n^*. \tag{14}$$

This result was extended, without any sure basis, to the hydrogen atoms in plasma.

We suppose that the homogeneous electric field of strength F [cgs] is in the positive z-direction, so that the force acts on the electron in the negative z-direction. The motion

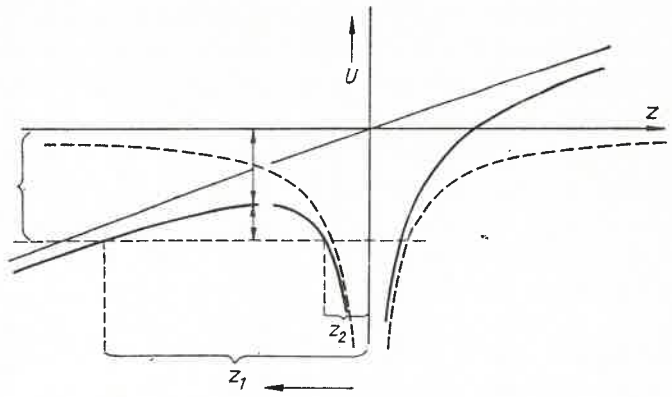

Fig. 3. The course of the potential energy function (15) (solid lines) along the z-axis. The arrow below shows the direction of the force acting on an electron

of the electron in the field of nucleus of charge $+e$ and in this external field is determined by the potential energy function

$$U = -e^2/r + eFz; \tag{15}$$

(z is the component of the distance r in the direction of the field; the changes of (15) along the z-axis is shown in Fig. 3). The Schrödinger equation, suitable for this case, reads

$$\frac{\hbar^2}{2\mu} \nabla^2 \Psi + (e^2/r - eFz + E)\Psi = 0, \tag{16}$$

where E is the total energy of the system and μ is reduced mass. This equation can be easily separated in a parabolic co-ordinate system. However, for our purpose the simplest description of the dynamics of the motion in the presence of the external homogeneous field can be obtained in the cylindrical co-ordinate system, where the perturbation is connected with the z co-ordinate only.

It is a well-known fact that in these co-ordinates the space variables cannot be fully separated[3]. After conventional separation of (16) with relation to φ, we get an equation dependent on ϱ and z. This equation, however, under the condition $\varrho \approx 0$ can be divided into two equations:

$$\frac{d^2R}{d\varrho^2} + \frac{1}{\varrho}\frac{dR}{d\varrho} + \left(\varepsilon_1 - \frac{m^2}{\varrho^2}\right)R = 0, \tag{17}$$

$$\frac{d^2\psi}{dz^2} + \frac{2\mu}{\hbar^2}\left(E_2 + \frac{e^2}{|z|} - eFz\right)\psi = 0, \quad (-\infty < z < +\infty), \tag{18}$$

where $\varepsilon_1 = \dfrac{2\mu E_1}{\hbar^2}$, $E_1 + E_2 = E$, and $\Phi(\varrho, z) = R(\varrho)\psi(z)$. Equation (17) gives the exact partial solution of Eq. (16) along the z-axis and in its surroundings only. From the standpoint of the tunnel effect this does not lessen the generality of the considerations, because the dynamics of the perturbation is exactly described by the Eq. (18); Eq. (17) is a "tacit spectator" only.

In Eqs (17) and (18) ε_1 and E_2 are the separation constants. Owing to the local character of Eq. (17), ε_1 cannot be established from the boundary conditions. This equation has two independent solutions; for $\varepsilon_1 \neq 0$ they are given by the Bessel functions. The first solution is the Bessel function of order m, namely, $J_m(\sqrt{\varepsilon_1}\varrho)$ or $I_m(\sqrt{\varepsilon_1}\varrho)$ for $\varepsilon_1 > 0$ and $\varepsilon_1 < 0$, respectively (see, *e.g.*, McLachlan 1955). The second solution, $Y_m(\sqrt{\varepsilon_1}\varrho)$ or $K_m(\sqrt{\varepsilon_1}\varrho)$, respectively, tends to $\pm\infty$ for $\varrho \to 0$ and, therefore, should be omitted. In particular, for $m = 0$ we get $J_0(0) = I_0(0) = 1$, independently of the sign of ε_1.

In the case of $\varepsilon_1 = 0$, Eq. (17) reduces to the Euler equation. As is well known, this last equation — after the substitution of a now variable t by means of the relation $\varrho = e^t$ — can be solved elementarily; its particular integrals are $R = e^{\pm mt} = \varrho^{\pm m}$. (The solution that tends to infinity for $\varrho \to 0$ — similarly as in the above case — should be omitted.) In particular, for $m = 0$, we get again along the z-axis $R(0) = 1$.

For further purposes it is more important to determine the separation constant E_2 in Eq. (18). This equation, after omitting the term $-eFz$, is formally equivalent to a radial equation

$$\frac{d^2\chi}{dr^2} + \left[\frac{2\mu}{\hbar^2}\left(E_n^0 + \frac{e^2}{r}\right) - \frac{l(l+1)}{r^2}\right]\chi = 0 \tag{19}$$

for the hydrogen atom in the state $l = 0$. (We notice that the values $E_n^0 = -\mu^2 Z^2 e^4/2\hbar^2 a_0^2$ follow from the condition imposed on the function $\chi(r)$ at $r \to +\infty$ (*cf.* Landau *et al.* 1948, § 36.)) The solution of Eq. (18) with $F = 0$ for $z \geqslant 0$ is identical to the solution of Eq. (19) for $l = 0$. Because the operator d^2/dz^2, and the potential energy function $-e^2/|z|$, are unvariable with respect to the reflection $z \to -z$, the above solution can be (exact

[3] Equation (16) without the Coulomb term is separable with respect to z, ϱ, φ and has a solution (Oppenheimer 1928) given in cylindrical functions.

to the factor ± 1) extended on the negative z-semi-axis (see, *e.g.*, Landau *et al.* 1948 § 19). Thus, for $F = 0$, we have $E_2 = E_n^0$. For $F \neq 0$ the perturbed eigenvalues $E_n' = E_n'(F)$ can be calculated by means of the standard perturbation theory, starting from the condition $E_n' \approx E_n^0$ when $F \approx 0$. For the hydrogen atoms we have the known results $E_n'(F) = E_{nl}'(F) = E_n^0 \pm \delta_{nl}(F)$ and $\langle E_{nl}' \rangle = E_n^0$ (the first-order Stark effect alterations of energy states, $\delta_{nl}(F)$, are all diminishing quantities due to the averaging over l).

Under plasma conditions the value E_n^0 represents the "centre of gravity" of the broadened n-th energy level and approximately of spectral line for which this level is the higher one. Thus, E_n^0 represents the mean feature of the whole line in any fields (or electron densities), except the extreme high fields of the order of 10^5 to 10^6 V/cm (because then the second-order Stark effect becomes important).

So, the consideration of the tunnel effect in cylindrical co-ordinate system is reduced to the consideration of one-dimensional motion described by equation

$$\frac{d^2\psi}{dz^2} + \frac{2\mu}{\hbar^2}\left(E_n^0 + \frac{e^2}{|z|} - eFz\right)\psi = 0, \quad (-\infty < z < +\infty). \tag{20}$$

This equation cannot be solved exactly. Its solution in WKB approximation is showed in Fig. 4. The upper part of this figure shows the potential energy function (15); schematically outlined below is the real part of $\psi(z)$. The solid lines represent the physically accep-

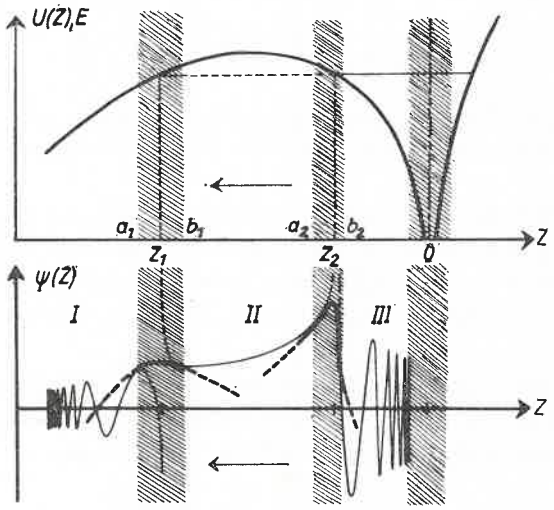

Fig. 4. The schematic discussion of the solution of Eq. (20). The arrows show direction of the force (see Fig. 3)

table solution of Eq. (20); the thin ones show the course of the asymptotic solution, the thick lines the locally exact solutions. (For a detailed discussion see book by Schiff or Landau and Lifshitz.) The hatched strips are transitional regions.

The coefficient of transparency of the barrier, D, is determined by the solutions of Eq. (20) in the regions I and III, and is given by (Landau-Lifshitz 1948, § 50):

$$D_n(F) = \exp\left[-\frac{2}{\hbar}\int_{z_1}^{z_2}\sqrt{2\mu(e^2/z+eFz-E_n^0)}\,dz\right], \quad e>0,\ z<0. \tag{21}$$

This result is only valid when: 1) in the whole region of the motion the field U satisfies the quasi-classical conditions, 2) the exponent in (21) is large, whereby D is small. In this paper, both these conditions — except in the surroundings of $z = 0$ — should rather be satisfied. Near $z \approx 0$ the region of applicability of the WKB approximation is restricted

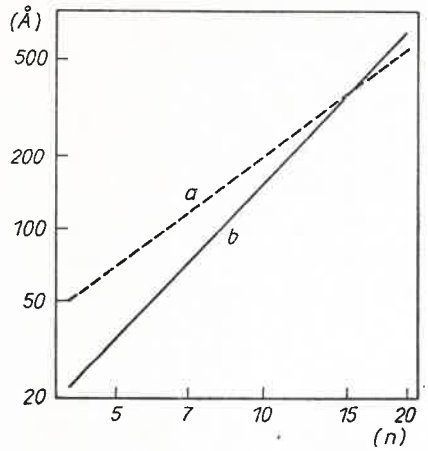

Fig. 5. The "thickness" of the barrier (curve *a*), and the distance of the "inside classical return point" z_2 from the atomic nucleus (curve *b*) *versus* the principal quantum number n

to distances larger than the Bohr "radius" a_0. From the Fig. 5 (curve *b*) we see that even in lower states, in relation to which this restriction is the sharpest, the distance of the "internal classical return points" are larger at least by a factor of 10. Similarly, owing to the large thickness of the barrier (Fig. 5, curve *a*), the asymptotic solution between the "return points" z_1 and z_2 could be used.

The probability (in s^{-1} units) of the penetration of the electron through the potential barrier is given by the product $D_n(F) \times \nu_n^*$, where ν_n^* is a frequency of quasi-classical oscillations of the atomic electron. For the n-th level of the hydrogen atom is

$$\nu_n^* = \frac{e}{2\pi a_0 n^3}\frac{1}{(\mu a_0)^{1/2}} = \frac{0.658\times 10^{16}}{n^3}\ [\mathrm{s}^{-1}].$$

Now, the probability A_n (in s^{-1} units) of the depopulation of the n-th level owing to spontaneous transitions to the lower k-th levels is given by $A_n = \sum_{k=1}^{n-1} A_{nk}$. (The values of A_{nk} were taken from the compilation by Wiese *et al.* 1966.)

The sharp dependence $D = D_n(F)$, such as for the 15-th level of the hydrogen atom, is shown in Fig. 6. For the determination of the critical field F_c, for which the tunnel depopulation is equal to the spontaneous one of a given level, the equation $D_n(F) \times \nu_n^* = A_n$ was solved numerically with respect to F. The electron densities N_e, appropriate for the

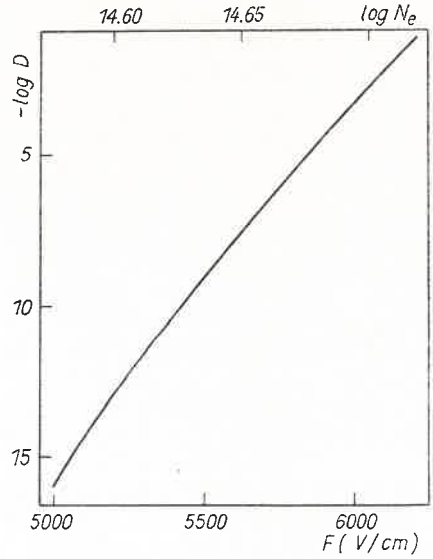

Fig. 6. The examplifying illustration (for $n = 15$) of the sharp dependence $D = D_n(F)$

values of F_c, can approximately be assigned by means of the relation $F_c = F_0\langle\beta\rangle_{r=0}$ or by the equivalent one,

$$\log N_e = 1.5 \log (F_c)_{\text{cgs}} + 12.74.$$

Between the values $\log N_e$ (or $\log F_c$) and $\log n^*$ there are the nearly linear relations

$$\begin{aligned} \log N_e &= 21.45 - 5.78 \log n^*, \\ &\pm.010 \quad \pm.010 \end{aligned} \tag{22}$$

$$\begin{aligned} \log (F_c)_{\text{V/cm}} &= 8.28 - 3.85 \log n^*, \\ &\pm.007 \quad \pm.006 \end{aligned} \tag{23}$$

which were obtained by the least square method. (All calculations in this paper were performed on an "Odra 1204" computer.) Equation (22) corresponds to the following depression of the ionization energy:

$$\log \Delta\chi_{\text{eV}} = 0.346 \log N_e - 6.29. \tag{24}$$

4. Conclusions

The relations (14) and (22) are compared in Fig. 2. We notice that these two results were obtained with the use of different physical models: Eq. (14) from an analysis of the one-dimensional motion of an electron in the fields of the atomic nucleus and of the

perturbing proton; Eq. (22) from an analysis of the three-dimensional motion in a homogeneous external electric field. The high similarity of these results permits us to presume that the model errors in both cases are not large, especially when the depression of the ionization energy is considered. It seems, also, that the best results (among the approximate methods of determining electron density) are given by the formulae (7) and (8).

REFERENCES

Cowley, C. R., *The Theory of Stellar Spectra*, Gordon and Breach Science Publishers, New York 1970.
Griem, H. R., *Astrophys. J.*, **132**, 883 (1960).
Griem, H. R., *Plasma Spectroscopy*, Mc Graw-Hill Book Co., New York 1964.
Grabowski, B., *Acta Astron.*, **19**, 23 (1969).
Grabowski, B., *Acta Astron.* (in preparation, 1973).
Inglis, D. R., Teller, E., *Astrophys. J.*, **90**, 439 (1939).
Ivanov-Kholodny, G. C., Nikolsky, G. M., Gulayev, R. A., *Astron. Zh.* (USSR), **37**, 799 (1960).
Ivanov-Kholodny, G. C., Nikolsky, G. M., *Astron. Zh.* (USSR), **38**, 455 (1961).
Kurochka, L. N., *Astron. Zh.* (USSR), **44**, 368 (1967).
Lanczos, C., *Z. Phys.*, **68**, 204 (1931).
Landau, L., Lifshitz, E., *Kvantovaya Mechanika*, Moscow 1948.
Lochte-Holtgreven, W., *Plasma Diagnostics*, Chapt. 3, ed. W. Lochte-Holtgreven, North-Holand Publ. Co., Amsterdam 1968.
McLachlan, N. W., *Bessel Functions for Engineers*, 2nd ed., Clarendon Press, Oxford 1955.
Mohler, F. L., *Astrophys. J.*, **90**, 429 (1939).
Mozer, B., Baranger, M., *Phys. Rev.*, **118**, 626 (1960).
Oppenheimer, J. R., *Phys. Rev.*, **31**, 66 (1928).
Pannekoek, A., *Monthly Not. Roy. Astron. Soc.*, **91**, 139 (1930).
Pannekoek, A., *Monthly Not. Roy. Astron. Soc.*, **98**, 694 (1938).
Robertson, H. P., Dewey, J. M., *Phys. Rev.*, **31**, 973 (1928).
Schiff, L. I., *Quantum Mechanics*, 2nd ed., McGraw-Hill Book Co., New York 1955.
Underhill, A. B., Waddell, J. A., *Nat. Bur. Stand. (USA), Circ.*, 603 (1959).
Unsöld, A., *Z. Astrophys.*, **24**, 355 (1948).
Unsöld, A., *Physik der Sternatmosphären*, 2nd ed., Springer-Verlag, Berlin 1955.
Vidal, C. R., *J. Quant. Spectrosc. Radiative Transfer*, **6**, 461 (1966).
Wiese, W. L., *Plasma Diagnostic Techniques*, Chapt. 6, ed. R. H. Huddlestone and S. L. Leonard, Academic Press, New York 1965.
Wiese, W. L., Schmidt, M. W., Glennon, B., *Atomic Transition Probabilities*, Vol. I, NSRDS-NBS 4, Washington 1966.