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EXCHANGE ENERGY OF BIAXIAL FILMS

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The dependence of exchange energy on the angle α between the magnetization vectors of the particular constituents of the film has been found for biaxial films. This dependence is E_A = const α^2 for the whole range of α where the constant is a function of the film thickness and the angle between the easy directions of the particular constituents.

1. Introduction

The knowledge of the dependence of exchange energy on the angle α between the magnetization vectors of multilayer systems in ferromagnetic thin films is of much use in the determination of the total interaction energy [1].

Goto and Yelon [2,3] postulated this dependence to be of the form $E_A = \text{const} \times \cos \alpha$, which in case of small angles (strong coupling) yields $E_A = \text{const} \times \alpha^2$. Lubecka and Maksymowicz [4] have found the asymptotic expression $E_A = \text{const} \times \alpha^{3/2}$ for small angles below 2° to be valid. The present paper gives the results of numerical calculations made for biaxial system with the angle $\varnothing$ between the easy axes equal to 90° nad 60° for arbitrary values of the angle α which depends on the magnetic field applied. The results obtained were: the relationship between E_A and α in the form E_A = const α^2, and the distribution of the magnetization vector in the film.

2. Calculation of the exchange energy

The calculations of the exchange energy have been made under the following assumptions. The z-component of the magnetizations vector $M_z = 0$, *i. e.* the magnetization vector lies in the film plane, there is no texture, the spacimen is policrystalline, the thicknesses of the two layers are d_1 and d_2, the uniaxial anisotropy constants are K_U^1 and K_U^2 and the two magnetizations are equal.

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In such a case the total energy density is given by:

$$\varepsilon = K_U g(\varphi) + A\left(\frac{d\varphi}{dz}\right)^2, \tag{1}$$

where the angle φ is defined as in Fig. 1.

The function $g(\varphi)$ is:

$$g(\varphi) = -2h\cos\varphi - 1/2\cos(\varnothing - 2\varphi)$$

and consist of a term which describes the uniaxial anisotropy energy density and another term which describes interaction with the field, K_U is the uniaxial anisotropy constant, and $K_U = K_U^1 = K_U^2$, h is the external reduced field $h = H/H_U$, $H_U = 2K_U/M$ and the magnetic field is directed along the x-axis, A is the exchange constant.

We assume that the magnetization distribution is uniform in arbitrary cross-section, *i. e.*, $\varphi = \varphi(z)$.

The Euler equation becomes the form:

$$\frac{K_U}{A}\frac{dg(\varphi)}{d\varphi} - 2\frac{d}{dz}\left(\frac{d\varphi}{dz}\right) = 0, \tag{2}$$

or, after transformation

$$\frac{K_U}{A}\frac{dg(\varphi)}{dz} - \frac{d}{dz}\left[\left(\frac{d\varphi}{dz}\right)^2\right] = 0. \tag{3}$$

After integration Eq. (3) we obtain

$$\frac{K_U}{A} g(\varphi) - \left(\frac{d\varphi}{dz}\right)^2 = \text{const.} \tag{4}$$

In the next stage we shall discuss the case in which the magnetic field is directed along the x-asis. Then $\varphi(-z) = -\varphi(z)$. Taking as boundary conditions $\left.\frac{d\varphi}{dz}\right|_{z=d} = 0$ where d is the thickness of the film, and substituting

$$\varphi(d) = \varphi_0 \text{ and } g(\varphi_0) = g_0$$

we obtain that the constant in Eq. (4) can be expressed in the form

$$\text{const} = \frac{K_U}{A} g_0.$$

Eq. (4) can be written in the form

$$\frac{d\varphi}{dz} = \left(\frac{K_U}{A}\right)^{1/2} [g(\varphi) - g_0]^{1/2} \tag{5}$$

and after substituting $\eta = \left(\frac{K_U}{A}\right)^{1/2} z$ and integration of (5) we obtain

$$\eta = \int_0^{\varphi} \frac{d\xi}{[g(\xi) - g_0]^{1/2}}, \tag{6}$$

The value of φ_0 can be determined from the relationship

$$\eta_0 = \int_0^{\varphi_0} \frac{d\xi}{[g(\xi) - g_0]^{1/2}} \tag{7}$$

where:

$$\eta_0 = \left(\frac{K_U}{A}\right)^{1/2} \cdot d.$$

Eq. (6) permits the distribution of the magnetization vector in the film to be determined, while from the expression (5) we can calculate the dependence of the exchange energy on the angle φ and further indirectly using the relationship

$$\langle \cos \varphi \rangle = \cos \alpha/2 \tag{8}$$

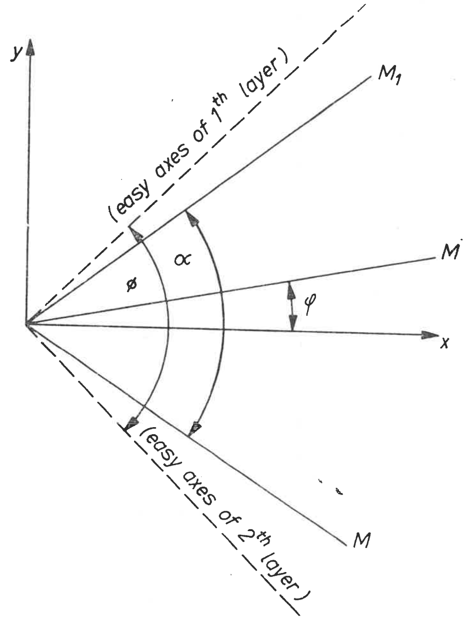


Fig. 1. The magnetization vectors in a thin film. The (x, y) plane coincides with the interface of both constituents: the x-axis in given by the direction of mean easy axis, the angle $\varphi = \varphi(z)$ determines the direction of local magnetization, and M_1, M_2 are the mean magnetizations of both constituents

we can find the dependence on the angle α. Zhe mean magnetization in the direction of the applied field (for field directions along the x-axis) is

$$\langle M \rangle = M_s \langle \cos \varphi \rangle. \tag{8a}$$

On the other hand $\alpha/2$ is the angle between the mean magnetization of one of the constituent of the double film and the x-axis (Fig. 1), and the resultant magnetization is

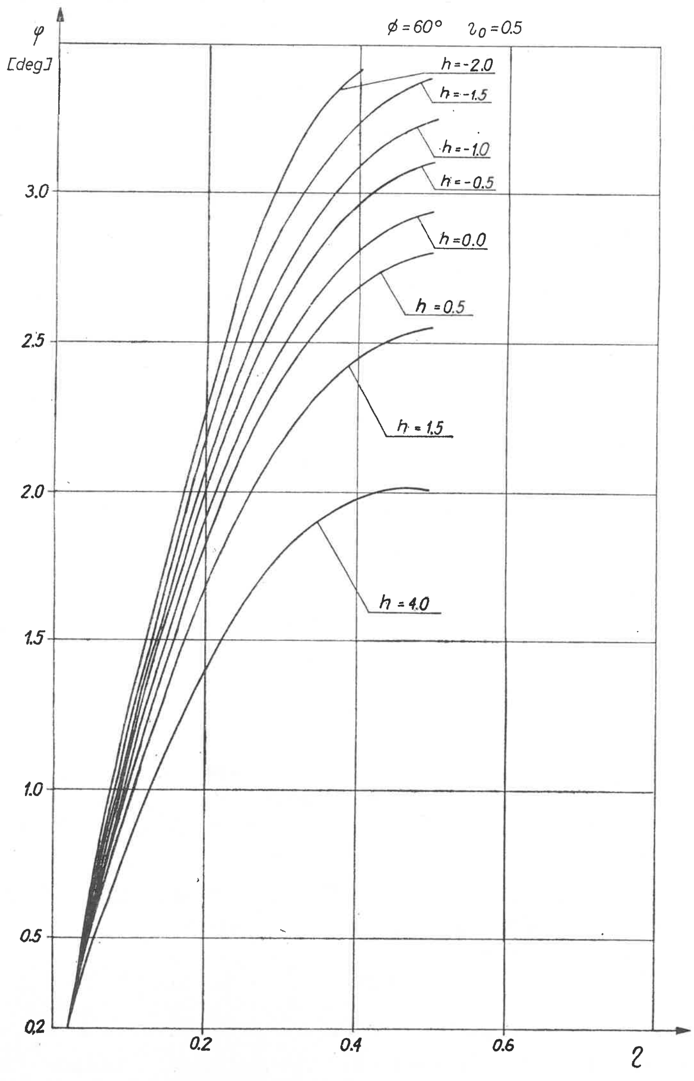
$$\langle M \rangle = M_s \cos \alpha/2 \tag{8b}$$

The comparison of Eqs (8a) and (8b) gives Eq. (8). The mean value of exchange energy as a function of the angle for fixed $\varnothing$ (Fig. 1) can be obtained from the expression

$$(K_U)^{-1}\langle E_A\rangle = \langle g(\varphi)\rangle - g_0, \tag{9}$$

where:

$$\langle g(\varphi)\rangle = \frac{1}{\eta_0}\int_0^{\varphi_0} \frac{g(\varphi)}{[g(\varphi)-g_0]^{1/2}}\,d\varphi. \tag{10}$$



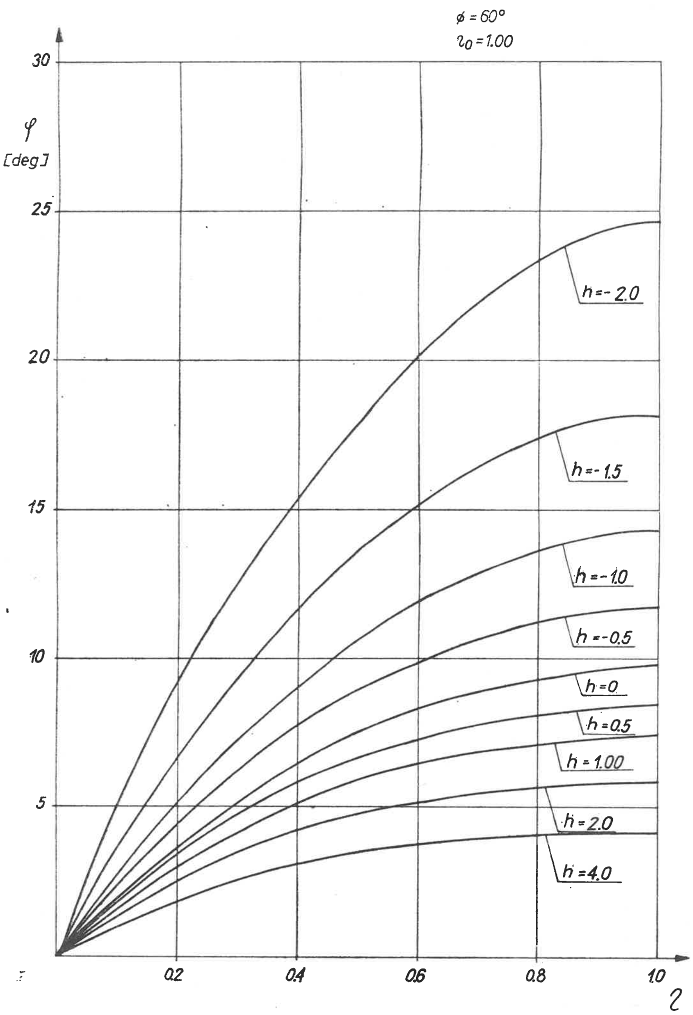
a

On the other hand the mean value of cos φ is calculated from the expression

$$\langle\cos\varphi\rangle = \frac{1}{\eta_0}\int_0^{\varphi_0} \frac{\cos\varphi}{[g(\varphi)-g_0]^{1/2}}\,d\varphi. \tag{11}$$

3. Results

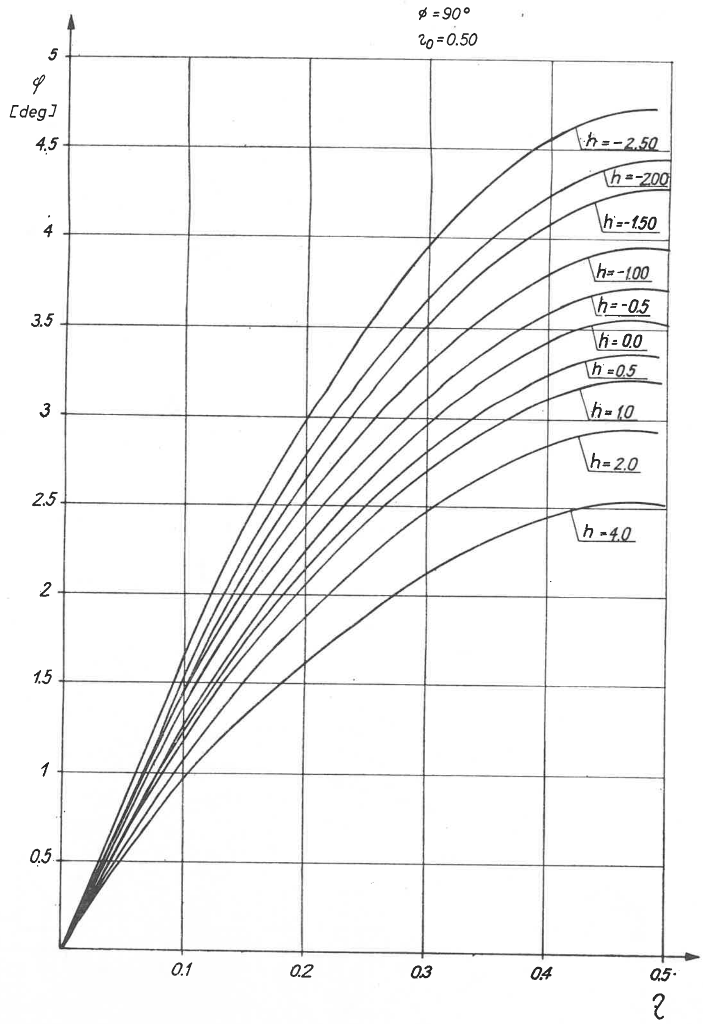
The calculation have been made for permalloy ($K_U = 10^3$ erg/cm^3, $A = 10^{-6}$ erg/cm) with the use of IBM 7094 computer. Figs 2 and 3 show the distribution of the magnetization vector inside the film for the angles $\varnothing = 90°$ and $\varnothing = 60°$, for various thickness



b

Fig. 2. Distribution of the magnetization vector in a thin film for $\varnothing = 60°$. a) $\eta_0 = 0.5$; b) $\eta_0 = 1.0$

values and fixed external field. The obtained curves of φ *vs* η dependences usually differ from the assumed distributions of step-function form, for which magnetizations in individual layers are uniform. Measurement of the real distribution of magnetization in two-layer film is difficult. We can only calculate the expected diffraction contrast for transmission electron micrographs for different proposed $\varphi(\eta)$ distributions and then by comparing it with obtained electron micrograph we verify the individual models. One should expect, that magnetization distribution that differs from the usually assumed step-function form influences on the threshold curve, the torque curve, the susceptibility and histeresis loop. In this paper the mean magnetization was calculated for a distribution founded.



a

Figs 4 and 5 show the dependence of ln $\langle 100E_A \rangle$ on ln α for $\varnothing = 90°$, and 60° and for $\eta_0 = 0.5$, 1.0, 2.0. As it can be seen from the figures, the relationship is linear with a slope of (2 ∓ 0.06). The above resuls indicate that the mean exchange energy is proportional to α squared. The proportional factor is strongly dependent on η_0 (film thickness) and practically does not depend on $\varnothing$, for $\varnothing = 60°$ and 90° and for $\eta_0 \geqslant 1$ which corresponds to the thickness $d = 3160$ Å.

The model presented in this paper is valid in the entire range of α. In particular, the result obtained is in agreement with the results obtained in Refs [2, 3] in case of strong coupling, *i. e.* small values of α. It is however in disagreement with the results obtained in Ref. [4] where the dependence on α was presented as $\alpha^{3/2}$, since the latter relationship has been derived as an asymptotic form for small values of α estimated as smaller than 2°.

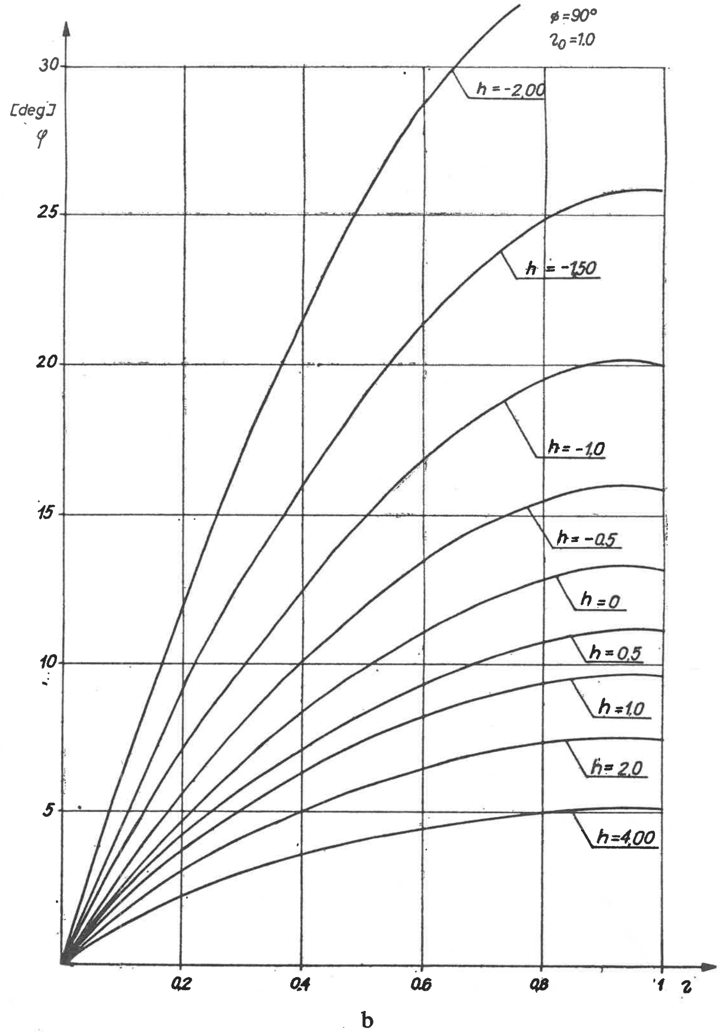


Fig. 3. Distribution of the magnetization vector in a thin film for $\varnothing = 90°$. a) $\eta_0 = 0.5$; b) $\eta_0 = 1.0$

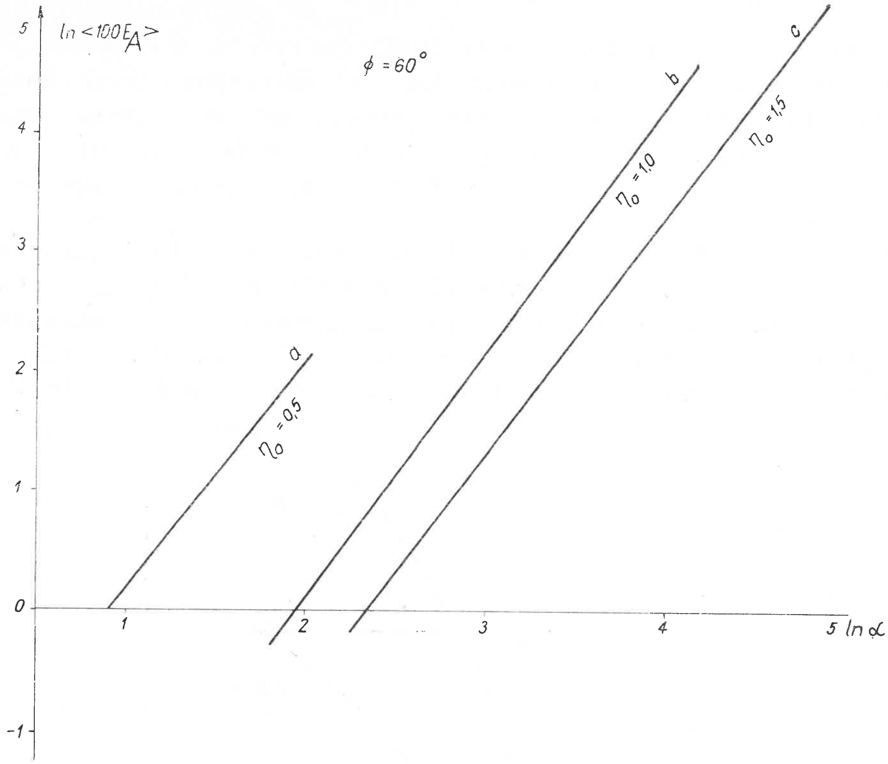


Fig. 4. Dependence of ln ⟨$100E_A$⟩ on ln α for $\varnothing = 60°$. a) $\eta_o = 0.5$; b) $\eta_o = 1.0$; c) $\eta_o = 2.0$

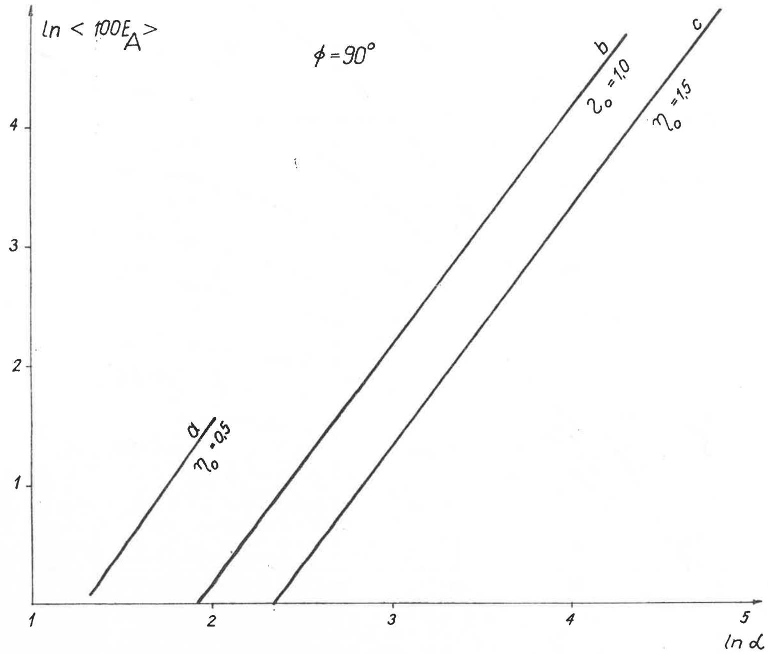


Fig. 5. Dependence of ln ⟨$100E_A$⟩ on ln α for $\varnothing = 90°$. a) $\eta_o = 0.5$; b) $\eta_o = 1.0$; c) $\eta_o = 2.0$

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