PHENOMENOLOGICAL SPIN WAVE THEORY OF UNIAXIAL FERROMAGNETS WITH PLATE-LIKE DOMAIN STRUCTURE

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By employing the method of approximate second quantization a general formula is derived for the excitation energy spectrum of a uniaxial ferromagnet with simple plate-like domain structure in the presence of a homogeneous external magnetic field. The conditions are examined under which the phenomenological Hamiltonian of the system can be diagonalized in the non-interacting-magnons approximation. It is shown that the bounding condition for the solutions of the Hill equation restricts the dependence of the system's excitation energy on the wave vector to only three types.

1. *Introduction*

The purpose of this paper is to give a general procedure for applying the phenomenological spin wave theory [1, 2] in determining the approximate energies of the elementary excitations of a uniaxial ferromagnet with a simple plate-like Shirobokov-type [4] domain structure in the presence of a homogeneous external magnetic field. This problem has recently been the subject of extensive theoretical investigations (*e.g.*, see [7, 12–15]), partly because of the specific thermodynamic properties and resonance characteristics of magnetic materials with domain structure. Also, knowledge of the spin wave energy spectrum is relevant to NMR studies of ferromagnetic materials [16–18].

The starting point for our considerations is a phenomenological [1] Hamiltonian constructed of the magnetization vector operator and including the exchange, anisotropy and Zeeman terms. The magnetostatic self-energy (demagnetization) is not taken into account. The zero-temperature spatial distribution of the direction of the magnetization vector — which is subsequently chosen as the (local) direction of quantization — is determined by minimizing the Hamiltonian in the quasi-classical approach (approximate ground state). Upon performing the Holstein-Primakoff mapping [3] in its lowest approximation we obtain a Hamiltonian which is bilinear in Bose operators. In diagonalizing it, we show that for a one-dimensional domain structure model the problem leads to a system of

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Hill-type differential equations, which is a consequence of the periodicity of the domain structure. This, in turn, leads to the conclusion that regardless of the direction of the external magnetic field the there exist only three types of dispersion laws for the spin wave energy. This explains the results obtained in Refs [7, 12] in the field-free limit.

2. *Approximate ground state*

We assume that the magnetic energy of a unial ferromagnet is described by the Hamiltonian of the form (*e.g.*, see [1, 2])

$$H = \int_v \{A^{\alpha\beta}M_\alpha(x)M_\beta(x)+B^{\alpha\beta}_{\mu\nu}M_{\alpha,\mu}(x)M_{\beta,\nu}(x)-h^\alpha M_\alpha(x)\}d^3x, \tag{1}$$

where the components $M_\alpha(x)$ of the magnetization operator fulfill the commutation relations

$$[M_\alpha(x), M_\beta(x')] = 2i\mu_0\delta(x-x')\varepsilon_{\alpha\beta\gamma}M_\delta(x). \tag{2}$$

The notation used here is: $i = \sqrt{-1}$, $2\mu_0 = g\mu_B$, μ_B — Bohr magneton, g — spectroscopic splitting factor, $\varepsilon_{\alpha\beta\gamma}$ — Levi-Civita tensor, $M_{\alpha,\mu}(x) = \partial M_\alpha(x)/\partial x_\mu$, and h^α — vector of the intensity of the external (uniform) magnetic field. The convention of summing over repeating Greek indices $\alpha, \beta, \ldots = 1, 2, 3$ is applied.

Wanting to describe the properties of a non-uniformly magnetized ferromagnet (with domain structure), we resort to a local system of coordinates being a function of x, wherein the local direction of quantization is that of the versor γ_α. In this reference frame the operator $M_\alpha(x)$ is expressed as

$$M_\alpha(x) = \gamma_\alpha(x)M_3(x)+A_\alpha(x)M^+(x)+A^*_\alpha(x)M^-(x), \tag{3}$$

where

$$M^\pm(x) = M_1(x)\pm iM_2(x). \tag{4}$$

The quantities $\gamma_\alpha(x)$, $A_\alpha(x)$ and $A^*_\alpha(x)$ satisfy the conditions [3]

$$\gamma_\alpha(x)\gamma_\alpha(x) = 1, \quad \gamma_\alpha(x) = \gamma^*_\alpha(x) \tag{5}$$

$$A_\alpha(x)A^*_\alpha(x) = \tfrac{1}{2}, \quad A_\alpha(x)A_\alpha(x) = A_\alpha(x)\gamma_\alpha(x) = 0.$$

$$|A_\alpha(x)|^2 = \tfrac{1}{4}(1-\gamma^2_\alpha(x)), \quad [\gamma(x)\times A(x)]_\alpha = iA_\alpha(x). \tag{6}$$

In order to find the approximate ground state of the Hamiltonian (1) we perform the quasi-classical approximation by substituting in first approximation the operators $M_\alpha(x)$ by the classical vectors $\mathscr{M}_\alpha(x)$ having a constant absolute value $\mathscr{M}_s$, *viz.*,

$$M_\alpha(x) \to \mathscr{M}_\alpha(x) = \gamma_\alpha \mathscr{M}_s. \tag{7}$$

Putting (7) into (1) we get the classical expression for the energy of the system,

$$H\{M_\alpha, M_{\alpha,\mu}\} \to E\{\gamma_\alpha, \gamma_{\alpha,\mu}\} = \int_v \mathscr{E}\{\gamma_\alpha(x), \gamma_{\alpha,\mu}(x)\}dx. \tag{8}$$

where

$$\gamma_{\alpha,\mu} = \partial\gamma_\alpha/\partial x\mu.$$

Taking the accessory condition (5) into account and requiring the functional $E\{\gamma_\alpha, \gamma_{\alpha,\mu}\}$ to take an extreme value yields the Euler-Lagrange equation determining $\gamma_\alpha(x)$,

$$\frac{\partial \mathscr{E}}{\partial \gamma_\alpha(x)} - \frac{\partial}{\partial x_\mu} \frac{\partial \mathscr{E}}{\partial \gamma_{\alpha,\mu}(x)} = 2\lambda(x)\gamma_\alpha(x). \tag{9}$$

Equations (9), together with periodicity conditions [4, 5] in the form $\gamma_\alpha(x+\Delta) = \gamma_\alpha(x)$ imposed on their solutions, describe the domain structure. By multiplying equations (9) by $\gamma_\alpha(x)$ and considering (5) we get the relation

$$\frac{\partial \mathscr{E}}{\partial \gamma_\alpha} \gamma_\alpha - \frac{\partial}{\partial x_\mu} \frac{\partial \mathscr{E}}{\partial \gamma_{\alpha,\mu}} \gamma_\alpha = 2\lambda, \tag{10}$$

for the Lagrange multiplier $\lambda(x)$.

3. Transition to the spin wave representation

In order to find the energy of elementary excitations of the system we put the operator $M_\alpha(x)$ in the form

$$M_\alpha(x) = \gamma_\alpha(x)\mathscr{M}_s + \delta M_\alpha(x), \tag{11}$$

where, in agreement with (3),

$$\delta M_\alpha(x) = \gamma_\alpha(x)\,[M_3(x) - \mathscr{M}_s] + A_\alpha(x)M^+(x) + A_\alpha^*(x)M^-(x). \tag{12}$$

We shall treat the operator $\delta M_\alpha(x)$ as a "small" operator complement to the classical vector $\mathscr{M}_\alpha(x)$. We expand the Hamiltonian into a power series with respect to the operators $\delta M_\alpha(x)$ and $\delta M_{\alpha,\mu}(x)$,

$$H = E\{\gamma_\alpha \mathscr{M}_s + \delta M_\alpha, \gamma_{\alpha,\mu}\mathscr{M}_s + \delta M_{\alpha,\mu}\} = E_0\{\gamma_\alpha, \gamma_{\alpha,\mu}\} + \mathscr{M}_s^{-1} \int\limits_v \frac{\partial \mathscr{E}}{\partial \gamma_\alpha} \delta M_\alpha dx +$$

$$+ \mathscr{M}_s^{-1} \int\limits_v \frac{\partial \mathscr{E}}{\partial \gamma_{\alpha,\mu}} \delta M_{\alpha,\mu} dx + \frac{1}{2} \mathscr{M}_s^{-2} \int\limits_v \frac{\partial^2 \mathscr{E}}{\partial \gamma_\alpha \partial \gamma_\beta} \delta M_\alpha \delta M_\beta dx +$$

$$+ \frac{1}{2} \mathscr{M}_s^{-2} \int\limits_v \frac{\partial^2 \mathscr{E}}{\partial \gamma_{\alpha,\mu} \partial \gamma_{\beta,\nu}} \delta M_{\alpha,\mu} \delta M_{\beta,\nu} dx + \ldots \tag{13}$$

where E_0 stands for the E of Eq. (8) after the solutions $\gamma_\alpha(x)$ from Eqs (9) are substituted. Hence, in first approximation E_0 determines the energy of the system's ground state. The second and third terms in Eq. (13) can be transformed by means of Eq. (9) into the following:

$$\int\limits_v \left\{ \frac{\partial \mathscr{E}}{\partial \gamma_\alpha} \delta M_\alpha + \frac{\partial \mathscr{E}}{\partial \gamma_{\alpha,\mu}} \delta M_{\alpha,\mu} \right\} \mathscr{M}_s^{-1} dx =$$

$$= \int 2\mathscr{M}_s^{-1} \lambda [M_3 - \mathscr{M}_s] dx. \tag{14}$$

Therefore, considering (13) and (14) we can rewrite the Hamiltonian (1) in the same approximation as

$$H = E_0+2\mathcal{M}_s^{-1}\int_v \lambda[M_3-\mathcal{M}_s]dx+\tfrac{1}{2}A^{\alpha\beta}\int_v \delta M_\alpha\delta M_\beta dx+$$
$$+\tfrac{1}{2}B^{\alpha\beta}_{\mu\nu}\int_v \delta M_{\alpha,\mu}\delta M_{\beta,\nu}dx. \tag{15}$$

Let us now perform the Holstein-Primakoff transformation [1–3, 6] (in the first approximation),

$$M^+(x) = (4\mu_0\mathcal{M}_s)^{\frac{1}{2}}a(x),$$
$$M^-(x) = (4\mu_0\mathcal{M}_s)^{\frac{1}{2}}a^+(x),$$
$$M_3(x) = \mathcal{M}_s-2\mu_0 a^+(x)a(x). \tag{16}$$

The operators $a(x)$ and $a^+(x)$ appearing here obey the following commutation relations:

$$[a(x), a^+(x')] = \delta(x-x'),$$
$$[a(x), a(x')] = [a^+(x), a^+(x')] = 0. \tag{17}$$

Taking only the bilinear part of the Hamiltonian in the operators $a(x)$ and $a^+(x)$ yields

$$H = E_0-4\mu_0\mathcal{M}_s^{-1}\int_v \lambda(x)a^+(x)a(x)dx+$$
$$+\tfrac{1}{2}A^{\alpha\beta}\int_v \tau_\alpha(x)\tau_\beta(x)dx+\tfrac{1}{2}B^{\alpha\beta}_{\mu\nu}\int_v \tau_{\alpha,\mu}(x)\tau_{\beta,\nu}(x)dx, \tag{18}$$

where

$$\tau_\alpha(x) = (4\mu_0\mathcal{M}_s)^{\frac{1}{2}}\{A_\alpha(x)a(x)+A^*_\alpha(x)a^+(x)\},$$
$$\tau_{\alpha,\mu}(x) = \partial\tau_\alpha(x)/\partial x_\mu. \tag{19}$$

For uniaxial ferromagnets we may take the tensors $A^{\alpha\beta}$ and $B^{\alpha\beta}_{\mu\nu}$ in the form [1]

$$A^{\alpha\beta} = K\delta_{\alpha 3}\delta_{\beta 3}, \qquad K < 0; \tag{20}$$

$$B^{\alpha\beta}_{\mu\nu} = C_\mu\delta_{\alpha\beta}\delta_{\mu\nu}. \tag{21}$$

Upon choosing the tensor $A^{\alpha\beta}$ in the form (20) the easy axis of magnetization is the x_3 axis. Let us also assume that the domain structure occurring in the ferromagnet is a flat structure and that rotation of the magnetization vector takes place about the x_1 axis, *i.e.*

$$\gamma_\alpha(x) = \gamma_\alpha(x_1), \qquad \gamma_1 = \text{const.}, \qquad A_\alpha(x) = A_\alpha(x_1). \tag{22}$$

We take the two-dimensional Fourier transformation of the operators $a(x)$ and $a^+(x)$ [7],

$$a(x) = \frac{1}{\sqrt{S}}\sum_\kappa b_\kappa(x_1)\exp(-i\kappa\varrho)$$

$$a^+(x) = \frac{1}{\sqrt{S}}\sum_\kappa b^+_\kappa(x_1)\exp(i\kappa\varrho) \tag{23}$$

where $\varrho = (0, x_2, x_3)$, $\kappa = (0, k_2, k_3)$ and S is the cross-sectional area of the sample in the plane $x_2 0 x_3$. Operators $b_\kappa(x_1)$ and $b_\kappa^+(x_1)$ satisfy the commutation relations

$$[b_\kappa(x_1), b_{\kappa'}^+(x_1')] = \delta_{\kappa\kappa'}\delta(x_1 - x_1'), \quad [b_\kappa(x_1), b_{\kappa'}(x_1')] = 0. \tag{24}$$

After putting (19)–(23) into (18) we get

$$H = E_0 + E_0' + \sum_\kappa \int_{L_1} dx_1 \Big\{ \alpha b_\kappa b_{-\kappa} + \psi_\kappa b_\kappa^+ b_\kappa + \alpha^* b_\kappa^+ b_{-\kappa}^+ + \\ + \eta_1 \left[\frac{db_\kappa}{dx_1}\frac{db_\kappa^+}{dx_1} + \frac{db_\kappa^+}{dx_1}\frac{db_\kappa}{dx_1} \right] + \vartheta \left[b_\kappa \frac{db_\kappa^+}{dx_1} + \frac{db_\kappa^+}{dx_1} b_\kappa \right] + \\ + \vartheta^* \left[\frac{db_\kappa}{dx_1} b_\kappa^+ + b_\kappa^+ \frac{db_\kappa}{dx_1} \right] \Big\}. \tag{25}$$

The notation used in Eq. (25) is:

$$E_0' = 2\mu_0 \mathscr{M}_s \sum_\kappa \int_{L_1} dx_1 \{ K A_3 A_3^* + C_1 \mathring{A}_\alpha \mathring{A}_\alpha^* + A_\alpha A_\alpha^* (C_2 k_2^2 + C_3 k_3^2) \} \tag{26}$$

$$\alpha = \alpha(x_1) = 2\mu_0 \mathscr{M}_s [C_1 \mathring{A}_\alpha(x_1) \mathring{A}_\alpha(x_1) + K A_3(x_1) A_3(x_1)],$$

$$\eta_\mu = \mu_0 \mathscr{M}_s C_\mu,$$

$$\psi_\kappa = \psi_\kappa(x_1) = 4\alpha(x_1) - 4\mu_0 \mathscr{M}_s^{-1} \lambda(x_1) + 2\eta_2 k_2^2 + 2\eta_3 k_3^2,$$

$$\vartheta = \vartheta(x_1) = 2\mu_0 \mathscr{M}_s C_1 \mathring{A}_\alpha(x_1) \mathring{A}_\alpha(x_1),$$

$$\mathring{A}_\alpha(x_1) = \frac{dA_\alpha(x_1)}{dx_1}. \tag{27}$$

4. *The spin wave energy spectrum*

The equation of motion of the operator $b_\kappa(x_1)$, with account taken of the Hamiltonian in the form (25) and the commutation relations (24), is

$$\frac{db_\kappa}{dt} = i/\hbar [H, b_\kappa] = -i/\hbar \left\{ \psi_\kappa b_\kappa + 2\alpha b_{-\kappa}^+ - 2\eta_1 \frac{d^2 b_\kappa}{dx_1^2} \right\}. \tag{28}$$

To diagonalize Hamiltonian (25) we carry out the transformation [7, 12]

$$b_\kappa(x_1) = u_\kappa(x_1) c_\kappa + v_\kappa^*(x_1) c_{-\kappa}^+,$$

$$b_\kappa^+(x_1) = u_\kappa^*(x_1) c_\kappa^+ + v_\kappa(x_1) c_{-\kappa}, \tag{29}$$

where the operators c_κ and c_κ^+ fulfill the commutation relations

$$[c_\kappa, c_{\kappa'}^+] = \delta_{\kappa\kappa'}; \quad [c_\kappa, c_{\kappa'}] = [c_\kappa^+, c_{\kappa'}^+] = 0. \tag{30}$$

Among other things, relations (30) lead to the following conditions for the functions $u_\kappa(x_1)$ and $v_\kappa(x_1)$:

$$\int (|u_\kappa|^2 - |v_\kappa|^2) dx_1 = 1, \tag{31}$$

$$u_\kappa(x_1) = u_{-\kappa}(x_1); \quad v_\kappa(x_1) = v_{-\kappa}(x_1). \tag{32}$$

The condition for the Hamiltonian (25) to be diagonal in terms of operators c_κ and c_κ^+ is

$$i\hbar \frac{dc_\kappa}{dt} = \mathscr{E}_\kappa c_\kappa; \quad i\hbar \frac{dc_\kappa^+}{dt} = -\mathscr{E}_\kappa c_\kappa^+ \tag{33}$$

From relations (28), (29) and (33) we get the system of equations for the functions $u_\kappa(x_1)$ and $v_\kappa(x_1)$,

$$2\eta_1 \frac{d^2 u_\kappa(x_1)}{dx_1^2} - 2\alpha^*(x_1) v_{-\kappa}(x_1) - [\psi_\kappa(x_1) - \mathscr{E}_\kappa] u_\kappa(x_1) = 0, \tag{34}$$

$$2\eta_1 \frac{d^2 v_\kappa^*(x_1)}{dx_1^2} - 2\alpha^*(x_1) u_{-\kappa}^*(x_1) - [\psi_\kappa(x_1) + \mathscr{E}_\kappa] v_\kappa^*(x_1) = 0.$$

For our assumed plate-like domain structure with 180° Bloch walls and magnetization vector lying in the x_2, 0, x_3 plane, the coefficients $A_\alpha(x_1)$ and $\gamma_\alpha(x_1)$ of transformation (3) take the simple form

$$A_1 = -i/2, \quad A_2(x_1) = -\tfrac{1}{2}\gamma_3(x_1), \quad A_3 = \tfrac{1}{2}\gamma_2(x_1);$$

$$\gamma_1(x_1) = 0, \quad \gamma_2(x_1) = \sin \varphi(x_1), \quad \gamma_3(x_1) = \cos \varphi(x_1). \tag{35}$$

The angle $\varphi = \varphi(x_1)$ is that between the x_3 — axis and the local quantization direction in the x_2, 0, x_3 plane; it is defined by Eq. (9). For this type of domain structure we have

$$\alpha(x_1) = \alpha^*(x_1), \quad \psi_\kappa(x_1) = \psi_\kappa^*(x_1), \tag{36}$$

whereby (see Eq. (27)) the coefficients of equations (34) are real. As both equations of the system (34) have the same form, their solutions should differ only by a constant multiplier. We take, hence, the function $u_\kappa(x_1)$ and $v_\kappa(x_1)$ in the form

$$u_\kappa(x_1) = C_\kappa^{(1)} f_\kappa(x_1); \quad v_\kappa(x_1) = C_\kappa^{(2)} f_\kappa(x_1). \tag{37}$$

It can be shown that assumption (37) does not violate the canonicity conditions (31) and (32). In agreement with (34), functions $f_\kappa(x_1)$ satisfy the equations

$$\frac{d^2 f_\kappa}{dx_1^2} - \left[\frac{C_\kappa^{(2)}}{C_\kappa^{(1)}} \tilde{\alpha}^{(1)}(x_1) + \tilde{\psi}^{(1)}(x_1)\right] f_\kappa - \left[\frac{C_\kappa^{(2)}}{C_\kappa^{(1)}} \tilde{\alpha}^{(0)} + \tilde{\psi}_\kappa^{(0)} - e_\kappa\right] f_\kappa = 0$$

$$\frac{d^2 f_\kappa}{dx_1^2} - \left[\frac{C_\kappa^{(1)}}{C_\kappa^{(2)}} \tilde{\alpha}^{(1)}(x_1) + \tilde{\psi}^{(1)}(x_1)\right] f_\kappa - \left[\frac{C_\kappa^{(1)}}{C_\kappa^{(2)}} \tilde{\alpha}^{(0)} + \tilde{\psi}_\kappa^{(0)} + e_\kappa\right] f_\kappa = 0 \tag{38}$$

where

$$\tilde{\alpha}(x_1) = \frac{\alpha(x_1)}{\eta_1} = \tilde{\alpha}^{(1)}(x_1)+\tilde{\alpha}^{(0)}, \qquad e_\kappa = \frac{\mathscr{E}_\kappa}{\eta_1},$$

$$\tilde{\psi}_\kappa(x_1) = \frac{\psi_\kappa(x_1)}{2\eta_1} = \tilde{\psi}^{(1)}(x_1)+\tilde{\psi}_\kappa^{(0)}. \tag{39}$$

The quantities $\tilde{\alpha}^{(0)}$ and $\tilde{\psi}_\kappa^{(0)}$ appearing in Eq. (38) are independent of the variable x_1.

Each of the equations in the system (38) is of the Sturm-Liouville type [8, 9]; therefore, the parameters $C_\kappa^{(1)}$ and $C_\kappa^{(2)}$ fulfill the system of equations

$$\left\{\tilde{\psi}_\kappa^{(0)} - e_\kappa + \int_0^{L_1}\left[\left|\frac{df_\kappa}{dx_1}\right|^2 + \tilde{\psi}^{(1)}(x_1)|f_\kappa|^2\right]dx_1\right\}C_\kappa^{(1)} +$$

$$+\{\tilde{\alpha}^{(0)} + \int_0^{L_1} \tilde{\alpha}^{(1)}(x_1)|f_\kappa|^2 dx_1\}C_\kappa^{(2)} = 0,$$

$$\{\tilde{\alpha}^{(0)} + \int_0^{L_1} \tilde{\alpha}^{(1)}(x_1)|f_\kappa|^2 dx_1\}C_\kappa^{(1)} +$$

$$+\left\{\tilde{\psi}_\kappa^{(0)} + e_\kappa + \int_0^{L_1}\left[\left|\frac{df_\kappa}{dx_1}\right|^2 + \tilde{\psi}^{(1)}(x_1)|f_\kappa|^2\right]dx_1\right\}C_\kappa^{(2)} = 0. \tag{40}$$

Here, the functions $f_\kappa(x_1)$ solving equations (38) form a system of orthonormal functions defined as

$$\int_0^{L_1} f_\kappa^*(x_1)f_{\kappa'}(x_1)dx_1 = \delta_{\kappa\kappa'}. \tag{41}$$

The system of equations (40) has a non-trivial solution when the determinant of the system is equal to zero. This condition lets the excitation energy to be found, *viz.*,

$$\mathscr{E}_\kappa = 2\eta_1\left\{\left[\tilde{\psi}_\kappa^{(0)} + \int_0^{L_1}\left(\left|\frac{df_\kappa}{dx_1}\right|^2 + \tilde{\psi}^{(1)}(x_1)|f_\kappa|^2\right)dx_1\right]^2 + \right.$$

$$\left. - [\tilde{\alpha}^{(0)} + \int_0^{L_1}\tilde{\alpha}^{(1)}(x_1)|f_\kappa|^2 dx_1]^2\right\}^{\frac{1}{2}}. \tag{42}$$

Let us yet note that the parameters $C_\kappa^{(1)}$ and $C_\kappa^{(2)}$ are unambiguous because in addition to Eqs (40) there are the equations

$$|C_\kappa^{(1)}|^2 - |C_\kappa^{(2)}|^2 = 1$$

which stem from the canonicity conditions (31) with Eq. (41) taken into account.

The Hamiltonian (25) expressed in terms of the operators c_κ and c_κ^+ thus takes the final form

$$H = E_0 + E_0' + \Delta H + \sum_\kappa \mathscr{E}_\kappa c_\kappa^+ c_\kappa. \tag{43}$$

The terms E_0 and E_0' are defined by expressions (13) and (26), whereas the term ΔH is given by the formula

$$\Delta H = -\sum_\kappa \mathscr{E}_\kappa \int_0^{L_1} |v_\kappa(x_1)|^2\, dx_1 = -\sum_\kappa \mathscr{E}_\kappa |C_\kappa^{(2)}|^2. \tag{44}$$

The solution of the system (38) for specific physical models may present considerable analytic difficulties. Notwithstandig, even without solving these equations a number of their solutions' properties can be stated. They follow from the periodicity conditions imposed on solutions to the system of equations (9), namely,

$$\gamma_\alpha(x_1 + 2n\Delta) = \gamma_\alpha(x_1); \quad n = 1, 2, 3, \ldots$$

where Δ is the domain width. These conditions reduce each of the equations of system (38) to Hill equations,

$$\frac{d^2 f_\kappa(x_1)}{dx_1^2} - F_\kappa(x_1) f_\kappa(x_1) = 0 \tag{45}$$

where

$$F_\kappa\ (x_1 + \tau) = F_\kappa(x_1); \quad \tau = \begin{cases} n\Delta \ \text{for}\ h^\alpha = 0 \\ 2n\Delta \ \text{for}\ h^\alpha \neq 0 \end{cases} \tag{46}$$

In accordance with Floquet's theorem [8, 10, 11], the solutions of Hill equations are separated into bound and unbound ones. In turn, the bound solutions are divided into three types, *viz.*, real periodical solutions of periods τ or 2τ and non-periodical complex solutions. The so-called normal non-trivial solutions of a Hill equation in the form (45) satisfy the condition

$$f_\kappa(x_1 + \tau) = \sigma f_\kappa(x_1), \tag{47}$$

with the characteristic factor σ given by the equation [10, 11]

$$\sigma^2 - a\sigma + 1 = 0. \tag{48}$$

The parameter a in Eq. (48) is real constant predetermined by the numerical parameters appearing in Eq. (45). The roots σ_1 and σ_2 of equation (48) fulfill the condition $\sigma_1\sigma_2 = 1$.

Equation (45) has a bound solution in the following cases:

A) $\sigma_1 = \sigma_2 = 1$ — real periodical solutions with period τ equal to the period of the function $F_\kappa(x_1)$ in Eq. (45);

B) $\sigma_1 = \sigma_2 = -1$ — real periodical solutions with period 2τ;

C) $\sigma_1 = \sigma_2^*$ — non-periodical complex solutions.

The demand for the solutions to Eq. (45) to be bound ($|\sigma_1| = |\sigma_2| = 1$) is the condition determining the excitation energy ε_κ.

5. *Concluding remarks*

When considering a domain structure of the Shirobokov type deformed by the action of an external magnetic field [4, 5], the function $F_\kappa(x_1)$ in Eq. (45) is expressed in two specific cases as

$$F_\kappa(x_1) = B_{1,\kappa}\left[\frac{\omega \text{ sn } qx_1+1}{\text{sn } qx_1+\omega}\right]^2 + B_{2,\kappa}\frac{\omega \text{ sn } qx_1+1}{\text{sn } qx_1+\omega} + B_{3,\kappa}, \tag{49}$$

or

$$F_\kappa(x_1) = B_{1,\kappa}\left[\frac{\omega \text{ cn } qx_1+1}{\text{cn } qx_1+\omega}\right]^2 + B_{2,\kappa}\frac{\omega \text{ cn } qx_1+1}{\text{cn } qx_1+\omega} + B_{3,\kappa} \tag{50}$$

depending on whether the external magnetic field acts along the easy axis of magnetization (uniaxial ferromagnet) or perpendicularly to this direction and in the Bloch wall plane. In the simplest case, when the external magnetic field is equal to zero, Eq. (45) reduces to the Lamé equation

$$\varphi^2 \frac{d^2 f_\kappa(x_1)}{dx_1^2} - (2k^2 \text{ sn}^2 \, qx_1 + B_{3,\kappa}) f_\kappa(x_1) = 0. \tag{51}$$

This case has been studied in Refs [7, 12].

We have shown here that the periodicity conditions imposed on solutions to the system of equations (9) lead the system of equations (34) to Hill equations. Hence, in ferromagnets having domain structure there may occur only three types of relationships between the energy of elementary excitations ε_κ and wave vector, which correspond to three bound solutions of equation (45) scrutinized in the preceding section of this paper.

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