DERIVATION OF A SCHRÖDINGER TYPE EQUATION WITH DAMPING FOR A SYSTEM OF INTERACTING PARTICLES

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The system of equations for the average values of Bose or Fermi amplitudes and kinetic equation for average value of particle number operator are derived by means of the nonequilibrium statistical operator method. The connection of this method with the linear response method of Kubo is discussed.

1. Introduction

Any small subsystem of a system of many particles interacts with its environment. (We have in mind such subsystems as a Brownian particle with its surrounding, a quasiparticle or a collective mode.) Since the environment has many more degress of freedom than the subsystem, it is possible to consider it as a thermal bath in which we embed the small subsystem. In general, the subsystem does not have any definite temperature.

Because of the very complicated character of interactions, we can assume that the influence of the thermal bath leads to a dissipation of the subsystem energy and to appearence of random forces influencing this subsystem. Thus the subsystem exhibits some features of statistical systems saving at the same time some properties of mechanical systems. This means that the random forces and terms with damping occur in the equation of motion for dynamical variable operators of subsystem. In this way it is possible to consider for example some collective mode (*i. e.* spin wave) as a form of Brownian motion under an influence of other degrees of freedom [1].

It is possible to find the damping in a different way, with the help of an equation of motion for correlation functions of dynamical variables of a subsystem (*e. g.* Green function method), or equation of motion for average values of these dynamical variables. A very convenient method of derivation of such equations is the nonequilibrium statistical operator (NSO) method [2]. This method is based on Bogolubov's idea of the existence of

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a hierarchy of characteristic times and the reduction of the number of parameters describing a system.

For the kinetic stage, the Hamiltonian of our many-particle system can be written in the following form

$$H = H_0 + H_{int},$$

where

$$H_0 = E_i a_i^+ a_i + H_T$$

is the Hamiltonian of the subsystem and thermal bath,

$$H_{int} = a_i^+ V_i^+ + V_i a_i$$

is the operator of interaction of the subsystem with thermal bath, a_i and a_i^+ are the annihilation and creation operators of particle i. Operators V_i and V_i^+ are connected with the variables of the thermal bath only. Suppose that at the considered stage of an evolution it is possible to describe the system using a set of average values of operators $\langle a_i \rangle^t$, $\langle a_i^+ \rangle^t$, $\langle n_i \rangle^t$ and $\langle H_T \rangle$. Decomposition of the Hamiltonian into the free part and interaction is also connected with the stage of evolution of the system [3]. In the framework of the NSO method, eliminating the influence of the thermal bath in the second order of an interaction H_{int} (we require the smallness of interaction between the subsystem and the thermal bath) leads to an equation of motion for the average values of our set of operators. Now this influence of thermal bath will appear as an effect of friction of the particle in the environment, i. e. as a dissipation of their energy.

Exactly the same method of derivation of the equation of motion for $\langle a_i \rangle^t$, $\langle a_i^+ \rangle^t$, $\langle n_i \rangle^t$ was used already in [4], where equations for $\langle a_i \rangle^t$, $\langle a_i^+ \rangle^t$ were called Schrödinger-type, as they exhibit mechanical and statistical features. The authors considered a small subsystem in thermal bath. This subsystem consisted of non-interacting particles of some kind (e. g. excitons). The particles of another kind formed the thermal bath (e. g. phonons). The relaxation of the subsystem is connected with the interaction of different kind of particles. In our work the thermal bath consists of the same kind of particles as those of the subsystem and particles of other kinds or impurities. In such a way it is possible to extend the class of systems which can be considered.

Section 2 is devoted to the derivation of the Schrödinger-type equations and the kinetic equation for $\langle n_i \rangle^t$. In Section 3 we shall give some examples and in Section 4 we shall consider connection with the linear response method of Kubo.

2. The general theory

Let us consider the system of N particles with the Hamiltonian

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{V}, \tag{1}$$

where

$$\mathscr{H}_0 = \sum_i E_i a_i^+ a_i.$$

E_i is one-particle energy, a_i^+ and a_i are creation and annihilation operators of fermions or bosons, $\mathscr{V}$ describes either the interaction between particles or particles with impurities. Hamiltonian (1) is equivalent, to terms of the order of N^{-1}, to

$$H = H_0 + H_{int}, \tag{2}$$

where

$$H_0 = E_i a_i^+ a_i + H_T, \tag{3}$$

$$H_{int} = a_i^+ V_i^+ + V_i a_i \tag{4}$$

and the notation used is the same as that in the Introduction. Examples of construction of H will be given in Section 3.

According to the NSO method we construct the statistical operator of the nonequilibrium system $\varrho(t, 0)$ with the help of the quasiequilibrium operator

$$\varrho_q(t, 0) = \exp\{-S(t, 0)\}, \tag{5}$$

where

$$S(t, 0) = \Omega(t) + \sum_m P_m F_m(t), \tag{6}$$

$$\Omega(t) = \ln \operatorname{Tr} \exp\{-\sum_m P_m F_m(t)\}. \tag{7}$$

Let us take the Hamiltonian of the thermal bath H_T and operators a_i, a_i^+, $n_i = a_i^+ a_i$ describing the nonequilibrium state of subsystem as the operators P_m. This choice of the set of operators P_m enables us to give a dynamical description of the system. The functions $F_m(t)$ are thermodynamical parameters conjugated to $\langle P_m \rangle^t$ in the sense of nonequilibrium statistical thermodynamics. We denote the parameters corresponding to $\langle a_i \rangle^t$, $\langle a_i^+ \rangle^t$ and $\langle n_i \rangle^t$ by $f_i(t), f_i^*(t)$ and $F_i(t)$ respectively. The mean value of $\langle H_T \rangle$ is connected with the reciprocal temperature of thermal bath β.

The invariant part of $\varrho_q(t, 0)$ gives us the nonequilibrium statistical operator of the system

$$\varrho(t, 0) = \varepsilon \int_{-\infty}^0 dt_1 e^{\varepsilon t_1} e^{it_1 L} \varrho_q(t+t_1, 0), \tag{8}$$

and $\varepsilon \to 0^+$, when the thermodynamical limit is taken. Here L denotes Liouville's operator

$$iLA = \frac{1}{i\hbar}[A, H], \qquad e^{itL}A = e^{\frac{itH}{\hbar}} A e^{-\frac{itH}{\hbar}}.$$

Integration by part of the right-hand side of Eq. (8) gives

$$\varrho(t, 0) = \varrho_q(t, 0) - \int_{-\infty}^0 dt_1 e^{\varepsilon t_1} e^{it_1 L} \left\{ iL_1 \varrho_q(t+t_1, 0) + \right.$$

$$\left. + \left(\frac{\partial}{\partial t_1} + iL_0\right) \varrho_q(t+t_1, 0)\right\}, \tag{9}$$

where

$$iL_0 A = \frac{1}{i\hbar}[A, H_0], \quad iL_1 A = \frac{1}{i\hbar}[A, H_{\text{int}}].$$

Note that our set of operators P_m satisfies the relation $[P_m, H_0] = \sum_k \alpha_{mk} P_k$, where α_{mk} are c-numbers, so

$$\frac{\partial \varrho_q(t, 0)}{\partial t} + iL_0 \varrho_q(t, 0) =$$

$$= \sum_m \frac{\delta \varrho_q(t, 0)}{\delta \langle P_m \rangle^t} \, \text{Tr} \left\{ \frac{1}{i\hbar}[P_m, H_{\text{int}}]\varrho(t, 0) \right\}. \tag{10}$$

We are interested in the approximate form of $\varrho(t, 0)$, linear in H_{int}. Since the second term under the integral (9) is proportional to H_{int}, the evolution of the system can be considered to be free in our approximation. With the help of (10) we obtain for $\varrho(t, 0)$ an integral equation, which can be solved by iteration. At the first step, limiting ourselves to the system with

$$\langle V_i \rangle_q^t \equiv \text{Tr}\,(V_i \varrho_q(t, 0)) = 0 \tag{11}$$

we obtain

$$\varrho(t, 0) = \varrho_q(t, 0) - \frac{1}{i\hbar} \int_{-\infty}^{0} dt_1 e^{\varepsilon t_1}[\varrho_q(t, 0), H_{\text{int}}(t_1)], \tag{12}$$

where we used the relation

$$e^{it_1 L_0} \varrho_q(t + t_1, 0) = \varrho_q(t, 0).$$

The condition (11) is equivalent to the assumption of "randomness" of forces V_i, V_i^+.

We require that average values of operators P_m satisfy the conditions

$$\text{Tr}\,(P_m \varrho(t, 0)) \equiv \langle P_m \rangle^t = \langle P_m \rangle_q^t \tag{13}$$

which can be used for the determination of parameters $F_m(t)$. Averaging operators P_m with the statistical operator (12), and the differentiating them with respect to time, we obtain the kinetic equation

$$i\hbar \frac{\partial \langle P_m \rangle^t}{\partial t} = \sum_k \alpha_{mk} \langle P_k \rangle^t +$$

$$+ \frac{1}{i\hbar} \int_{-\infty}^{0} dt_1 e^{\varepsilon t_1} \langle [[P_m, H_{\text{int}}], H_{\text{int}}(t_1)] \rangle_q^t. \tag{14}$$

For our set of operators P_m we obtain from Eq. (14) two following equations

$$i\hbar \frac{\partial \langle a_i \rangle^t}{\partial t} = E_i \langle a_i \rangle^t + \frac{1}{i\hbar} \int\limits_{-\infty}^{0} dt_1 e^{\varepsilon t_1} \times$$

$$\times \langle [[a_i, H_{int}], H_{int}(t_1)] \rangle_q^t, \tag{15}$$

$$\frac{\partial \langle n_i \rangle^t}{\partial t} = -\frac{1}{\hbar^2} \int\limits_{-\infty}^{0} dt_1 e^{\varepsilon t_1} \langle [[n_i, H_{int}], H_{int}(t_1)] \rangle_q^t, \tag{16}$$

and the equation complex conjugate to (15). In the following we assume that

$$\langle V_i V_i(t) \rangle = \langle V_i^+, V_i^+(t) \rangle = 0$$

which is usually fulfilled due to conservation laws, $\langle ... \rangle = \mathrm{Tr} \left(... \dfrac{\exp(-\beta H_T)}{\mathrm{Tr}\exp(-\beta H_T)} \right)$ denotes an average over equilibrium distribution of the thermal bath. Introducing the Fourier transforms of correlation functions

$$\langle V_i(t) V_i^+ \rangle = \int\limits_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i\omega t} \langle V_i V_i^+ \rangle_\omega, \tag{17a}$$

$$\langle V_i^+ V_i(t) \rangle = \int\limits_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i\omega t} \langle V_i V_i^+ \rangle_\omega e^{\beta \hbar \omega} \tag{17b}$$

and calculating the double commutators in Eqs (15) and (16) the kinetic equations can be written in a simple form

$$i\hbar \frac{\partial \langle a_i \rangle^t}{\partial t} = (E_i + \Delta(E_i)) \langle a_i \rangle^t - \tfrac{1}{2} i\hbar \gamma(E_i) \langle a_i \rangle^t, \tag{18}$$

$$\frac{\partial \langle n_i \rangle^t}{\partial t} = -\gamma(E_i) (\langle n_i \rangle^t - \bar{n}_i), \tag{19}$$

where

$$\Delta(E_i) = -P \int\limits_{-\infty}^{\infty} \frac{d\omega}{2\pi} \langle V_i V_i^+ \rangle_\omega \frac{e^{\beta \hbar \omega} \mp 1}{\hbar \omega - E_i}, \tag{20a}$$

$$\gamma(E_i) = \frac{1}{\hbar^2} \langle V_i V_i^+ \rangle_{\hbar \omega = E_i} (e^{\beta E_i} \mp 1), \tag{20b}$$

and

$$\bar{n}_i = (e^{\beta E_i} \mp 1)^{-1}$$

is the mean number of particles (bosons or fermions) in the state i. From Eqs (20a) and (20b) it follows that the energy shift $\Delta(E_i)$ and damping $\gamma(E_i)$ are connected by the Kramers-Kronig relation

$$\Delta(E_i) = -\frac{\hbar}{2\pi} P \int\limits_{-\infty}^{\infty} d\omega \frac{\gamma(\omega)}{\omega - E_i}. \tag{21}$$

The equation (18) has both statistical and mechanical features, so following [4] we call it a Schrödinger-type equation. This equation differs from the corresponding equation in [4] by the definition of "random" forces V_i, V_i^+. Note that equation of the form (18) and (19) has been derived for the anharmonic oscillator by Lax [6].

3. Examples

i) Let us consider an interacting phonon system. In this case the particles of subsystem and the thermal bath are of the same kind. The Hamiltonian of the system is

$$\mathscr{H}_0 = \sum_Q \omega_Q n_Q, \tag{22a}$$

$$\mathscr{V} = \sum_{Q_1,Q_2,Q_3} V(Q_1, Q_2, Q_3) A_{Q_1} A_{Q_2} A_{Q_3}, \tag{22b}$$

where ω_Q are frequencies of phonons, $Q = (q, j)$, q is a wave vector, j is the polarization and phonon mode index, $\overline{Q} = (-q, j)$, $V(Q_1, Q_2, Q_3)$ describes the matrix element of three-phonon interaction, $A_Q = a_Q + a_{\overline{Q}}^+$ and a_Q^+, a_Q are creation and annihilation operators of phonons. In this case the subsystem consists of one phonon mode Q, the other phonon modes play the role of the thermal bath. Let us consider an equivalent Hamiltonian H which can be constructed using (22)

$$H = H_0 + H_{\text{int}}$$

where

$$H_T = \sum_{P \neq Q} \omega_P n_P + \sum_{P_1,P_2,P_3 \neq Q} V(P_1, P_2, P_3) A_{P_1} A_{P_2} A_{P_3}, \tag{23}$$

$$H_{\text{int}} = a_Q^+ V_Q^+ + V_Q a_Q,$$

$$V_Q = 3 \sum_{P_1,P_2 \neq Q} V(Q, P_1, P_2) A_{P_1} A_{P_2}.$$

For the Bravais lattice with $\langle V_Q \rangle = 0$, after some algebra, we obtain

$$i \frac{\partial \langle a_Q \rangle^t}{\partial t} = (\omega_Q + \Delta(\omega_Q)) \langle a_Q \rangle^t - i \frac{\hbar}{2} \gamma(\omega_Q) \langle a_Q \rangle^t, \tag{24}$$

$$\frac{\partial \langle n_Q \rangle^t}{\partial t} = -\gamma(\omega_Q) (\langle n_Q \rangle^t - \bar{n}_Q), \tag{25}$$

and the complex conjugate equation to (24). For $\Delta(\omega_Q)$ and $\gamma(\omega_Q)$ we find

$$\Delta(\omega_Q) = \frac{1}{\hbar} \operatorname{Im} \int_{-\infty}^{0} dt_1 e^{-\frac{it_1}{\hbar}(\omega_Q + i\varepsilon)} \langle [V_Q^+, V_Q(t_1)] \rangle, \tag{26}$$

$$\gamma(\omega_Q) = \frac{2}{\hbar^2} \operatorname{Re} \int_{-\infty}^{0} dt_1 e^{-\frac{it_1}{\hbar}(\omega_Q + i\varepsilon)} \langle [V_Q^+, V_Q(t_1)] \rangle. \tag{27}$$

These expressions were obtained in a similar manner in [1], [7] and [8]. They coincide with ones derived from self-energy operator of the retarded Green function $\langle\langle A_Q(t); A_Q^+(t_1)\rangle\rangle$. Eqs (26) and (27) may be used for the study of attenuation of sound in superfluid helium dielectric crystals [9].

ii) As a next example we shall consider systems for which operator V_i appearing in Eq. (4) can be written in the form

$$V_i = \sum_k V_{ki} a_k^+$$

where V_{ik} do not involve operators a and a^+. This is the case of scattering of electrons by impurities with an inner degrees of freedom. In this case correlation functions (17) can be approximated as follows

$$\langle V_i(t) V_i^+ \rangle = \sum_{kl} \langle V_{ki}(t) a_k^+(t) V_{li}^+ a_l \rangle \approx \sum_k \bar{n}_k \langle V_{ki}(t) V_{ki}^+ \rangle e^{iE_k t}.$$

Then instead of Eq. (20) we obtain

$$\Delta(E_i) = \sum_k{}' \bar{n}_k P \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \langle V_{ki} V_{ki}^+ \rangle_\omega \frac{e^{\beta(\hbar\omega + E_k)} - 1}{\hbar\omega - E_i + E_k}$$

and

$$\gamma(E_i) = \frac{1}{\hbar^2} \sum_k \frac{\bar{n}_k}{\bar{n}_i} \langle V_{ki} V_{ki}^+ \rangle_{\hbar\omega = E_i - E_k}.$$

The correlation function $\langle V_{ki} V_{ki}^+ \rangle$ describes fluctuations of scatterers. As an example let us take conduction electrons interacting with the magnetic impurity. The Hamiltonian of the systems is

$$\mathcal{H} = \sum_{k_1, s_1} \varepsilon_{k_1 s_1} C_{k_1 s_1}^+ C_{k_1 s_1} - \frac{1}{2N} \sum_{\substack{k_1 s_1 \\ k_2 s_2}} J_{k_1 k_2} \mathbf{S} \cdot \boldsymbol{\sigma}_{s_1 s_2} C_{k_1 s_1}^+ C_{k_2 s_2}$$

where $C_{k_1 s_1}^+$, $C_{k_1 s_1}$ are operators of electrons with momentum k_1 and spin s_1 $\boldsymbol{\sigma}$ is a vector with Pauli matrix components. Here the subsystem consists of an electron with momentum

q and spin s, the rest of the system forms the thermal bath. The equivalent Hamiltonian is of the form (2), where

$$H_{\text{int}} = C_{qs}^+ V_{qs}^+ + \text{h.c.}$$

and

$$V_{qs}^+ = -\frac{1}{2N} \sum_{(k_1 s_1) \neq (qs)} J_{qk_1} \mathbf{S} \cdot \boldsymbol{\sigma}_{ss_1} C_{k_1 s_1}.$$

The Hamiltonian of the thermal bath is

$$H_T = \sum_{(k_1 s_1) \neq (qs)} \varepsilon_{k_1} C_{k_1 s_1}^+ C_{k_1 s_1} - \frac{1}{2N} \sum_{(k_1 s_1), (k_2 s_2) \neq (qs)} J_{k_1 k_2} \mathbf{S} \cdot \boldsymbol{\sigma}_{s_1 s_2} C_{k_1 s_1}^+ C_{k_2 s_2}.$$

If we do not make any further approximation we obtain formulas for $\Delta(\varepsilon_q)$ and $\gamma(\varepsilon_q)$ by simply replacing in (20a, b) V_i^+ by V_{qs}. A further simplification can be made by decoupling the correlation function in the manner mentioned above, taking into account the symmetry of the system. So finally we obtain

$$\Delta(\varepsilon_q) = -\frac{3}{8N^2} \sum_{(k_1 s_1) \neq (qs)} |J_{qk_1}|^2 \bar{n}_{k_1} P \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \langle S^+ S^- \rangle_\omega \frac{1 - e^{\beta(\hbar\omega - \varepsilon k_1)}}{\hbar\omega - \varepsilon_q + \varepsilon_{k_1}}$$

and

$$\gamma(\varepsilon_q) = \frac{3\pi}{8\hbar N^2} \sum_{(k_1 s_1) \neq (qs)} |J_{qk_1}|^2 \frac{\bar{n}_{k_1}}{n_q} \langle S^+ S^- \rangle_{\hbar\omega = \varepsilon q - \varepsilon k_1}.$$

These equations relate the energy shift and relaxation time $\tau_q = \gamma^{-1}(\varepsilon_q)$ of conduction electrons to the fluctuations of the magnetic moment of the impurity [10].

iii) The last example concerns two-particle interaction of magnons [11]

$$\mathcal{H} = \sum_k \varepsilon_k a_k^+ a_k + \sum_{k_1, k_2, k_3, k_4} \psi_S(k_1, k_2; k_3, k_4) \Delta(k_1 + k_2 - k_3 - k_4) a_{k_1}^+ a_{k_1}^+ \times$$

$$\times a_{k_3} a_{k_4} + \text{h.c.}$$

The Hamiltonian of the subsystem and thermal bath is

$$H_0 = \varepsilon_q a_q^+ a_q + \sum_{k_1 k_2 k_3 k_4 \neq q} \psi_S(k_1, k_2; k_3, k_4) \Delta(k_1 + k_2 - k_3 - k_4) a_{k_1}^+ a_{k_2}^+ a_{k_3} a_{k_4} + \text{h.c.}$$

and interaction between them is

$$H_{\text{int}} = a_q^+ \Big\{ 2 \sum_{k_2 k_3 k_4 \neq q} \psi_S(q, k_2; k_3, k_4) \Delta(q + k_2 - k_3 - k_4) a_{k_2}^+ a_{k_3} a_{k_4} \Big\} + \text{h.c.}$$

We calculate the relaxation time assuming that the evolution of the subsystem is described by the free Hamiltonian

$$H^{(0)} = \sum_k \varepsilon_k n_k.$$

Then we have (*cf.* [11])

$$\frac{1}{\tau(\varepsilon_q)} = \frac{16}{\pi\hbar} \sum_{k_2 k_3 k_4 \neq q} |\psi_S(q, k_2; k_3, k_4)|^2 \Delta(q + k_2 - k_3 - k_4) \times$$

$$\times \delta(\varepsilon_q + \varepsilon_{k_2} - \varepsilon_{k_3} - \varepsilon_{k_4}) \{\bar{n}_{k_2}(1 + \bar{n}_{k_3})(1 + \bar{n}_{k_4}) - (1 + \bar{n}_{k_2})\bar{n}_{k_3}\bar{n}_{k_4}\}.$$

4. Connection with theory of linear response

Now we shall show that equations similar to (18) can be derived from the linear response theory of Kubo [2]. Let us consider a linear response of the phonon system which we have already considered in Section 3, for an external disturbance of the form

$$H_t = \sum_{i\alpha} F_\alpha(X_i; t) u_{i\alpha}, \quad i = 1, \ldots, N, \quad \alpha = 1, 2, 3,$$

where X_i is the radius vector of the i-th atom and $\boldsymbol{u}_i$ describes the displacement operator of this atom. From the linear response theory it follows that

$$\langle a_Q \rangle^t = \int\limits_{-\infty}^{\infty} dt_1 \langle\langle a_Q(t); H_{t_1}(t_1) \rangle\rangle^r =$$

$$= \sum_{i,\alpha} \int\limits_{-\infty}^{\infty} dt_1 F_\alpha(X_i, t_1) \langle\langle a_Q(t); u_{i\alpha}(t_1) \rangle\rangle^r.$$

Now we expand the displacement operator u_i in plane waves, introduce the Fourier transform $G_Q(\omega)$ of the retarded Green function $\langle\langle a_Q(t); a_Q^+(t')^+ \rangle\rangle^r$ and neglect mixing of polarization. Then

$$\langle a_Q \rangle = \int\limits_{-\infty}^{\infty} dt_1 F_Q(t_1) \int\limits_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t_1)} G_Q(\omega - i\varepsilon), \tag{28}$$

where

$$F_Q(t_1) = \sum_i F(X_i, t) e^{-iq \cdot X_i} \frac{e_Q}{\sqrt{2MN\omega_Q}},$$

e_Q is polarization vector and M is the mass of atoms. In the pole approximation the Green function is equal to

$$G_Q(\omega - i\varepsilon) = (\omega - \tilde{\omega}_Q - i\tfrac{1}{2}\gamma(\omega_Q))^{-1} \tag{29}$$

where

$$\tilde{\omega}_Q = \omega_Q + \Delta(\omega_Q).$$

Inserting (29) into (28) and integrating over ω we get

$$\langle a_Q \rangle^t = -i \int_{-\infty}^{t} dt_1 F_Q(t_1) e^{-i(t-t_1)\omega_Q - (t-t_1)\frac{1}{2}\gamma(\omega_Q)}. \tag{30}$$

The measured quantity $\langle u_i \rangle^t$ is given by a linear combination of $\langle a_i^+ \rangle^t$ and $\langle a_i \rangle^t$. In experiments with sound attenuation one introduces into a crystal a single short-duration (relative to the time taken for one round trip of the pulse in the sample) pulse of stress into a solid, normal to two parallel faces of the sample. The resulting multiple reflections of this pulse from the two parallel faces indicate how rapidly, in term of time or distance traveled, do the successive echoes decrease in amplitude. Thus we observe the system after a time much longer than the duration of the impulse [12]. We can assume that $F_Q(t) \sim \delta(t-\tau)$ where τ denotes the moment of switching on of the external field. Differentiating Eq. (30) with respect to time t we get

$$i\hbar \frac{\partial \langle a_Q \rangle}{\partial t} = \hbar F_Q(t) + (\hbar\omega_Q - \tfrac{1}{2}i\hbar\gamma(\omega_Q)) \langle a_Q \rangle^t. \tag{31}$$

Equation (31) describes the behaviour of the system after a time much longer than the time of transient processes, but not too long (we make a pole approximation). In such a time interval the system is characterized by average values of a_Q, a_Q^+, n_Q, H_T and equations (18) and (31) have the same form. But there are considerable differences between them. In Eq. (31) the forces $F(t)$ are fixed, while Eq. (18) describes the evolution. This perturbation is in some sense "self-consistent" for the parameters $f_Q(t)$ being determined from (13).

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