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FERROMAGNETIC RESONANCE IN A FERROMAGNET WITH SINGLE-ION ANISOTROPY

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The Green functions method is used for calculating the magnetic susceptibility of a ferromagnet with single-ion anisotropy of the type $-\mathcal{K}\sum_f (S_f^z)^2$.

1. *Introduction*

In this paper the Green function technique is used for calculating the magnetic susceptibility of a magnetic crystal which is described by the Hamiltonian

$$H = -h\sum_f S_f^z - \tfrac{1}{2}\sum_{fg} J_{fg}\boldsymbol{S}_f\cdot\boldsymbol{S}_g - \mathcal{K}\sum_f (S_f^z)^2, \tag{1}$$

where $h = \mu g\mathcal{H}$ (μ being the Bohr magneton, g the gyromagnetic factor and $\mathcal{H}$ the magnetic field strength), J_{fg} is the exchange integral and $\mathcal{K}$ is the single-ion anisotropy constant.

Application of the Green function method in this case encounters difficulties. Because of the anisotropy term a Green function of the type $\langle\langle S^zS^+ + S^+S^z|B\rangle\rangle$ appears. The ordinary decoupling procedure [1] cannot be applied. Lines [2] has given a decoupling scheme for this function. His paper discusses the sensitivity of the Curie temperature to crystal field anisotropy and his approach is valid for arbitrary spin.

Recently, Potapkov [3] considered the thermodynamic properties of the same ferromagnet for spin $S = 1$. His method is based solely on spin algebra and yields a closed set of equations for the Green functions. The most general method based on spin algebra, valid for arbitrary spin, has been developed by Devlin [4]. Potapkov and Devlin use the usual decoupling procedure [1]; hence, their methods are equivalent to the RPA--approximation. The papers [2, 3, 4] do not contain calculations of the magnetic susceptibility χ.

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Tyablikov [1] has presented a technique of findig χ for an isotropic ferromagnet. His calculations do not take into account the part of the demagnetization energy which is due to the spin operators in the same lattice sites.

In this paper we find χ by the method described in [1], and take into account the terms which Tyablikov had omitted. The decoupling procedure of Lines is used for arbitrary spin and the method of Potapkov-Devlin for spin $S = 1$. In Section 2 we present, using the Lines decoupling procedure the calculations of the magnetization σ and $\langle (S^z)^2 \rangle$. In Section 3 we recall some results obtained by Potapkov and Devlin, and in Section 4 we calculate the magnetic susceptibility in both cases.

2. *The method of Lines*

In order to calculate the magnetization σ, $\langle (S^z)^2 \rangle$ and χ we have to know the Green function (*cf.* [1]):

$$G_{mn}(t) = \langle\langle S_m^+(t) | B_n(0) \rangle\rangle^{(r)} = \Theta(t) \langle [S_m^+(t), B_n(0)] \rangle, \tag{2}$$

where

$$B_n(0) = (S_m^z)^{k'} S_n^-, \qquad k' = 0, 1, \ldots, 2S-1.$$

The equation of motion is

$$i \frac{d}{dt} G_{mn}(t) = i\delta(t) \langle [S_m^+, B_n] \rangle + hG_{mn}(t) + \sigma J(0) G_{mn}(t) -$$
$$- \sigma \sum_f J_{fm} G_{fn}(t) - \mathscr{K} \langle\langle S_m^z(t) S_m^+(t) + S_m^+(t) S_m^z(t) | B_n(0) \rangle\rangle. \tag{3}$$

The simplest decoupling scheme [1] is used to obtain Eq. (3). According to Lines [2] we can write

$$\langle\langle S_m^z(t) S_m^+(t) + S_m^+(t) S_m^z(t) | B_n(0) \rangle\rangle = \Phi(B_n) G_{mn}(t), \tag{4}$$

where $\Phi(B_n)$ is a constant which can be easily calculated by putting $t = 0$. This decoupling is valid for arbitrary spin. Taking the Fourier time and space transformations we obtain

$$G_k^{(r)}(\Omega) = \frac{i \langle [S^+, B] \rangle}{\Omega - h - \sigma(J(0) - J(k)) - \mathscr{K} \Phi(B) + i\delta}. \tag{5}$$

In the same way as in [1] we may calculate the magnetization σ and $\langle (S^z)^2 \rangle$. For spin $S = 1$ we have

$$\sigma = \frac{1 + 2P_1}{1 + P_0 + 2P_1 + 3P_0 P_1}, \tag{6a}$$

$$\langle (S^z)^2 \rangle = \frac{1 + 2P_1 + 2P_0 P_1}{1 + P_0 + 2P_1 + 3P_0 P_1}, \tag{6b}$$

where

$$P_{k'} = \frac{1}{N} \sum_{k} \frac{1}{\exp\{1/kT[h+\sigma(J(0)-J(k))+\mathscr{K}\Phi_{k'}]\}-1}$$

and

$$\Phi_{k'} = \Phi(B) = \begin{cases} \dfrac{3\langle(S^z)^2\rangle - 2}{\sigma} = \dfrac{\lambda}{\sigma}, & \text{for } k' = 0, \\ -1 \qquad\qquad , & \text{for } k' = 1. \end{cases}$$

The nonlinear set of equations (6a) and (6b) is valid for arbitrary temperature and in principle is solvable.

As a special case we can consider σ and $\langle(S^z)^2\rangle$ at $T = 0$. We obtain then, including the terms proportional to $T^{3/2}$, the following formula

$$\sigma = \langle(S^z)^2\rangle = 1 - \frac{v}{[(\bar{\delta})^2]^{\frac{3}{2}}} \left(\frac{3kT}{2\pi J(0)}\right)^{\frac{3}{2}} Z_{\frac{3}{2}} \left[\frac{1}{kT}(h - \mathscr{K}\Phi(S^-))\right], \tag{7}$$

where

$$(\bar{\delta})^2 = \frac{\sum f^2 J(f)}{J(0)}, \qquad v = \frac{V}{N}, \qquad z_p(x) = \sum_{n=1}^{\infty} n^{-p} e^{-nx}.$$

Assuming $\dfrac{\mathscr{K}}{J(0)} \ll 1$ we can calculate from (6a) and (6b) the Curie temperature

$$kT_C = \frac{2J(0)}{3C} - \frac{3}{2}\mathscr{K}, \tag{8}$$

where C is a constant.

3. *The method of Potapkov and Devlin*

The method of Devlin [4] is general and is valid for arbitrary spin S. The number of equations which are to be solved is $2S$. For $S = 1$ Devlin's method is equivalent to that of Potapkov [3]. We recall some results presented in [3]. In the low temperature region (including the terms proportional to $T^{3/2}$) an identical result (7) is obtained for σ and $\langle(S^z)^2\rangle$. For the Curie temperature (using similar approximations) one yields

$$kT_C = \frac{2J(0)}{3C} + \frac{\mathscr{K}}{3C}. \tag{9}$$

Hence, in the low temperature region both methods (Lines and Potapkov-Devlin) are in good agreement. At higher temperatures, however, the agreement is not so good.

4. Calculations of the magnetic susceptibility

a) In our opinion, calculation of the magnetic susceptibility χ according to the decoupling approximation of Lines should prove useful.

Consider a spheroidal ferromagnetic sample. The Hamiltonian including the demagnetization energy is

$$H = S(S+1)NB-(h-B)\sum_{f} S_f^z - \tfrac{1}{2}\sum_{fg}(J_{fg}-2C)S_f^z S_g^z -$$

$$- \tfrac{1}{2}\sum_{fg}(J_{fg}-2B)S_f^+ S_g^- -(\mathscr{K}+B-C)\sum_{f}(S_f^z)^2, \tag{10}$$

where N is the number of spins,

$$B = \frac{\mu^2}{2N}M_1, \qquad C = \frac{\mu^2}{2N}M_2, \tag{11}$$

M_1, M_2 are the demagnetization factors. The sample is placed into a magnetic field alternating with frequency Ω. The χ-tensor can be calculated from the expression (*cf.* [1])[1]

$$\chi_{\alpha\beta}(\Omega) = i\mu^2\langle\langle S^\alpha | S^\beta\rangle\rangle_\Omega^{(r)}, \tag{12}$$

where

$$\alpha, \beta = 1, 2, \qquad S^\alpha = \sum_f S_f^\alpha,$$

$$\chi_{\alpha 3} = \chi_{3\alpha} = 0, \text{ for } \alpha = 1, 2,$$

$$\chi_{33} = \chi_{st.}\,.$$

To get (12) we need the Green functions $\langle\langle S^+|S^-\rangle\rangle_\Omega^{(r)}$ and $\langle\langle S^-|S^+\rangle\rangle_\Omega^{(r)}$. These functions can be easily obtained by (5) interchanging

$$h \to h-B, \qquad J(\boldsymbol{k}) \to J(\boldsymbol{k})-2BN\delta(\boldsymbol{k}),$$

$$J(0) \to J(0)-2CN, \qquad \mathscr{K} \to \mathscr{K}+B-C. \tag{13}$$

Thus

$$\langle\langle S^+|S^-\rangle\rangle_\Omega^{(r)} = NG_0(\Omega) \tag{14}$$

and according to (5) and (13)

$$G_0(\Omega) = \frac{2i\sigma}{\Omega-\Omega_0+i\delta}, \tag{15}$$

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[1] The factor 2π in (12) is omitted because it is dealt with by the definition of the Fourier time transformation.

where

$$\Omega_0 = h-B+(\mathscr{K}+B-C)\Phi(S^-)+2N\sigma(B-C), \tag{16}$$

$$\Phi(S^-) = \frac{3\langle(S^z)^2\rangle - S(S+1)}{\sigma}. \tag{17}$$

In a similar way we find

$$\langle\langle S^- | S^+ \rangle\rangle_\Omega^{(r)} = -N \frac{2i\sigma}{\Omega+\Omega_0+i\delta}. \tag{18}$$

The χ-tensor (12) can be expressed in the form

$$\chi_{\alpha\beta}(\Omega) = \chi'_{\alpha\beta}(\Omega)+i\chi''_{\alpha\beta}(\Omega), \tag{19}$$

where

$$\chi'_{11}(\Omega) = \chi'_{22}(\Omega) = \frac{\mu^2 N\sigma}{2}\left[P\left(\frac{1}{\Omega+\Omega_0}\right)-P\left(\frac{1}{\Omega-\Omega_0}\right)\right],$$

$$\chi'_{21}(\Omega) = -\chi'_{12}(\Omega) = \frac{\mu^2 N\pi\sigma}{2}\left[\delta(\Omega+\Omega_0)+\delta(\Omega-\Omega_0)\right],$$

$$\chi''_{11}(\Omega) = \chi''_{22}(\Omega) = \frac{\mu^2 N\pi\sigma}{2}\left[\delta(\Omega-\Omega_0)-\delta(\Omega+\Omega_0)\right],$$

$$\chi''_{21}(\Omega) = -\chi''_{12}(\Omega) = \frac{\mu^2 N\sigma}{2}\left[P\left(\frac{1}{\Omega+\Omega_0}\right)+P\left(\frac{1}{\Omega-\Omega_0}\right)\right]. \tag{20}$$

The formulae (20) are valid for arbitrary spin. Unknown σ and $\langle(S^z)^2\rangle$ for $S = 1$ are obtainable from (6a) and (6b) with (13) taken into account.

b) To obtain the formulae for the Green functions in Devlin's method we must solve the set of $2S$ equations. Unfortunately, to get a general solution is quite impossible. Therefore, we restrict ourselves to the case $S = 1$, following Potapkov [3]. Using the Green functions given in [3] and taking into account (13) and (14) we get

$$\langle\langle S^+ | S^- \rangle\rangle_\Omega^{(r)} = \frac{iL_1}{\Omega-\Omega_++i\delta}+\frac{iL_2}{\Omega-\Omega_-+i\delta}, \tag{21}$$

where

$$L_1 = \frac{N}{2\beta}\left[2(\mathscr{K}+B-C)\lambda-\sigma^2(J(0)-2BN)+2\sigma\beta\right],$$

$$L_2 = \frac{N}{2\beta}\left[-2(\mathscr{K}+B-C)\lambda+\sigma^2(J(0)-2BN)+2\sigma\beta\right],$$

$$\beta = \left[(\mathscr{K}+B-C)\,(\mathscr{K}+B-C-\lambda(J(0)-2BN))+\tfrac{1}{4}\sigma^2(J(0)-2BN)^2\right]^{1/2},$$

$$\Omega_\pm = h-B+\tfrac{1}{2}\sigma J(0)+N\sigma(B-2C)\pm\beta. \tag{22}$$

Similarly, we find

$$\langle\langle S^-|S^+\rangle\rangle_\Omega^{(r)} = -\frac{iL_1}{\Omega+\Omega_+ + i\delta} - \frac{iL_2}{\Omega+\Omega_- + i\delta}. \tag{23}$$

Using (12), (21) and (23), (19) we obtain the χ-tensor:

$$\chi'_{11} = \chi'_{22} = \frac{\mu^2}{4}\left\{L_1\left[P\left(\frac{1}{\Omega+\Omega_+}\right) - P\left(\frac{1}{\Omega-\Omega_+}\right)\right] + \right.$$

$$\left. + L_2\left[P\left(\frac{1}{\Omega+\Omega_-}\right) - P\left(\frac{1}{\Omega-\Omega_-}\right)\right]\right\},$$

$$\chi'_{21} = -\chi'_{12} = \frac{\pi^2\mu^2}{4}\{L_1[\delta(\Omega+\Omega_+)+\delta(\Omega-\Omega_+)]+L_2[\delta(\Omega+\Omega_-)+\delta(\Omega-\Omega_-)]\},$$

$$\chi''_{11} = \chi''_{22} = \frac{\pi^2\mu^2}{4}\{L_1[\delta(\Omega-\Omega_+)-\delta(\Omega+\Omega_+)]+L_2[\delta(\Omega-\Omega_-)-\delta(\Omega+\Omega_-)]\},$$

$$\chi''_{21} = -\chi''_{12} = \frac{\mu^2}{4}\left\{L_1\left[P\left(\frac{1}{\Omega+\Omega_+}\right) + P\left(\frac{1}{\Omega-\Omega_+}\right)\right] + \right.$$

$$\left. + L_2\left[P\left(\frac{1}{\Omega+\Omega_-}\right) + P\left(\frac{1}{\Omega-\Omega_-}\right)\right]\right\}. \tag{24}$$

The unknown σ and $\langle(S^z)^2\rangle$ are given in [3], but for our purposes we must use the substitution (13). By putting $\mathscr{K} = 0$ in (20) or (24) we find the χ-tensor for the isotropic ferromagnet.

Unlike the formulae in [1], the formulae (20) and (24) contain the terms of demagnetization energy due to the spin operators in the same lattice sites.

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