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PARTIALLY SPACE COHERENT DIFFRACTION BY AN ANNULAR APERTURE WITH AMPLITUDE FILTER

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A theoretical study is made of the effect of partial coherence on the diffraction of a defocused annular aperture having an amplitude filter and illuminated by partially coherent light. It is shown that the aberrant images can be improved with the help of an appropriate amplitude filter. The intensity distribution has been compared for cases with and without filter. The effect of the filter is more marked when the obstruction ratio is small and becomes less significant for the higher value of the central obstruction ratio. Two forms of the mutual coherence function, three values of the aberration coeficient and four values of the correlation intervals have been assumed.

1. Introduction

The criteria such as the resolving power, Strehl intensity encircled energy and optical transfer functions *etc.* have been used to assess the system's performance. These criteria appear somewhat arbitrary in nature since they take into account the transfer characteristics of the imaging system and assume the absence of aberrations in the optical system but this disturbing effect of aberrations is not always absent, hence one has to study the effect of aberration on the image formation in an optical system and effort should be made to reduce them by means of various devices. If no due care of the effect of aberrations is taken, then these aberrations are very likely to impair the sharpness of the images to such an extent that either the optical system becomes unserviceable or, in the case of simultaneous focusing on two objects at defferent distances from the objective, disturbing effects are produced, unless the relative aperture is reduced [1]. In fact, we find successful examples of a filter for this type of compensation in the correcting plate of Schmidt camera and the aspheric mirror for the reflecting objectives of microscopes used in the ultraviolet region [2].

The annular aperture is of high importance because of its use in the optical systems employing reflecting components. Welford [3] pointed out some applications where the enhanced side lotes in the diffraction pattern of an annular aperture, are not of much significance

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and the consideration of the central pattern becomes important there. An account of the previous investigations concerning the importance of annular aperture has already been given in detail by Singh and Dhillon [4], Singh and Kavatheker [5] and Katti *et al.* (6). The interest in the subject continues to grow and some additional papers [7–17] may also be mentioned in connection with annular aperture. In recent times Lit [18] has given the concept of the centre-π phased lens (CPPL) which is a specific form of the centre blocked annular aperture in which the inner zone has a phase retardation of π radian with respect to the outer annular zone by the use of phase coating. This is found to give better performance then the centre blocked lens.

The present paper deals with an anuular aperture suffering from defect of focus. An appropriate amplitude filter is used in the central peak in order to remove the defect of focus in partially coherent illumination. It is seen that the effect of defocusing is minimized by the use of such an amplitude filter. The intensity distribution has been calculated in the central Airy pattern of a defocused annular aperture and illuminated by partially coherent light. The same has also been done with filter in order to predict the improvement in the abnormal behaviour of the intensity distribution. The calculations provided a more detailed graphical description of the performance characteristics of the defocused optical system. The filter which will be discussed in the present paper is widely used for making apodizers which are frequently used for the improvement of the intensity distribution in various optical instruments.

2. *Theory*

The theory of the present paper deals with quasimonochromatic partially space coherent fields, *i.e.* the case when the spectral spread of radiations is small compared to the mean frequency of radiation. This approximation allows us to ignore variations with frequency. Use has been made of Schell's theorem [19–20] which establishes the relationship between the far-field intensity and the Fourier-transform of the product of the source auto-correlation function and the correlation interval function across the diffracting aperture. Here we assume that the aperture radius is much larger than the mean wavelength and therefore the angles of diffraction are small. The intensity distribution in the image plane is given by:

$$I(\theta) = \frac{2\pi^2 a^4}{\bar{\lambda}^2} \int_0^2 c(\varrho)\gamma(\alpha\varrho)J_0(\bar{k}a\theta\varrho)\varrho d\varrho \tag{1}$$

where θ — is the diffraction angle, a — the aperture radius, $\bar{\lambda}$ — the mean wavelength, $\bar{k} = 2\pi/\bar{\lambda}$ — the wave number, $c(\varrho)$ — the transfer function of the aperture amplitude, α — the number of correlation intervals contained in the aperture plane, $\gamma(\alpha\varrho)$ — the normalized mutual coherence function.

To calculate the auto-correlation function of the optical system (which by definition is also the transfer function of the optical system) suffering from the defect of focus, we make use of the sampling theorem technique of Barakat [21]. According to him, the transfer function for a circular aperture with radially symmetric aberration is given by

$$C(\varrho) = \sum_{n=1} [J_1^2(A_n)]^{-1} t(A_n/2) \cdot J_0(A_n\varrho/2) \tag{2}$$

where J_0 and J_1 are the zeroth order and first order Bessel functions respectively, A_n are the positive zeros of J_0 and $t(A_n/2)$ is the point spread function evaluated at $A_n/2$.

The point spread function for an annular aperture in the presence of defocusing is given by

$$t(A_n/2) = \left| \frac{2}{1-\varepsilon^2} \exp(ik W_{020}\varrho_1^2) J_0(A_n\varrho_1/2)\varrho_1 d\varrho_1 \right|^2 \tag{3}$$

where W_{020} is the coefficient of defocusing measured in the units of wavelength. Exp $(ik W_{020}\varrho_1^2)$ is the pupil function and ε is the aperture obstruction ratio.

For a complex degree of coherence we assume the following two forms:

$$\gamma(\alpha\varrho) = \exp(-\alpha^2\varrho^2) \text{ Gaussian-correlation}$$

$$\gamma(\alpha\varrho) = 2J_1(\alpha\varrho)/(\alpha\varrho) \text{ Besinc-correlation.}$$

It is well known that the Gaussian form comes into picture at the turbulent medium and the Besinc form when the source is a circular one. At this stage it is interesting to note that the Besinc form was derived in Ref. [22] on the basis of resonator theory. The various forms of mutual coherence function have been frequently used in the Refs [23–24].

3. *Amplitude filter in a defocused optical system with annular aperture*

The imaging characteristics of the defocused optical system can be improved by placing a filter of amplitude transmission factor $T(\varrho_1)$ at the pupil of the objective. The pupil function of the system becomes then

$$F(\varrho_1) = T(\varrho_1) \exp(ik W_{020}\varrho_1^2). \tag{4}$$

The pupil function is constant if the optical system is without aberration. Hence in order to remove the abnormal behaviour of the intensity distribution due to the harmful effect of defocusing on the image, $T(\varrho_1)$ must take the form

$$T(\varrho_1) = \exp(-ik W_{020}\varrho_1^2).$$

Such a filter is a phase filter which gives inverse phase retardation to that of defocusing but its practical use in the optical system for the removal of defocusing is very difficult and this can be discarded from its practical utility point of view.

To avoid the above difficulty a filter, whose transmission factor is real, is suggested by Tsujiuchi [25]. This type of filter is defined by

$$T(\varrho_1) = \tfrac{1}{2}\{1+\cos(k W_{020}\varrho_1^2+\delta)\} \tag{5}$$

where the value of the parameters δ depends upon the magnitude of W_{020}. Following Tsujiuchi

$$\delta = (2-W_{020})\pi \quad \text{for} \quad 0 \leqslant W_{020} < 1$$
$$= (3-W_{020})\pi \quad \text{for} \quad 1 \leqslant W_{020} < 2. \tag{6}$$

Now if we use the pupil function defined by

$$F(\varrho_1) = \tfrac{1}{2}\{1+\cos(\bar{k}\, W_{020}\, \varrho_1^2+\delta)\} \exp(i\bar{k}\, W_{020}\, \varrho_1^2) \tag{7}$$

in the place of $\exp(i\bar{k}\, W_{020}\varrho_1^2)$ in equation (3) we obtain the modified intensity distribution.

4. *Results and discussion*

The integral expressed by equation (1) has been evaluated on an ICT 1909 electronic computer. In order to have a matched accuracy, the transfer function has been calculated using a 64 point Gaussian quadrature method [26]. Expression for $C(\varrho)$ given by equation(2) was used together with the expression for the forms of degree of coherence $\gamma(\alpha\varrho)$. The entire set of calculation was performed for the two forms of correlation function, two values of obstruction ratio $\varepsilon = 0.25$, 0.75, four values of the coefficient of defocusing W_{020} ($W_{020} = 0.0$. 0.5 and 1.0) and four values of $\alpha = 0.0$, 0.5, 1.0 and 2.0). The calculation was performed with and without filter in the defocused optical system.

The behaviour of the normalized central intensity distribution (obtained by dividing the intensity at each point by the intensity at the point $\bar{k}a\theta/\pi = 0.0$ when $\alpha = 0.0$ and $W_{020} = 0.0$) per unit solid angle for $\varepsilon = 0.25$ and $\alpha = 0.0$, 0.5 and 1.0, is shown in figures (1a–1c) with Besinc correlation and for $\alpha = 0.5$ and 1.0 is shown in figures (2a–2b) with Gaussian correlation. Figure for $\alpha = 0.0$ is the same for both the forms. In Figs 1 and 2 the solid and the dotted lines represent the results without and with amplitude filter respectively. In each of these figures a set of curves illustrates the effect of varying W_{020} while α is held constant. From the trend of the curves drawn in figures 1 and 2 it is clear that as W_{020} increases, the central fringe widens, the central intensity decreases and the diffraction pattern tends towards the form of incoherent limit. The Gaussian curves tend more rapidly towards incoherent limit than Besinc curves. It is seen from figures 1 and 2 that the maximum intensity in the central peak is higher with filter than without filter which is the net improvement in the central intensity due to the presence of an amplitude filter. The effect of amplitude filter is more marked for higher values of W_{020}. For example the improvement is more pronounced when $W_{020} = 0.50$ than $W_{020} = 1.0$ in both forms of correaltion function and for the same value of α.

Figs 2a and 2b present the normalized central intensity distribution for Besinc form of correlation and for $\alpha = 0.5$ and $\alpha = 2.0$. Here we have not shown the curve for $\alpha = 0.0$ because this is found to be the same for Besinc correlation as that for Gaussian correlation and hence common to both correlations shown in Fig. 1a. Comparing these graphs with earlier ones, we reach the conclusion that the effect of the filter is the same in both cases except that the Gaussian curves tend more rapidly towards the incoherent limit than the Besinc curves.

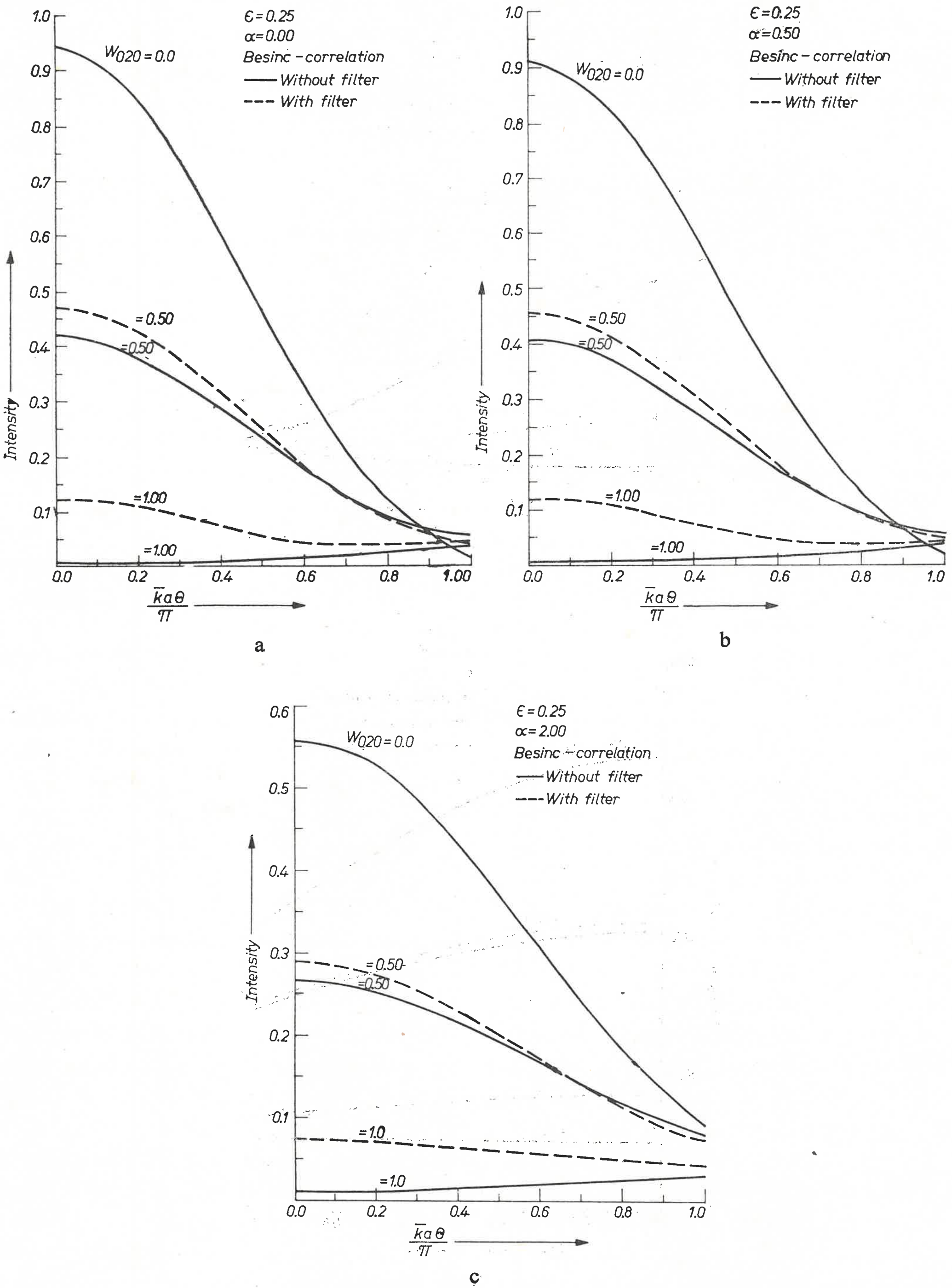

Fig. 1. Normalized central intensity for Besinc correlation, $\varepsilon = 0.25$ and $W_{020} = 0.0$, 0.5 and 1.0.
a) $\alpha = 0.0$; b) $\alpha = 0.5$ and c) $\alpha = 2.0$

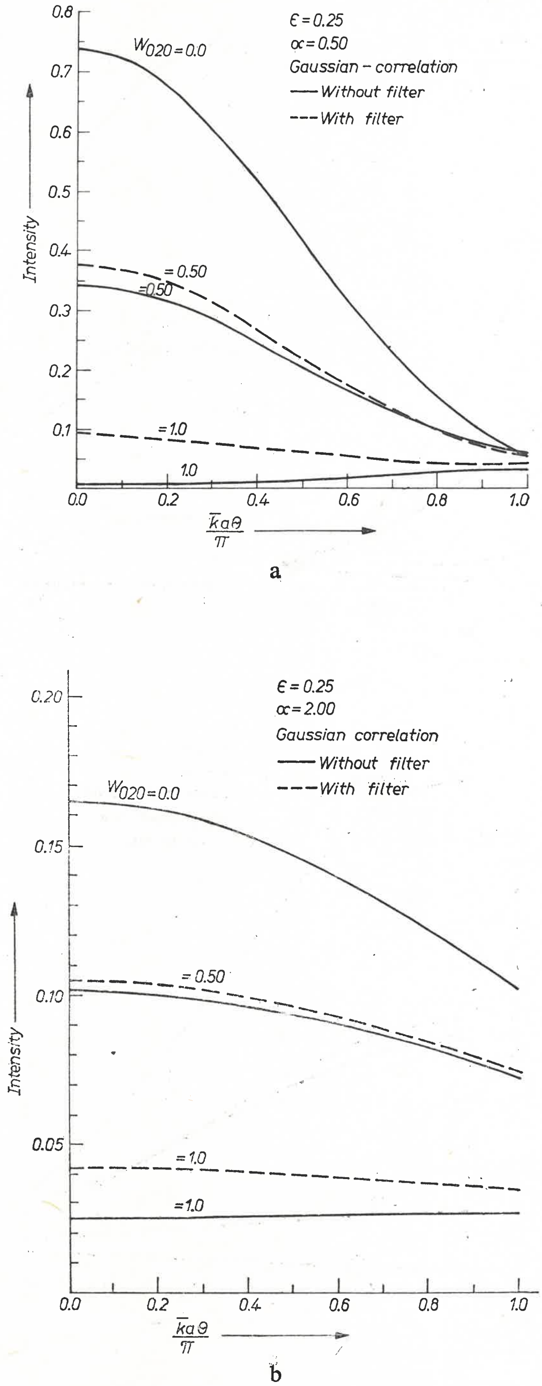

Fig. 2. Normalized central intensity for Gaussian form of correlation, $\varepsilon = 0.25$ and $W_{020} = 0.0$, 0.5 and 1.0. a) $\alpha = 0.5$ and b) $\alpha = 2.0$

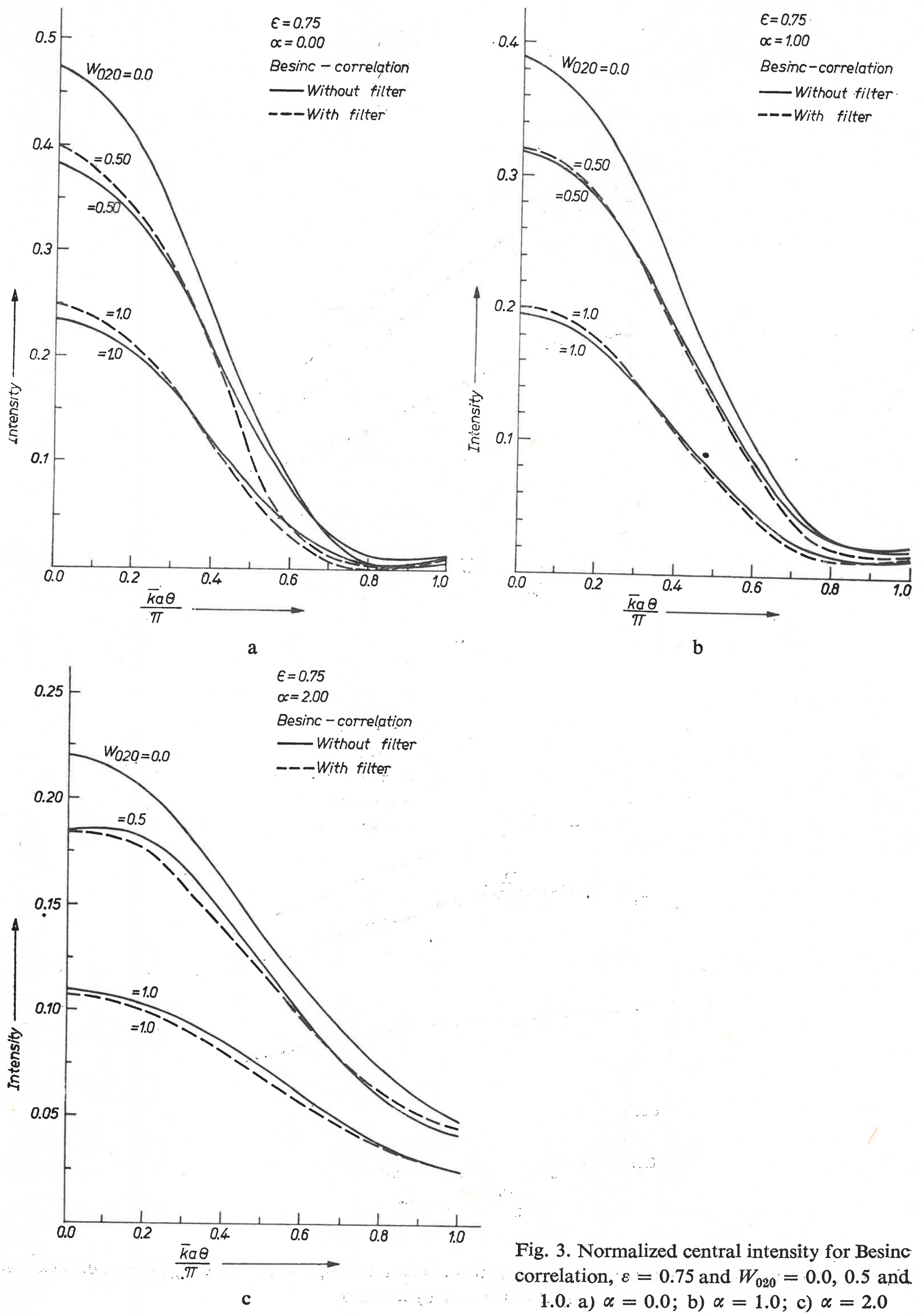

Fig. 3. Normalized central intensity for Besinc correlation, $\varepsilon = 0.75$ and $W_{020} = 0.0$, 0.5 and 1.0. a) $\alpha = 0.0$; b) $\alpha = 1.0$; c) $\alpha = 2.0$

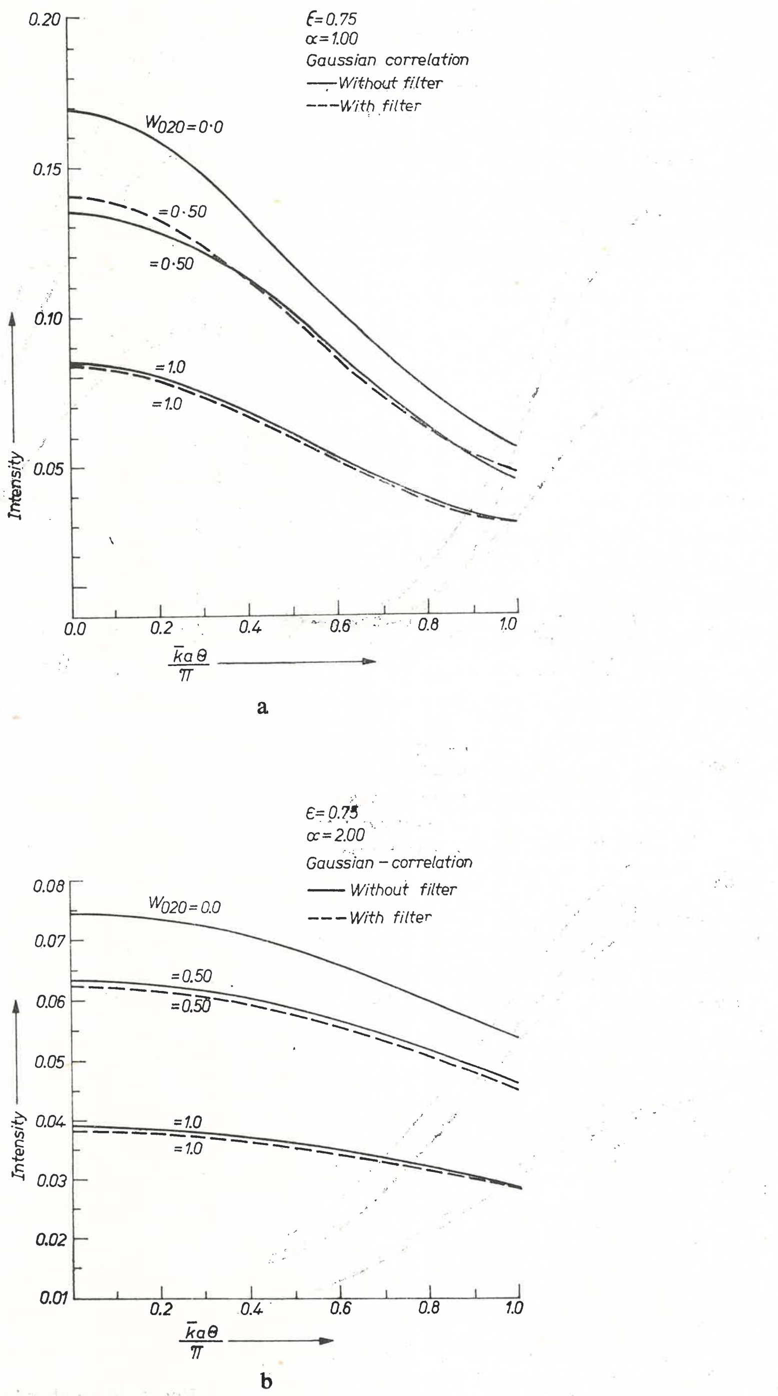

Fig. 4. Normalized central intensity for Gaussian form of correlation, $\varepsilon = 0.75$ and $W_{020} = 0.0, 0.5$ and 1.0. a) $\alpha = 1.0$ and b) $\alpha = 2.0$

Figs 3 and 4 show the variation in the central intensity *vs* W_{020} for Besinc and Gaussian forms of correlation at the obstruction ratio $\varepsilon = 0.75$ and $\alpha = 0.0$, 1.0, and 2.0. The value of α is held constant in each curve and W_{020} is made to vary. As the value of α increase the effect of the filter becomes less significant and the curves tend towards the incoherent limit. In this case there is a remarkable reduction in the fall of the central intensity which is due to the increase in W_{020}. Very little is gained up to $\alpha \geqslant 1.0$ when an appropriate amplitude filter is used in both forms of correlation function. It is therefore advisable not to use an amplitude filter with an annular aperture at such a high obstruction ratio and at a large value of correlation intervals when the sytem is suffering from the defect of focus. Hence an attempt to improve the system's performance by increasing the value of α with amplitude filter is not very effective at high obstruction ratio of an annular aperture.

Comparing the Figs 1 and 2 with 3 and 4, it is observed that the fall in the intensity due to increase in ε is reduced with and without filter compared to $\varepsilon = 0.25$. The improvement in the systems' performance on account of increase in α is also reduced. From the results obtained by other workers we reach the following conclusions:

1) The trend of the results obtained here is in agreement with the earlier results obtained by Singh and Dhillon [4, 27] without filter and with annular aperture. The amplitude filter has been used to show the improvement in the abnormal behaviour of the intensity distribution due to the defect of focus.

2) Though the results for exponential and Besinc forms of correlation have already been shown by Shore [28] and Shore *et al.* [29] without filter, their curves do not provide a very good idea of the instensity distribution in the main peak because the intensity has been plotted on the semi-logarithmic scale to bring out clearly the changes in the side lobes. Secondly, the problem considered by them was ideal, *i.e.* no defocusing was present whereas our main aim in presenting these results is to show clearly the changes that may arise due to defect of focus and then to improve this defect to some extent by the use of an appropriate amplitude filter in Gaussian and Besinc forms of correlation.

Figs 5 and 6 show the central intensity variation with α for Gaussian and Besinc correlations. The central intensity has been plotted with and without filter for cases of Gaussian and Besinc correlations, on the same graph. Two values of the obstruction ratio $\varepsilon = 0.25$, 0.75 have been assumed. Fig. 5 shows that the fall in the central intensity with α is more rapid with and without filter in the case of Gaussian correlation than that for Besinc correlation. The improvement by the introduction of an amplitude filter is more for $W_{020} = 1.0$ than $W_{020} = 0.5$ in both forms of correlation function. Fig. 6 for $\varepsilon = 0.75$ shows that the filter is only effective in the central intensity region for lower values of $\alpha \leqslant 1.0$ whereas for higher values of α the filter is not very effective.

Tables I and II list the normalized values of the central intensity for $\varepsilon = 0.25$ and 0.75 respectively and for both the functions of correlation with and without filter. The normalized values were obtained by using the actual values shown in the bracket. It is seen that as the value of α increases the requirement of W_{020} becomes less stringent. The depth of focus, as judged by the relative intensity at the centre of the out of focus Airy pattern is increased. In partially coherent light the focus shifts towards the aperture and

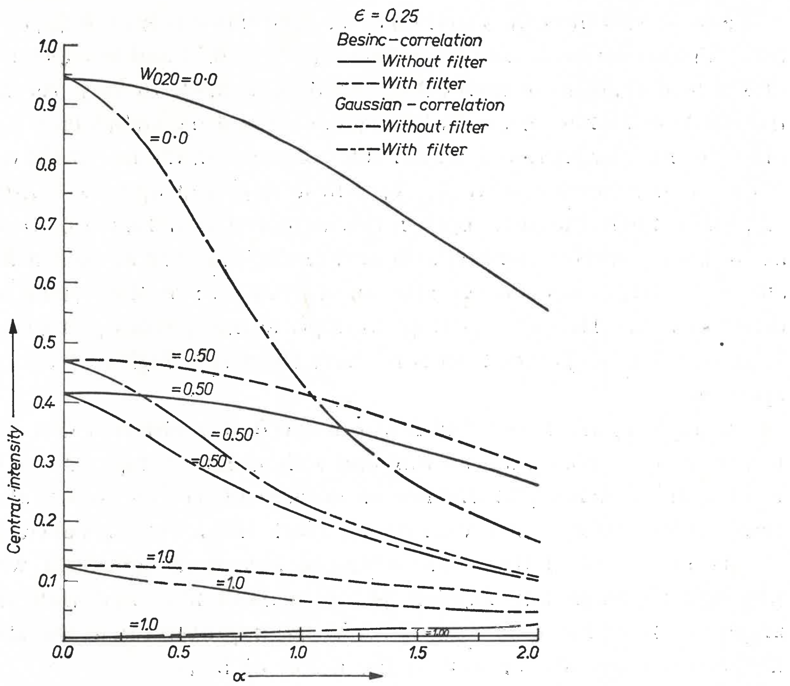

Fig. 5. Normalized central peak intensity for Gaussian and Besinc correlations, $\varepsilon = 0.25$ and $W_{020} = 0.0$, 0.5 and 1.0

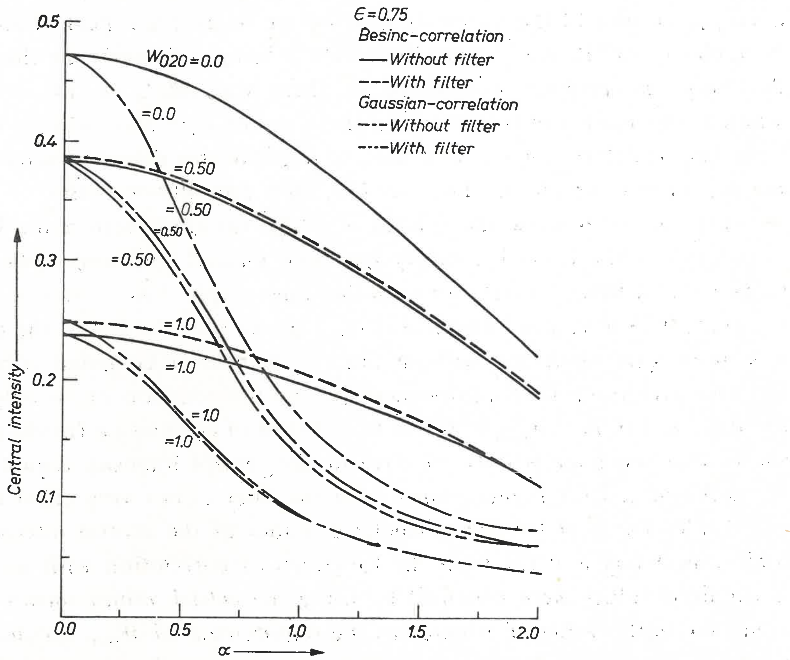

Fig. 6. Normalized central peak intensity for Gaussian and Besinc correlations, $\varepsilon = 0.75$ and $W_{020} = 0.0$, 0.5 and 1.0

TABLE I

Central intensity obstruction ratio $\varepsilon = 0.25$

α	W_{020}	Forms of correlation function			
		Besinc-correlation		Gaussian-correlation	
		Without filter	With filter	Without filter	With filter
0.0	0.0	1.0 (= 0.4810)	1.0 (= 0.4810)	1.0 (= 0.4810)	1.0 (= 0.4810)
	0.5	0.4415	0.4991	0.4415	0.4991
	1.0	0.0045	0.1326	0.0045	0.1326
0.5	0.0	1.0 (= 0.4653)	1.0 (= 0.4653)	1.0 (= 0.3766)	1.0 (= 0.3766)
	0.5	0.4429	0.4996	0.4614	0.5042
	1.0	0.0045	0.1315	0.0071	0.1279
1.0	0.0	1.0 (= 0.4215)	1.0 (= 0.4215)	1.0 (= 0.2201)	1.0 (= 0.2201)
	0.5	0.4474	0.5013	0.4925	0.5306
	1.0	0.0049	0.1283	0.0349	0.1508
2.0	0.0	1.0 (= 0.2875)	1.0 (= 0.2875)	1.0 (= 0.0839)	1.0 (= 0.0839)
	0.5	0.4709	0.5130	0.6197	0.6388
	1.0	0.0142	0.1276	0.1549	0.2574

TABLE II

Central intensity obstruction ratio $\varepsilon = 0.75$

α	W_{020}	Forms of correlation function			
		Besinc-correlation		Gaussian-correlation	
		Without filter	With filter	Without filter	With filter
0.0	0.0	1.0 (= 0.2408)	1.0 (= 0.2408)	1.0 (= 0.2408)	1.0 (= 0.2408)
	0.5	0.8118	0.8151	0.8118	0.8151
	1.0	0.5012	0.5311	0.5012	0.5311
0.5	0.0	1.0 (= 0.2293)	1.0 (= 0.2293)	1.0 (= 0.1693)	1.0 (= 0.1693)
	0.5	0.8137	0.8163	0.8458	0.8239
	1.0	0.5010	0.5499	0.5014	0.5121
1.0	0.0	1.0 (= 0.1982)	1.0 (= 0.1982)	1.0 (= 0.0863)	1.0 (= 0.0863)
	0.5	0.8178	0.8203	0.7311	0.8331
	1.0	0.5010	0.5191	0.5052	0.4959
2.0	0.0	1.0 (= 0.1124)	1.0 (= 0.1124)	1.0 (= 0.380)	1.0 (= 0.0380)
	0.5	0.8407	0.8354	0.8552	0.8394
	1.0	0.5026	0.4893	0.5236	0.5131

therefore the partially coherent illumination compensates to some extent the effect of defocusing. This fact is in agreement with Auria and Solimini [30–31] who pointed out that in partially coherent light the focus displaces towards the aperture. For $\varepsilon = 0.75$ the improvement with filter due to the change in W_{020} is not marked for higher values of α as it is more marked in case of $\varepsilon = 0.25$, a conclusion obvious from the table. From the table it is also clear that the improvement in the intensity distribution by the introduction of an amplitude filter is appreciably different for different correlations and is higher in the case of Besinc correlation with and without filter than for Gaussian correlation.

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