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MOMENTUM TRANSFER IN FRAUNHOFER DIFFRACTION OF LIGHT

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Basing on the analogy of the dependence of maximum current on the magnetic flux in a Josephson junction, as well as on the relation defining the Fraunhofer diffraction of light by a slit,a concept of the treatment of light diffraction is proposed which accentuates the corpuscular aspect of this phenomenon.

Making use of the idea of complex momentum and energy of the microparticle introduced by the author in another paper it is found that the wave equation is the continuity equation for the complex densities of momentum and energy of the photon beam. For the case of a monochromatic, uniform photon beam falling perpendicularly on an opaque plane screen with apertures, it is shown that the diffraction can be treated as the result of complex momentum transfer to the photons from the aperture. The process of interaction of photons with the aperture described here, enables us to conclude that the Fraunhofer diffraction of light by multi-slit diffraction gratings is accompanied by simultaneous interference.

The dependence of the maximum supercurrent $I_{\max}$ in a Josephson junction of the magnetic flux Φ has the characteristic form of the relation which describes the Fraunhofer diffraction by a slit

$$I_{\max} = I_c \frac{\sin (\pi\Phi/\Phi_0)}{\pi\Phi/\Phi_0} \tag{1}$$

where Φ_0 denotes the magnetic flux quantum, and I_c is a constant. One is tempted to assume that this similarity is not merely accidental but has a deeper physical meaning, since it comes from the similarity of processes occurring in the motion of supercurrent carriers through a tunnel junction and the motion of photons through a slit in an opaque screen. In the first case we deal with the summation of current densities whose magnitudes and directions depend on the local value of the vector potential. Since this motion of Cooper pairs is associated with momentum we deal here also with the summation of variable momentum densities in the region of a tunnel junction [2]. Similarly in the diffra-

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ction of light beam by a narrow slit one can assume that owing to the interaction of photons with the screen there occurs the summation of momenta. This is the starting point to the attempt of describing the process of the diffraction of light by distinguishing its corpuscular aspect. According to the earlier published concept of the author [3] the motion of the microparticle can be described by means of a complex function which represents the space-time vibration and which has been called activity [1]. The behaviour of a many-particle system is determined by the density of the activity ψ which is obtained by summing up the activities of the particular particles of the system and which has all the features of a quantum-mechanical wave function, the product $\psi^*\psi$ being interpreted as the density of the active (genuine) presence of the particles.

Basing on the definitions accepted for microparticles with non-vanishing rest masses we can assume that the wave function of a photon beam φ which fulfils the wave equation

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = 0 \tag{2}$$

is the density of the activity of the photon beam.

Under this assumption the complex energy density is

$$E_c = i\hbar\frac{\partial\varphi}{\partial t} \tag{3}$$

and the density of complex momentum

$$\bar{p}_c = -i\hbar\nabla\varphi. \tag{4}$$

Making use of Eqs (3) and (4) we can write the wave equation in the form

$$\operatorname{div}\vec{p}_c + \frac{1}{c^2}\frac{\partial E_c}{\partial t} = 0 \tag{5}$$

which has the form of a continuity equation

$$\operatorname{div}\vec{p}_c + \frac{\partial\varrho}{\partial t} = 0 \tag{6}$$

where the density of complex mass is $\varrho = \frac{E_c}{c^2}$. From the comparison with the continuity equation of classical hydrodynamics it follows that the densities of momentum and mass have been replaced by corresponding complex densities. Hence we obtain a practical rule: in problems concerning the wave-corpuscular properties of photons one should consider instead of classical energy and momentum values, the complex energy and complex momentum, respectively. If one accentuates the corpuscular aspect in the diffraction phenomenon at the edge of an aperture in a screen, one can assume that the deviation of the photon beam from the primary direction is due to the interaction with the boundary region of the screen and to the complex momentum transfer between the photon beam and the screen. If follows that the additional outflow of complex momentum of photon beams after diffraction by the aperture in the screen is due to interaction of the aperture edge and should depend on its shape.

The essential role of the edge of the aperture in the diffraction of light has been pointed out by Rubinowicz who suggested that the diffraction process can be treated as a certain kind of reflection of light quanta on the edge of the aperture [4]. Basing on the idea of Young, Rubinowicz has given a rigorous proof that in the framework of the Kirchhoff theory for spherical incident wave, the motion behind the screen can be presented as the superposition of the directly incident wave and the edge wave emitted by the edge of the diffracting slit. As it turned out later, a similar representation is also possible in the case of Fraunhofer diffraction [5].

In order to apply our idea to quantitative description of Fraunhofer diffraction let us consider a monochromatic uniform photon beam with the wave vector $\vec{k}_0$ which falls perpendicularly on a thin opaque, plane screen having a small aperture O. The wave function of the photon beam at a given $t = 0$ is (according to the interpretation accepted in the present paper) the density of the activity and has the form

$$\varphi_0 = C_0 \exp\,[i(\vec{k}_0\vec{r})] \tag{7}$$

where C_0 is a constant defining the intensity of the incident beam and $\vec{r}$ is the radius vector. The photons which, owing to the interaction with the edge of the screen, have the wave vector $\vec{k}$ ($|\vec{k}| = |\vec{k}_0|$) have obtained additional momentum $\hbar(\vec{k}-\vec{k}_0)$ and thus the density of the additional complex momentum in such a beam has the form

$$\vec{p}_i = \hbar(\vec{k}-\vec{k}_0)\,\varphi_i \tag{8}$$

where

$$\varphi_i = C_k \exp\,[i(\vec{k}-\vec{k}_0)\vec{r}] \tag{9}$$

and C_k is a constant with respect to space coordinates. φ_i is the activity of the screen at the point $\vec{r}$ at $t = 0$ for the photon beam which obtains additional momentum $\hbar(\vec{k}-\vec{k}_0)$. The number of photons in the beam with the wave vector $\vec{k}$ is determined by the total density of the activities for the beam of these photons and, what follows, this number is also defined by the resultant inflow due to the components of additional complex momenta, which are perpendicular to the direction of the primary beam, and which give the final direction $\vec{k}$ the photons falling on the screen.

In the investigated case of the diffraction of light beam at the edge of the thin plane screen, the overall outflow P from the edge of the aperture has the form

$$P = \oint (\vec{p}_i)_N ds \tag{10}$$

where $(\vec{p}_i)_N$ denotes the projection of the density of complex momentum on the direction of the normal to the edge of the aperture. The integration has to be performed along the edge of the aperture (for a greater number of apertures the total outflow is the sum of those from the particular apertures).

By virtue of the Gauss theorem for two-dimensional case in the plane of the screen there is

$$\int_o \operatorname{div} \vec{p}_i\, d\sigma = \oint (\vec{p}_i)_N ds \tag{11}$$

where $d\sigma$ denotes the element of the area of the aperture while ds is the linear element of the edge. Basing on Eq. (9) and on the meaning of φ_i we have

$$\operatorname{div} \vec{p}_i = i\hbar(\vec{k}-\vec{k}_0)^2\varphi_i \tag{12}$$

and thus Eq. (11) for $(\vec{k}-\vec{k}_0) \neq 0$ can be brought to the form

$$\int\limits_{o} \varphi_i d\sigma = -\frac{iC_k}{|\vec{k}-\vec{k}_0|^2}\oint (\vec{k}-\vec{k}_0)_N \exp\,[i(\vec{k}-\vec{k}_0)\vec{r}]ds. \tag{13}$$

The left-hand side of Eq. (13) is up to a constant factor, equal to the Fraunhofer diffraction integral (Fraunhofersches Beugungsintegral). It is thus natural to presume that the right-hand side integral along the edge of the aperture is (again up to a constant factor) according to the theory of Rubinowicz the Fraunhofer diffraction wave (Fraunhofersche Beugungswelle). It has been shown by Rubinowicz in a paper on Fraunhofer diffraction [5] that the Fraunhofer integral transformed by Laue [6] to a form which does not essentially differ from Eq. (13), is identical with the Fraunhofer diffraction wave for the particular case, when the edge of the diffracting aperture is a plane curve.

The description of Fraunhofer diffraction presented in this paper which accentuates the corpuscular aspect of the problem leads thus in fact to the same results as does the classical theory of Kirchhoff based on the Huygens principle. The simple model of the diffraction of a photon beam with the transfer of complex momenta permits the role of the edge of the aperture to be visualized and the conclusion to be drawn that in case of Fraunhofer diffraction by a multi-slit diffraction grating the interference process occurs not in the detecting device but during the diffraction process after which there are no more changes in the parallel beams. Photons traversing singly (not at the same instant) the diffraction grating can thus also "interfere" [7] due to the transfer of complex momenta by the diffraction grating. It is namely clear that the spectrum of complex momenta transferred by the screen depends on the form and number of apertures in the screen.

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