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THERMODYNAMICAL BEHAVIOUR OF A ONE-DIMENSIONAL AMORPHOUS ISING MODEL

By K. Handrich, J. Hemmerling and N. Abdallah

Section of Physics, Technical University, Dresden*

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We consider a one-dimensional amorphous Ising magnet with positive exchange integrals $I_{ij} = \langle I \rangle + \Delta I_{ij}$. We assume that the fluctuations of the exchange integrals ΔI_{ij} are stochastical and small. Using a thermodynamical perturbation theory we obtain the change in free energy $\Delta f(T, H)$ in comparison to the corresponding crystalline case $(I_{ij} = \langle I \rangle)$. The structure fluctuations lead to a decrease of both the free energy and the magnetization for all magnetic fields H and temperatures T. The changes in entropy, specific heat, internal energy, and susceptibility oscillate in sign in dependence on the field and the temperature. The structure fluctuations cause a shift of the peaks of the specific heat and susceptibility to lower temperatures. The reason for the kind of changes are discussed qualitatively in some cases.

1. Introduction

In recent years amorphous ferromagnets have repeatedly been investigated both experimentally and theoretically. A summary on these works can be found in [1, 2]. Usually one starts from a lattice model in which the exchange integrals are considered as random quantities. An analysis of the Ising model shows some basic differences of the magnetic thermodynamic properties in the crystalline and amorphous case. Using a complicated mathematical method Fan and McCoy [3] investigated the thermodynamic properties of a one-dimensional Ising model with random exchange integrals. With the same approach Smith [4] studied the low temperature thermodynamics for the one-dimensional $X-Y-$model. In [5] the magnetization was numerically calculated for a finite amorphous one-dimensional Ising model.

We use a thermodynamic perturbation theory [6] to determine the change of the free energy $\Delta f(T, H)$, entropy $\Delta s(T, H)$, specific heat $\Delta c(T, H)$, internal energy $\Delta u(T, H)$, magnetization $\Delta m(T, H)$, and susceptibility $\Delta \chi(T, H)$ per spin due to small fluctuations of the exchange integrals. A shift of the peaks of the specific heat and susceptibility to lower temperatures is found.

* Address: TU Dresden, Sektion Physik, DDR-8027 Dresden, Zellescher Weg 12.

2. Basic equations

Starting from a thermodynamical perturbation theory of an n-dimensional amorphous Ising magnet with $S_i^z = \pm 1$ [6] we investigate the thermodynamic quantities for the one--dimensional case. In the amorphous case the exchange integrals are considered as $I_{ij} = \langle I \rangle + \Delta I_{ij} (I_{ij} > 0)$, where $\langle I \rangle$ is the mean value of the exchange integrals with regard to the structure and ΔI_{ij} is a fluctuating one. The system with $I_{ij} = \langle I \rangle$ is denoted as the corresponding crystalline system. For small stochastical fluctuations ΔI_{ij} the difference between the free energy per spin for the amorphous system (f) and the corresponding crystalline system (f_0) is given by

$$\Delta f(T, H) = f(T, H) - f_o(T, H) = -\tfrac{1}{2}\Delta^2 \langle I \rangle^2 \beta Z \left[1 - \left(\frac{1}{Z} \frac{\partial f_o(T, H)}{\partial \langle I \rangle} \right)^2 \right] \tag{1}$$

with

$$\frac{\langle \Delta I_{ij} \Delta I_{kl} \rangle}{\langle I \rangle^2} = \frac{\Delta^2}{2} (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{2}$$

(see [6]). For the one-dimensional Ising model the analytical terms $f_0(T, H)$, for the free energy per spin in the ordered case is well known, see *e. g.* [7]. From equation (1) one obtains $Z=2$ the changes

$$\Delta m(T, H) = -\left(\frac{\partial \Delta f(T, H)}{\partial H} \right)_T = -\frac{1}{2} \Delta^2 \beta (|u_0(T, H)| - m_o H) \frac{\partial m_o(T, H)}{\partial \langle I \rangle} \langle I \rangle, \tag{3}$$

$$\Delta \chi(T, H) = \left(\frac{\partial \Delta m(T, H)}{\partial H} \right)_T = -\frac{1}{2} \Delta^2 \beta \left[(|u_0(T, H)| - m_o H) \frac{\partial \chi_0(T, H)}{\partial \langle I \rangle} + \right.$$

$$\left. + \left(\frac{\partial m_o(T, H)}{\partial \langle I \rangle} \langle I \rangle \right)^2 \right] \tag{4}$$

for the magnetization and susceptibility per spin and analogous relations for $\Delta s(T, H)$, $\Delta c(T, H)$ and $\Delta u(T, H)$. The subscript o denotes the corresponding crystalline case.

3. Discussion

1) $\Delta f(T, H)$ (Fig. 1).

In agreement with the Bogolyubov theorem for amorphous magnetic systems [8] we find $\Delta f(T, H) \leqq 0$ for all T and H. The change of the free energy has a minimum and approximates zero as $-\frac{1}{T}$ for higher temperatures. With higher H-fields $|\Delta f(T, H)|$ decreases and the minimum being less sharp is displaced towards higher temperatures. The applied magnetic field causes a relative approach to the crystalline case.

2) $\Delta s(T, H)$ (Fig. 2).

As discussed in [9] the increasing of $\Delta s(T, H)$ at low temperatures is caused by the fluctuations with $\Delta I_{ij} < 0$ and the decrease at high temperatures by the fluctuations with

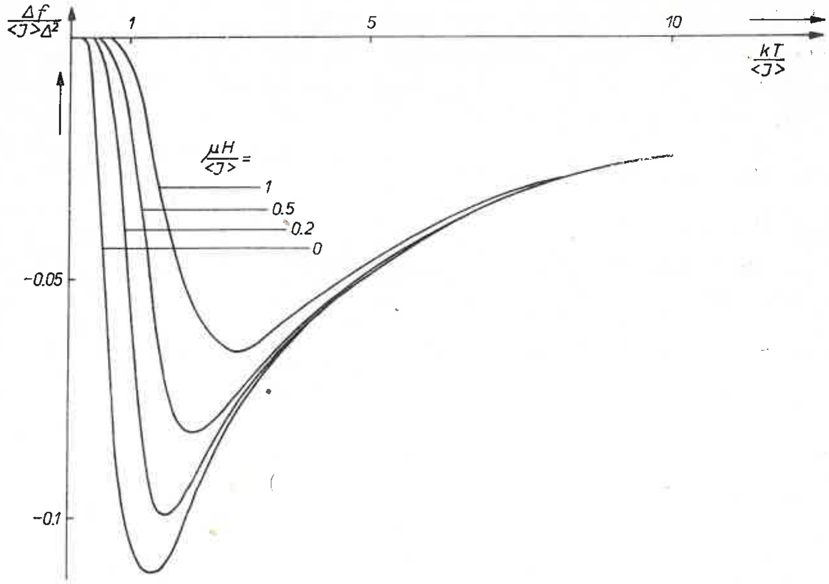

Fig. 1. Change of the free energy per spin *vs* temperature for various magnetic fields

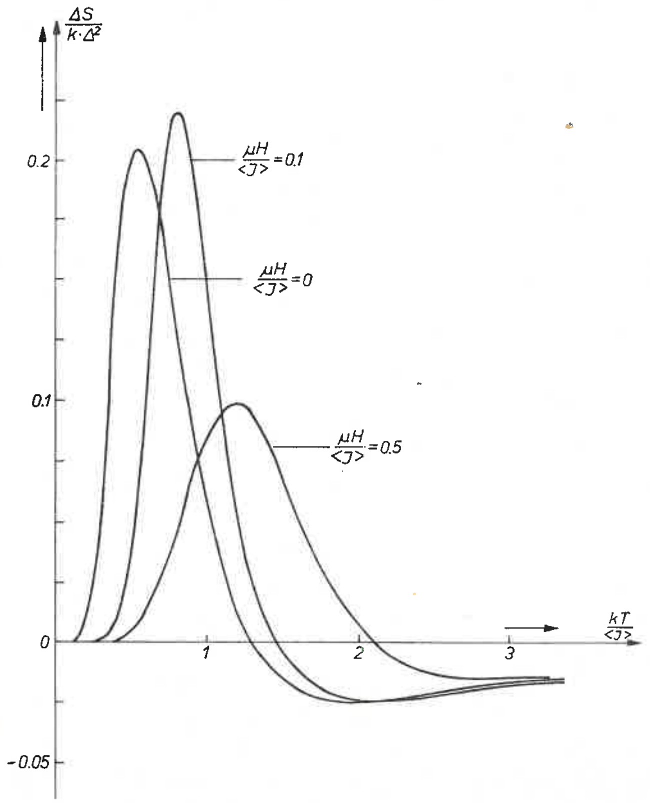

Fig. 2. Change of the entropy per spin *vs* temperature for various magnetic fields

$\Delta I_{ij} > 0$. In the first case the smaller exchange intergrals couple the spins weaker and the local magnetic disorder enlarges at low temperatures. In the second case the greater exchange integrals yield an additional local magnetic order at high temperatures. It is remarkable that with increasing H at first we obtain higher maxima and then smaller ones. For $T \to \infty$ $\Delta s(T, H)$ approaches zero as $-\frac{1}{T^2}$.

3) $\Delta c(T, H)$ (Fig. 3).

Analogously to $\Delta s(T, H)$ we find for increasing field at first higher maxima and then decreasing ones. Fan and McCoy [3] only stated a permanent reducing of these maxima. The change of the specific heat approaches zero for higher temperatures as $\frac{1}{T^2}$.

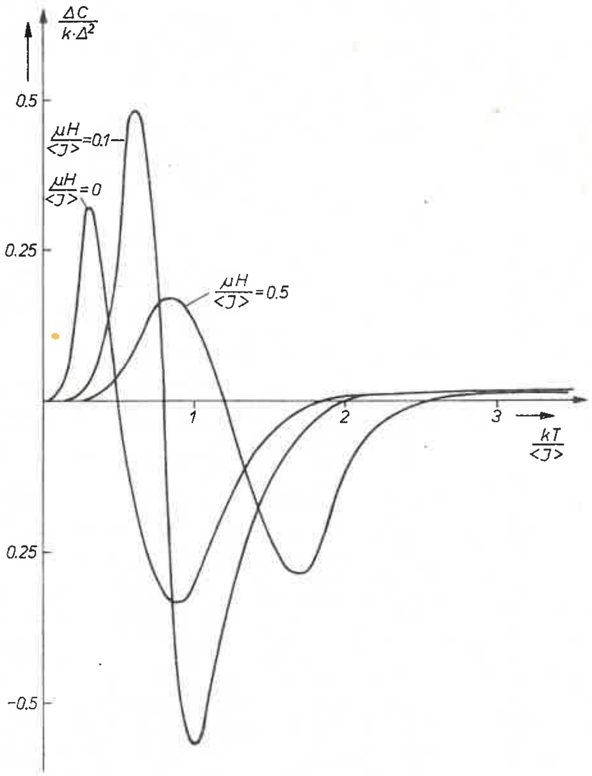

Fig. 3. Change of the specific heat per spin *vs* temperture for various magnetic fields

4) $c(T, \Delta^2)$ (Fig. 4).

In comparison with the crystalline case the maximum is reduced and shifted to lower temperatures.

5) $\Delta u(T, H)$ (Fig. 5).

Like $\Delta s(T, H)$ and $\Delta c(T, H)$ we find for increasing H at first a increase and then a decrease of the maxima of $\Delta u(T, H)$. Heinrich [10] has obtained an analogous behaviour for

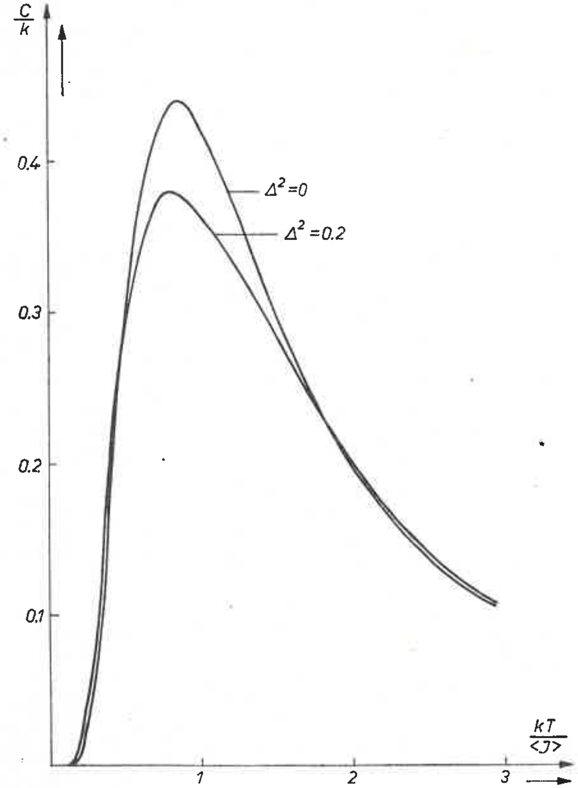

Fig. 4

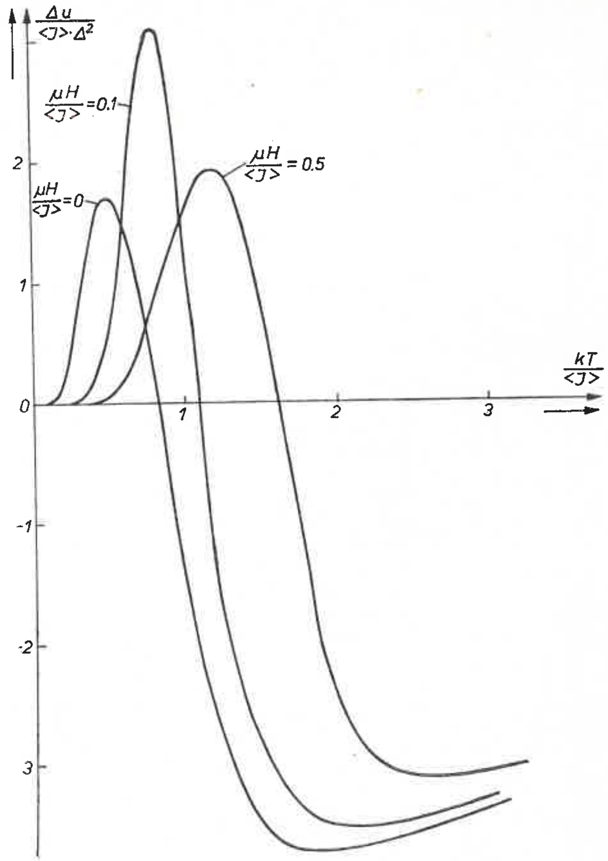

Fig. 5

Fig. 4. Specific heat per spin *vs* temperature for various mean square fluctuations of the exchange integrals Δ^2 and zero magnetic field

Fig. 5. Change of the internal energy per spin *vs* temperature for various magnetic fields

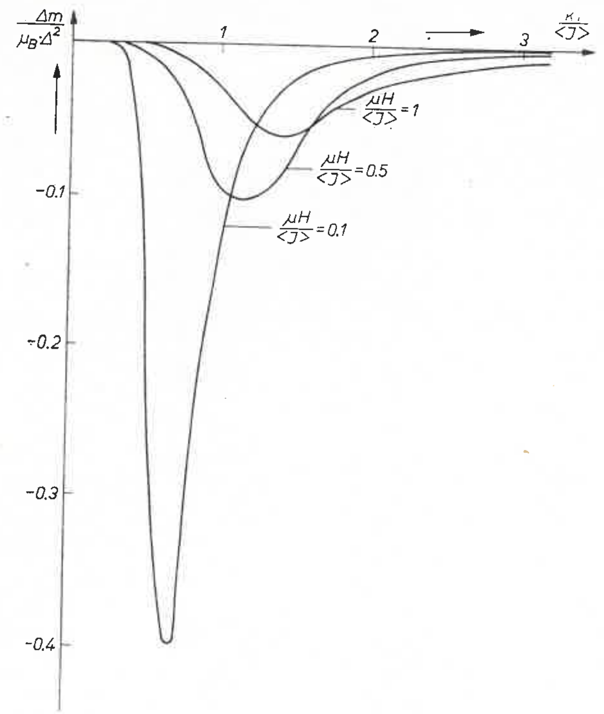

Fig. 6. Change of the magnetization per spin *vs* temperature for various magnetic fields

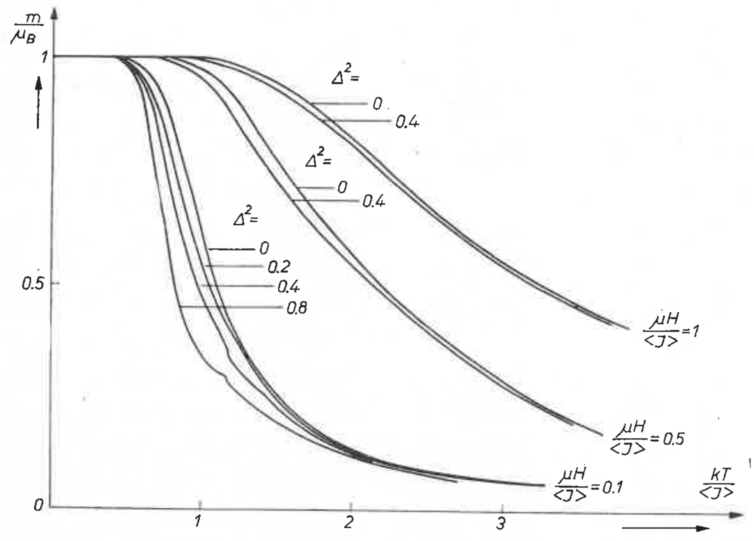

Fig. 7. Magnetization per spin *vs* temperature for various magnetic fields and mean square fluctuations of the exchange integrals Δ^2

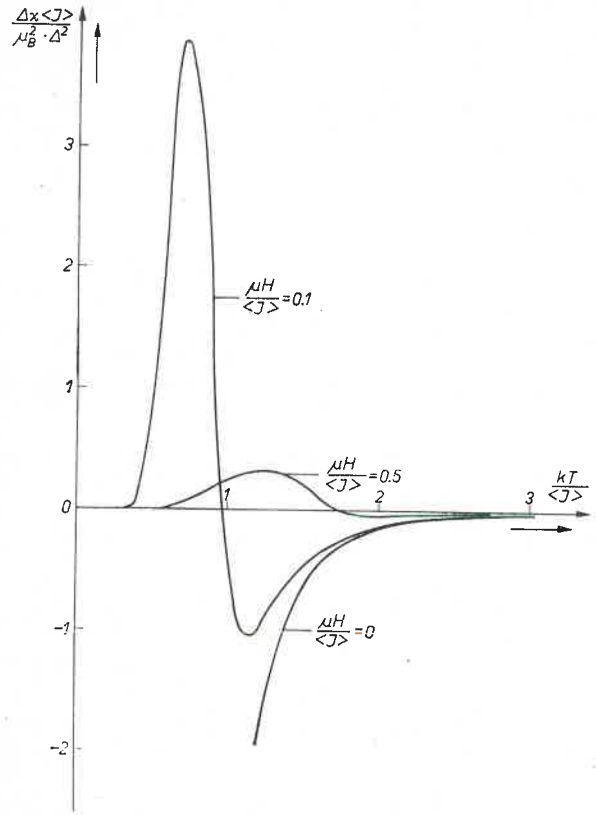

Fig. 8

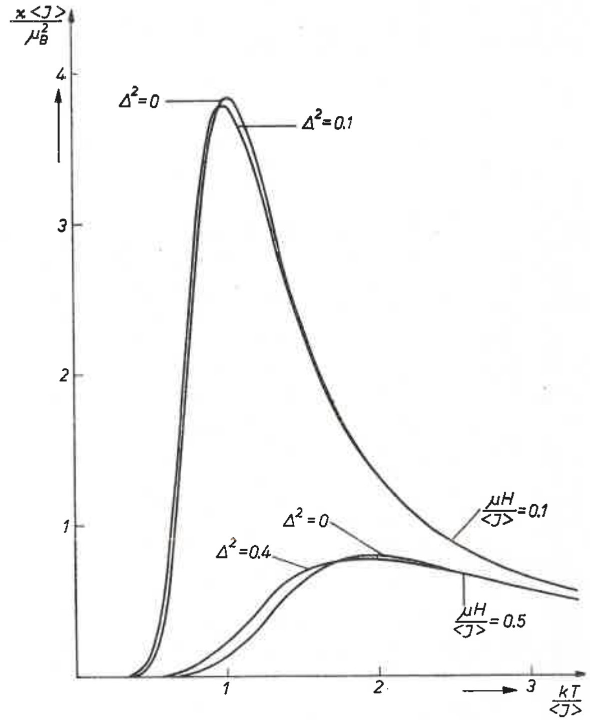

Fig. 9

Fig. 8. Change of the susceptibility per spin *vs* temperature for various magnetic fields

Fig. 9. Susceptibility per spin *vs* temperature for various magnetic fields and mean square fluctuations of the exchange integrals Δ^2

the change of the internal energy with a computer experiment for a finite two-dimensional amorphous Ising model. For $T \to \infty$ $\Delta u(T, H)$ approaches zero as $-\frac{1}{T^2}$.

6) $\Delta m(T, H)$ (Fig. 6).

The change of the magnetization is always negative and vanishes at zero field. With a higher field the minima of $\Delta m(T, H)$ being less sharp increase and move to higher temperatures.

7) $m(T, H, \Delta^2)$ (Fig. 7).

With increasing fluctuations of the exchange integrals ΔI_{ij} the magnetization reduces. Although the free energy for the one-dimensional Ising model is analytical at all temperatures, we get for greater ΔI_{ij} magnetization curves, the behaviour of which is not physically reasonable in a certain region of temperatures. That happens because the perturbation series does not converge well for these temperatures. At higher fields the magnetization is less sensitive to structure fluctuations than for small fields.

8) $\Delta\chi(T, H)$ (Fig. 8).

For $H \neq 0$ the change of the susceptibility oscillates in sign in dependence on the temperature and field, whereas for $H = 0$ it is always negative with a singular behaviour at $T = 0$. Using the equation (4) one can explain the oscillation for $H \neq 0$ because

$$\frac{\partial\chi_0(T, H)}{\partial\langle J\rangle} \begin{matrix}\leqq\\ \geqq\end{matrix}\ 0 \text{ for } \begin{matrix}\text{low } T\\ \text{high } T\end{matrix}, \text{ whereas for } H = 0 \text{ always } \frac{\partial\chi_0(T, H)}{\partial\langle J\rangle} \geqq 0.$$

With increasing H the maxima and minima will be less sharp and displaced to higher temperatures.

9) $\chi(T, H, \Delta^2)$ (Fig. 9).

Similar to the specific heat (Fig. 4) the peaks of the susceptibility for $H \neq 0$ are shifted to lower temperatures due to fluctuations of the exchange integrals ΔI_{ij}. With increasing ΔI_{ij} the maximum at first decreases and then increases. It seems that the latter has not phisiacal reasons but is a consequence of the used perturbation theory, which is not applicable for great Δ^2 (see also the behaviour of the magnetization).

Fan and McCoy [3] also calculated the changes $\Delta f(T, H)$, $\Delta c(T, H)$, $\Delta m(T, H)$ and $\Delta\chi(T, H)$, which agree with our results.

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