

# THEORY OF PHOTON-INDUCED HOPPING ON ACCEPTORS IN *p*-TYPE GERMANIUM

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The probability of photon-induced transition in the dipole approximation between the two-centre states of the acceptor pair in *p*-type Ge, is calculated. The envelope functions introduced by Schechter are used for the construction of the two-centre states.

## 1. Introduction

The far infrared absorption coefficient at very low temperatures for *p*-type germanium was measured by Smith *et al.* [1]. There are some differences between their experimental results and the consequences of Blinowski and Mycielski's theory for *n*-type germanium [2]:

- a) the maximum value of the absorption coefficient for the *p*-type is about six times higher than that predicted for the *n*-type,
- b) the maximum for the *p*-type occurs at shorter wave-lengths than that for the *n*-type.

There exists also another theory of absorption of microwaves in *p*-type materials developed by Tanaka *et al.* [3, 4]. The interpretation of the experimental data in their work is based on the assumption of a spherical eigenfunction of the acceptor ground state. The Bohr radius of this function was adjusted so that the theoretical curve agrees with Tanaka's experiment. Some approximations used by Tanaka *et al.* cause that this theory cannot be directly applied to the wavelength region 200–800  $\mu\text{m}$  investigated by Smith *et al.* The formula for transition rate given by Tanaka is however quite general for a hydrogenlike model.

An analysis of the formulas: these for *n*-type Ge [2] and these in Tanaka's model [3, 4] for the *p*-type shows that the transition rate is proportional to the square of the re-

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sonance energy. In our earlier paper [5] the resonance energy for a given pair in  $p$ -type Ge, calculated with the help of anisotropic Schechter functions has been obtained. This energy is about two times larger than the same energy calculated by means of the hydrogenlike functions [5, (Fig. 4)], and also is about two times larger than the corresponding energy in  $n$ -type germanium [6]. It seems that it can qualitatively explain the higher absorption in  $p$ -type Ge and also the shift of the maxima of the absorption towards shorter wavelengths. To obtain more rigorous conclusions one can should calculate however the transition rate starting from the Schechter functions [7] for isolated acceptors. As was shown by Smith there is no appreciable change in the absorption in the interesting wave-length range for temperatures varying between 2.0 and 4.2°K. Also Blinowski and Mycielski's theory shows that phonons have negligible influence on absorption in  $n$ -type Ge. Therefore one can expect that at very low temperatures the transitions in Ge of both types take place without absorption or emission of phonons.

The purpose of the present work is to calculate the transition rate for the photon-induced hopping without the participation of phonons using the Schechter functions of the isolated acceptor ground state.

In Section 2 the two-centre states are constructed by using the Schechter functions. The energies of these states as solutions of a suitable secular equation are also given.

In Section 3 the transition rate for the photon-induced hopping in the dipole approximation is calculated.

## 2. Hole states of the two-centre Hamiltonian

The effective mass equations for the acceptor states in valence semiconductors were given by Luttinger and Kohn [8]. In the general case these equations form a system of six differential equations. If the spin-orbit splitting of the valence bands goes to infinity, the six effective-mass equations decouple into two systems: one of four equations and one of two equations. This approximation of infinite spin-orbit splitting is expected to be a good approximation in germanium because of the considerable difference between the spin-orbit splitting (0.3 eV) and the ionization energy of the acceptor (0.01 eV).

In this approximation the acceptor wave functions have the form [7]

$$\Psi_{\mu}(\mathbf{r}) = \sum_{p=1}^4 F_{p}^{\mu}(\mathbf{r})\varphi_p(\mathbf{r}). \quad (2.1)$$

Here the index  $\mu$  denotes the  $\mu$ -th degenerate state of the acceptor level under consideration and  $p$  refers to the  $p$ -th degenerate band. The  $\varphi_p(\mathbf{r})$  are the Bloch functions at the top of the valence band,  $F_{\mu}(\mathbf{r})$  are the solutions of the effective-mass equations. The functions  $F_p(\mathbf{r})$  for the acceptor ground state can, to a good approximation, be represented by the functional form [7, 9]

$$F^1(\mathbf{r}) = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-\frac{r}{r_1}} + \frac{c_2}{2} \begin{pmatrix} 3z^2 - r^2 \\ 0 \\ -\sqrt{3}(x^2 - y^2) \\ 0 \end{pmatrix} e^{-\frac{r}{r_2}} + ic_3 \begin{pmatrix} 0 \\ z(x+iy) \\ xy \\ 0 \end{pmatrix} e^{-\frac{r}{r_2}}$$

$$\begin{aligned}
 F^2(\mathbf{r}) &= c_1 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{-\frac{r}{r_1} - \frac{c_2}{2} \begin{pmatrix} 0 \\ 3z^2 - r^2 \\ 0 \\ \sqrt{3}(x^2 - y^2) \end{pmatrix}} e^{-\frac{r}{r_2} - ic_3 \begin{pmatrix} z(x - iy) \\ 0 \\ 0 \\ -xy \end{pmatrix}} e^{-\frac{r}{r_2}}, \\
 F^3(\mathbf{r}) &= c_1 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{-\frac{r}{r_1} - \frac{c_2}{2} \begin{pmatrix} \sqrt{3}(x^2 - y^2) \\ 0 \\ 3z^2 - r^2 \\ 0 \end{pmatrix}} e^{-\frac{r}{r_2} - ic_3 \begin{pmatrix} xy \\ 0 \\ 0 \\ z(x + iy) \end{pmatrix}} e^{-\frac{r}{r_2}}, \\
 F^4(\mathbf{r}) &= c_1 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{-\frac{r}{r_1} + \frac{c_2}{2} \begin{pmatrix} 0 \\ -\sqrt{3}(x^2 - y^2) \\ 0 \\ 3z^2 - r^2 \end{pmatrix}} e^{-\frac{r}{r_2} + ic_3 \begin{pmatrix} 0 \\ -xy \\ z(x - iy) \\ 0 \end{pmatrix}} e^{-\frac{r}{r_2}}. \quad (2.2)
 \end{aligned}$$

The parameters  $c_1, c_2, c_3, r_1, r_2$  were calculated by the standard variational procedure [7].

We are now interested in obtaining the lowest hole states of the two-centre Hamiltonian in a nondeformed crystal lattice.

The total Hamiltonian has the form

$$H = T + V - \frac{e^2}{\kappa r_i} - \frac{e^2}{\kappa r_j}. \quad (2.3)$$

Here  $T$  denotes the Hamiltonian for a hole in an ideal crystal,  $V$  denotes the potential due mainly to the positively charged donor nearest to the acceptors  $i$  and  $j$ ,  $\kappa$  is the static dielectric constant and  $r_i$  and  $r_j$  are

$$r_i = r - R_i, \quad r_j = r - R_j,$$

where  $R_i$  and  $R_j$  are the positions of the two acceptors.

Following the procedure developed by Miller and Abrahams [6] for  $n$ -type germanium and silicon, we look for the lowest orthonormal states of the Hamiltonian (2.3) in the form

$$\Psi^\gamma(\mathbf{r}; \varepsilon_q) = \sum_{\mu=1}^4 [a_\mu^\gamma(\varepsilon_q)\Psi_\mu(\mathbf{r}_i) + b_\mu^\gamma(\varepsilon_q)\Psi_\mu(\mathbf{r}_j)]. \quad (2.4)$$

Here the index  $q$  labels the four energy states and the index  $\gamma$  distinguishes the degenerate functions corresponding to the energy  $\varepsilon_q$ . The expansion coefficients  $a_\mu^\gamma, b_\mu^\gamma$  are of the form [9]

$$\begin{aligned}
 a_1^\gamma(\varepsilon_q) &= M_{1q}\delta_{2\gamma}, \\
 a_2^\gamma(\varepsilon_q) &= -M_{2q}(r\delta_{1\gamma} + p^*\delta_{2\gamma}), \\
 a_3^\gamma(\varepsilon_q) &= M_{2q}(p\delta_{1\gamma} - r^*\delta_{2\gamma}), \\
 a_4^\gamma(\varepsilon_q) &= +M_{1q}\delta_{1\gamma}, \\
 b_1^\gamma(\varepsilon_q) &= -M_{3q}\delta_{2\gamma},
 \end{aligned}$$

$$\begin{aligned}
 b_2^{\gamma}(\varepsilon_q) &= M_{4q}(r\delta_{1\gamma} + p^*\delta_{2\gamma}), \\
 b_3^{\gamma}(\varepsilon_q) &= -M_{4q}(p\delta_{1\gamma} - r^*\delta_{2\gamma}), \\
 b_4^{\gamma}(\varepsilon_q) &= M_{3q}\delta_{1\gamma}
 \end{aligned} \tag{2.5}$$

where

$$\begin{aligned}
 M_{1q} &= -X^{-1}[k(cd - m^2) + m(|p|^2 + |r|^2)], \\
 M_{2q} &= X^{-1}(cd + km - |p|^2 - |r|^2), \\
 M_{3q} &= X^{-1}c(cd - m^2 - |p|^2 - |r|^2), \\
 M_{4q} &= X^{-1}c(k + m).
 \end{aligned} \tag{2.6}$$

The normalization factor is equal to

$$\begin{aligned}
 X^2 &= (cd - m^2 - |p|^2 - |r|^2) \times \\
 &\times [c^2 + k^2 + |p|^2 + |r|^2 - 2c(kS_{11} + p^*S_{12} + r^*S_{13})] + \\
 &\quad + 2(k + m)(cd - m^2 - |p|^2 - |r|^2) \times \\
 &\times [(|p|^2 + |r|^2)(k + m - cS_{11} - cS_{22}) - c(k + m)(p^*S_{12} + r^*S_{13})] + \\
 &\quad + (k + m)^2(|p|^2 + |r|^2) \times \\
 &\times [c^2 + m^2 + |p|^2 + |r|^2 - 2c(mS_{22} + p^*S_{12} + r^*S_{13})]
 \end{aligned} \tag{2.7}$$

and

$$\begin{aligned}
 c &\equiv c_q = \varepsilon_q + J + \frac{\Delta}{2}, \\
 d &\equiv d_q = \varepsilon_q + J - \frac{\Delta}{2}, \\
 k &\equiv k_q = \varepsilon_q S_{11} + L_{11}, \\
 m &\equiv m_q = \varepsilon_q S_{22} + L_{22}, \\
 p &\equiv p_q = \varepsilon_q S_{12} + L_{12}, \\
 r &\equiv r_q = \varepsilon_q S_{13} + L_{13}.
 \end{aligned} \tag{2.8}$$

The two-centre wave functions  $\Psi^{\gamma}(\mathbf{r}; \varepsilon_q)$  calculated in this way are normalized

$$\int \Psi^{\gamma*}(\mathbf{r}; \varepsilon_q) \Psi^{\gamma}(\mathbf{r}; \varepsilon_q) d_3r = \delta_{qq'} \delta_{\gamma\gamma'}.$$

In these formulas  $\Delta$  is the energy difference between the neighbouring impurity sites due to the local electric field

$$\Delta = V(\mathbf{r}_i) - V(\mathbf{r}_j) \tag{2.9}$$

and  $S_{\mu'\mu}$  and  $L_{\mu'\mu}$  are the the overlap integrals and the matrix elements of Coulomb potential, respectively. They are defined as follows

$$S_{\mu'\mu} = \langle F^{\mu'}(r_i) | F^{\mu}(r_j) \rangle,$$

$$L_{\mu'\mu} = \left\langle F^{\mu'}(r_i) \left| -\frac{e^2}{\kappa r_i} \right| F^{\mu}(r_j) \right\rangle. \quad (2.10)$$

The explicit expressions for  $S_{\mu'\mu}$  and  $L_{\mu'\mu}$ , are given in our earlier work [10].

The investigation of the hole energy  $\varepsilon_q$  in the lowest states requires the solution of a fourth-order equation (see Eq. (2) in the paper [5]). We are interested in the solution of this equation in the range of the separation between the acceptor pair for which the overlap of the localized wave-functions of the individual acceptor centers is small. We restrict our calculations to these separations  $2R$  for which  $|S_{\mu'\mu}|^2 < 0.03$ . The fourth-order equation can be solved analytically. The solutions with an accuracy to the second order terms in the overlap integrals are of the form

$$\varepsilon_{1,4} = \frac{-J + \left(S + \frac{\sqrt{B}}{2} A_5^1\right) \left(L + \frac{\sqrt{B}}{2} A_5^2\right) \pm \sqrt{\frac{\delta_1}{4}}}{1 - \left(S + \frac{\sqrt{B}}{2} A_5^1\right)^2},$$

$$\varepsilon_{2,3} = \frac{-J + \left(S - \frac{\sqrt{B}}{2} A_5^1\right) \left(L - \frac{\sqrt{B}}{2} A_5^2\right) \pm \sqrt{\frac{\delta_2}{4}}}{1 - \left(S - \frac{\sqrt{B}}{2} A_5^1\right)^2}, \quad (2.11)$$

where

$$\delta_1 = 4 \left\{ W_{R1}^2 + \frac{A^2}{4} \left[ 1 - \left(S + \frac{\sqrt{B}}{2} A_5^1\right)^2 \right] \right\},$$

$$\delta_2 = 4 \left\{ W_{R2}^2 + \frac{A^2}{4} \left[ 1 - \left(S - \frac{\sqrt{B}}{2} A_5^1\right)^2 \right] \right\}, \quad (2.12)$$

and the resonance energies  $W_{R1}$  and  $W_{R2}$  are equal to

$$W_{R1} = -J \left( S + \frac{\sqrt{B}}{2} A_5^1 \right) + \left( L + \frac{\sqrt{B}}{2} A_5^2 \right),$$

$$W_{R2} = -J \left( S - \frac{\sqrt{B}}{2} A_5^1 \right) + \left( L - \frac{\sqrt{B}}{2} A_5^2 \right), \quad (2.13)$$

and

$$S = c_1^2 A_1^1 + c_2^2 A_2^1 + c_3^2 A_3^1 + (c_3^2 - 3c_2^2) (l^2 m^2 + l^2 n^2 + m^2 n^2) A_4^1,$$

$$L = c_1^2 A_1^2 + c_2^2 A_2^2 + c_3^2 A_3^2 + (c_3^2 - 3c_2^2) (l^2 m^2 + l^2 n^2 + m^2 n^2) A_4^2,$$

$$J = \frac{e^2}{2\kappa R},$$

$$B = c_1^2 \{ c_2^2 [(3n^2 - 1)^2 + 3(l^2 - m^2)^2] + 4c_3^2 [n^2(1 - n^2) + l^2 m^2] \}. \quad (2.14)$$

Here  $l, m, n$  are the cosines of the angles between the vector  $2\mathbf{R}$  and the  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$  crystal directions, respectively. The functions  $A_v^1(\mathbf{R})$ ,  $A_v^2(\mathbf{R})$  ( $v = 1 \dots 5$ ) are given in the paper [10].

### 3. Transition rate

In the dipole approximation the transition rate of a hole from the energy state  $\varepsilon_q$  to the energy state  $\varepsilon_f$  with absorption of photon at 0°K is equal to

$$W_{qf} = \frac{2\pi e^2}{c\hbar^2 \kappa^{1/2}} I_0(\omega) \delta(\omega_{qf} - \omega) \sum_{\gamma'=1}^2 \left| \sum_{\gamma=1}^2 \langle \Psi_{f'}^{\gamma'} | \mathbf{r} \mathbf{N} | \Psi_q^{\gamma} \rangle \right|^2. \quad (3.1)$$

Here  $I_0(\omega)$  is the intensity of light,  $\mathbf{N}$  is the unit vector lying along electric vector of light wave,  $\omega$  is the frequency of light and  $\omega_{qf} = (\varepsilon_f - \varepsilon_q)/\hbar$ .

In the approximation of slowly varying envelope functions  $\mathbf{F}^{\mu}(\mathbf{r})$  the dipole matrix element is expressible in the form

$$\begin{aligned} \langle \Psi_{f'}^{\gamma'} | \mathbf{r} \mathbf{N} | \Psi_q^{\gamma} \rangle &= \\ &= \sum_{\mu=1}^4 \sum_{\mu'=1}^4 [a_{\mu'}^{\gamma'*}(\varepsilon_f) a_{\mu}^{\gamma}(\varepsilon_q) \langle \mathbf{F}^{\mu'}(\mathbf{r}_i) | \mathbf{r} \mathbf{N} | \mathbf{F}^{\mu}(\mathbf{r}_i) \rangle + \\ &\quad + b_{\mu'}^{\gamma'*}(\varepsilon_f) b_{\mu}^{\gamma}(\varepsilon_q) \langle \mathbf{F}^{\mu'}(\mathbf{r}_j) | \mathbf{r} \mathbf{N} | \mathbf{F}^{\mu}(\mathbf{r}_j) \rangle + \\ &\quad + a_{\mu'}^{\gamma'*}(\varepsilon_f) b_{\mu}^{\gamma}(\varepsilon_q) \langle \mathbf{F}^{\mu'}(\mathbf{r}_i) | \mathbf{r} \mathbf{N} | \mathbf{F}^{\mu}(\mathbf{r}_j) \rangle + \\ &\quad + b_{\mu'}^{\gamma'*}(\varepsilon_f) a_{\mu}^{\gamma}(\varepsilon_q) \langle \mathbf{F}^{\mu'}(\mathbf{r}_j) | \mathbf{r} \mathbf{N} | \mathbf{F}^{\mu}(\mathbf{r}_i) \rangle]. \end{aligned} \quad (3.2)$$

When we choose the origin of the coordinate system at the midpoint between the two acceptors

$$\mathbf{R}_i = \mathbf{R}, \quad \mathbf{R}_j = -\mathbf{R},$$

the matrix elements with both  $F$  functions centred at the same site reduce to the very simple form

$$-\langle \mathbf{F}^{\mu'}(\mathbf{r}_i) | \mathbf{r} \mathbf{N} | \mathbf{F}^{\mu}(\mathbf{r}_i) \rangle = \langle \mathbf{F}^{\mu'}(\mathbf{r}_j) | \mathbf{r} \mathbf{N} | \mathbf{F}^{\mu}(\mathbf{r}_j) \rangle = \mathbf{R} \mathbf{N} \delta_{\mu'\mu}. \quad (3.3)$$

The matrix elements with  $F$  functions centered at different sites

$$\begin{aligned} N_{\mu'\mu ij} &= \langle \mathbf{F}^{\mu'}(\mathbf{r}_i) | \mathbf{r} \mathbf{N} | \mathbf{F}^{\mu}(\mathbf{r}_j) \rangle \\ N_{\mu'\mu ji} &= \langle \mathbf{F}^{\mu'}(\mathbf{r}_j) | \mathbf{r} \mathbf{N} | \mathbf{F}^{\mu}(\mathbf{r}_i) \rangle \end{aligned} \quad (3.4)$$

have the property:

$$N_{\mu'\mu'ji} = N_{\mu\mu'ij}^*$$

The matrix  $N_{\mu\mu'}$  has the following symmetry:

$$\begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} \\ -N_{12}^* & N_{22} & N_{23} & N_{13} \\ -N_{13}^* & -N_{23}^* & N_{22}^* & N_{12} \\ -N_{14}^* & N_{13}^* & -N_{12}^* & N_{11}^* \end{bmatrix}. \quad (3.5)$$

Thus there are only six independent  $N_{\mu\mu'}$ .

For calculations of the integrals (3.4) the coordinate system in which the functions  $F^\mu(\mathbf{r})$  in formulas (2.1) are given, is rotated in such a way as to make  $z$ -axis parallel to vector  $2\mathbf{R}$  which in the nonrotated coordinate system has the form  $\mathbf{R} = R(l, m, n)$ . The spheroidal coordinates are used [11]

$$\begin{aligned} x_i = x_j &= R[(\xi^2 - 1)(1 - \eta^2)]^{1/2} \cos \varphi, \\ y_i = y_j &= R[(\xi^2 - 1)(1 - \eta^2)]^{1/2} \sin \varphi, \\ z_i &= R(\xi\eta + 1), \\ z_j &= R(\xi\eta - 1), \\ 0 &\leq \varphi < 2\pi; \quad 1 \leq \xi < \infty; \quad -1 \leq \eta \leq 1. \end{aligned} \quad (3.6)$$

The calculated  $N_{\mu\mu'}$  depend on the pair separation  $2R$  and its direction  $(l, m, n)$ . They are of the form

$$N_{11} = i\pi(mN_x - lN_y) \left\{ c_3^2[(1 - n^2)Y_1 - n^2Y_2] - \frac{\sqrt{3}}{2} c_2 c_3[(3 + n^2)Y_1 - (1 - n^2)Y_2] \right\},$$

$$\begin{aligned} N_{22} = -i\pi(mN_x - lN_y) &\left\{ c_3^2[(1 - n^2)Y_1 - n^2Y_2] + \right. \\ &\left. + \frac{\sqrt{3}}{2} c_2 c_3[(3 + n^2)Y_1 - (1 - n^2)Y_2] \right\}, \end{aligned}$$

$$\begin{aligned} N_{12} = \frac{\pi}{2} c_2 c_3 \{ &[-2lmn(Y_1 + Y_2)N_x + n(3l^2 + m^2 + 4)Y_1 + (1 - 2m^2 - 3n^2)Y_2]N_y - \\ &- m((5 - n^2)Y_1 - (1 + n^2)Y_2)N_z] + \\ &+ i[n((l^2 + 3m^2 + 4)Y_1 + (1 - 2l^2 - 3n^2)Y_2)N_x - 2lmn(Y_1 + Y_2)N_y - \\ &- l((5 - n^2)Y_1 - (1 + n^2)Y_2)N_z] \}, \end{aligned}$$

$$\begin{aligned} N_{13} = \frac{\pi}{2} (Y_1 + Y_2) \{ &2\sqrt{3} c_2^2[l(m^2 - n^2)N_x + m(n^2 - l^2)N_y + n(l^2 - m^2)N_z] + \\ &+ ic_2 c_3[m(1 - 2l^2 - 3n^2)N_x + l(1 - 2m^2 - 3n^2)N_y + 4lmnN_z] \}, \end{aligned}$$



$$\begin{aligned}
N_{14} = & \pi \left\{ c_3^2 [(1-l^2)Y_1 - l^2 Y_2] (nN_y - mN_z) + \frac{\sqrt{3}}{2} c_2 c_3 [2lmn(Y_1 + Y_2)N_x + \right. \\
& + n((1+n^2)Y_1 - (1-n^2)Y_2)N_y - m((l^2 - m^2 + 2)Y_1 - (m^2 - l^2)Y_2)N_z] + \\
& - i \frac{\sqrt{3}}{2} c_2 c_3 [n((1+n^2)Y_1 - (1-n^2)Y_2)N_x + 2lmn(Y_1 + Y_2)N_y - \\
& - l((m^2 - l^2 + 2)Y_1 - (l^2 - m^2)Y_2)N_z] - ic_3^2 [(1-m^2)Y_1 - m^2 Y_2] (nN_x - lN_z) \left. \right\}, \\
N_{23} = & \pi \left\{ -c_3^2 [(1-l^2)Y_1 - l^2 Y_2] (nN_y - mN_z) + \frac{\sqrt{3}}{2} c_2 c_3 [2lmn(Y_1 + Y_2)N_x + \right. \\
& + n((1+n^2)Y_1 - (1-n^2)Y_2)N_y - m((l^2 - m^2 + 2)Y_1 - (m^2 - l^2)Y_2)N_z] + \\
& + i \frac{\sqrt{3}}{2} c_2 c_3 [n((1+n^2)Y_1 - (1-n^2)Y_2)N_x + 2lmn(Y_1 + Y_2)N_y - \\
& - l((m^2 - l^2 + 2)Y_1 - (l^2 - m^2)Y_2)N_z] - ic_3^2 [(1-m^2)Y_1 - m^2 Y_2] (nN_x - lN_z) \left. \right\}. \quad (3.7)
\end{aligned}$$

Here

$$\begin{aligned}
Y_1 = & -\frac{r_2^8}{70} \frac{1}{\alpha^3} \left[ 1 + \frac{10}{\alpha} + \frac{45}{\alpha^2} + \frac{105}{\alpha^3} + \frac{105}{\alpha^4} \right] e^{-\alpha}, \\
Y_2 = & -\frac{r_2^8}{70} \frac{1}{\alpha^2} \left[ 1 + \frac{5}{\alpha} + \frac{5}{\alpha^2} - \frac{30}{\alpha^3} - \frac{105}{\alpha^4} - \frac{105}{\alpha^5} \right] e^{-\alpha}, \\
\alpha = & \frac{2R}{r_2}. \quad (3.8)
\end{aligned}$$

The calculated transition rate is

$$W_{af} = \frac{4\pi^2 e^2}{c\hbar^2 \kappa^{1/2}} I_0(\omega) \delta(\omega_{af} - \omega) (W_0 + W_1 + W_2), \quad (3.9)$$

where

$$\begin{aligned}
W_0 = & (RN)^2 \{ P_1^2 + P_2^2 (|p_f|^2 + |r_f|^2) (|p_q|^2 + |r_q|^2) + 2 \operatorname{Re} [P_1 P_2 (p_f p_q^* + r_f r_q^*)] \}, \\
W_1 = & (RN) \cdot 2 \operatorname{Re} \{ P_1 P_4 (p_f N_{12}^* + r_f N_{13}^*) + P_1 P_5 (p_q N_{12}^* + r_q N_{13}^*) + \\
& + P_2 P_4 (|p_f|^2 + |r_f|^2) (p_q N_{12}^* + r_q N_{13}^*) + \\
& + P_2 P_5 (|p_q|^2 + |r_q|^2) (p_f N_{12}^* + r_f N_{13}^*) \},
\end{aligned}$$



$$\begin{aligned}
W_2 = & P_3^2(|N_{11}|^2 + |N_{14}|^2) + [P_4^2(|p_f|^2 + |r_f|^2) + P_5^2(|p_q|^2 + |r_q|^2)] (|N_{12}|^2 + |N_{13}|^2) + \\
& + P_6^2(|p_f|^2 + |r_f|^2) (|p_q|^2 + |r_q|^2) (|N_{22}|^2 + |N_{23}|^2) + \\
& + 2 \operatorname{Re} \{ P_3 P_4 [N_{11}(p_f N_{12}^* + r_f N_{13}^*) + N_{14}^*(r_f N_{12} - p_f N_{13})] + \\
& - P_3 P_5 [N_{11}(p_q N_{12}^* + r_q N_{13}^*) + N_{14}^*(r_q N_{12} - p_q N_{13})] + \\
& + P_3 P_6 [N_{22} N_{14}^*(r_f p_q + r_q p_f) + N_{11} N_{22} (r_f r_q^* - p_f p_q^*) - N_{14} N_{23}^* r_f r_q^* - N_{14} N_{23} p_f p_q - \\
& - N_{11} N_{23} (p_f r_q^* + p_q r_f^*)] + \\
& + P_4 P_5 [N_{12}^2 p_f^* p_q^* + N_{13}^2 r_f^* r_q^* - |N_{12}|^2 r_f r_q^* - |N_{13}|^2 p_f p_q^* + \\
& + N_{12}^* N_{13}^* (r_f p_q + r_q p_f) + N_{12} N_{13}^* (r_f p_q^* + r_q p_f^*)] + \\
& + P_4 P_6 (|p_f|^2 + |r_f|^2) [N_{22} (N_{12}^* p_q + N_{13}^* r_q^*) + N_{23} (N_{13}^* p_q - N_{12}^* r_q^*)] + \\
& - P_5 P_6 (|p_q|^2 + |r_q|^2) [N_{22} (N_{12}^* p_f + N_{13}^* r_f^*) + N_{23} (N_{13}^* p_f - N_{12}^* r_f^*)] \}. \quad (3.10)
\end{aligned}$$

Here

$$\begin{aligned}
P_1 &= M_{3f} M_{3q} - M_{1f} M_{1q}, \\
P_2 &= M_{4f} M_{4q} - M_{2f} M_{2q}, \\
P_3 &= M_{3f} M_{1q} - M_{1f} M_{3q}, \\
P_4 &= M_{2f} M_{3q} + M_{4f} M_{1q}, \\
P_5 &= M_{1f} M_{4q} + M_{3f} M_{2q}, \\
P_6 &= M_{2f} M_{4q} - M_{4f} M_{2q}. \quad (3.11)
\end{aligned}$$

An analysis of the matrix elements  $N_{\mu\mu}$  shows that the order of their magnitude is mainly the same for all directions of the vector  $\mathbf{R}$ .  $N_{\mu\mu}$  are about eight orders of magnitude smaller than  $(\mathbf{R}N)$  for the smallest possible  $\mathbf{R}$  and their ratio  $(\mathbf{R}N)$  decreases as the pair separation  $R$  increases. The other functions depending on  $R$  — the functions  $p_q$ ,  $r_q$  and  $P_i$  are of the same order of magnitude for a given pair separation. Therefore in the formula for the transition rate,  $W_0$  is about eight orders of magnitude larger than  $W_1$ , and larger than  $W_2$  about sixteen orders. Hence, one can restrict oneself with sufficient accuracy to this first term  $W_0$ . Thus the transition rate can be presented in the form

$$W_{af} = \frac{4\pi^2 e^2}{c\hbar^2 \kappa^{1/2}} I_0(\omega) \delta(\omega_{af} - \omega) [P_1 + DP_2(\varepsilon_f A_5^1 + A_5^2) (\varepsilon_q A_5^1 + A_5^2)]^2 (\mathbf{R}N)^2 \quad (3.12)$$

where

$$D = \frac{1}{4} \{ 3c_1^2 c_2^2 (l^2 - m^2)^2 + 4c_1^2 c_3^2 (n^2(1 - n^2) + l^2 m^2) \}. \quad (3.13)$$

In the hydrogenlike approximation of the acceptor ground state function the second term in the square bracket vanishes, and  $P_1$  goes over into a form analogous to that for

the  $n$ -type.  $P_1$  is proportional to the square of the resonance energy, which for a pair with a given separation is about the two times larger for the  $p$ -type Ge than for the  $n$ -type. This causes an increase of the absorption in the  $p$ -type.

The second reason which probably causes an increase of the absorption is the existence of the term proportional to  $P_2$  which is e.g. two times larger than  $P_1$  for the  $l = m = 1/2$  direction. The larger value of the resonance energy in  $p$ -type can also explain the shift of the maximum of the absorption towards shorter wavelengths. In fact for some average pair separation equal to  $(\frac{4}{3}\pi N)^{-1/3}$ , where  $N$  is the majority concentration, the larger value of the resonance energy causes the larger value of the energy difference between the two-centre states of the pair and then necessitates the absorption of a more energetic photon by it.

One might ask, why do the discrepancies between  $n$ - and  $p$ -type Ge occur in the absorption, whereas the  $dc$  and  $ac$  conductivities in both materials are very similar? The answer is, that the dominant contribution to the hopping conductivity comes from the so-called polar pairs. It was shown in [12] that for these pairs the results of a hydrogenlike model of acceptors agree qualitatively with the results from the model based on the Schechter wave functions. For the so-called resonance pairs (with  $\Delta \approx 0$ ) which are most important for the absorption, predictions of both models differ substantially [5]. The agreement between the experiment and theory obtained by Tanaka for microwaves in  $p$ -type Ge is to some extent due to his procedure of adjusting the Bohr radius.

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