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SPIN WAVE THEORY OF THE FIELD-INDUCED MAGNETIC PHASES OF A UNIAXIAL TWO-SUBLATTICE NÉEL-TYPE ANTIFERRIMAGNET. II. TRANSVERSAL MAGNETIC FIELD

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In Part I of this paper the linear spin-wave theory was applied to a two-sublattice antiferrimagnet of Néel type with nearest-neighbour exchange anisotropy and external magnetic field parallel to the anisotropy axis, and the field-induced magnetic phase transitions were studied. It was shown that the reality and positiveness of the spin-wave energy spectrum cannot serve as sole criterion for the stability of a magnetic phase, as it often leads to much wider stability intervals for the external field strength than those following from the minimum conditions for the system's (approximate) ground state energy. Here, analogous calculations are carried through for the case when the external magnetic field is perpendicular to the anisotropy axis.

1. Introduction

In Part I of this paper [1], a two-sublattice antiferrimagnet of Néel type with nearest-neighbour uniaxial exchange anisotropy and external magnetic field parallel to the anisotropy axis has been considered. Using the non-interacting spin waves approximation the formulae for the ground state energy, for the spin-wave spectra and for the sublattice magnetizations in each magnetic phase have been derived. It was shown that in some cases the energy spectra are real and positive for fields far beyond the stability region of the system's approximate ground state. Therefore, the analysis of the spin-wave spectra alone, without paying any attention to the stability of the system's (approximate) ground state, can lead to erroneous results. In the present paper, we extend the considerations of Part I to the case when the external magnetic field is perpendicular to the anisotropy axis.

In a previous paper [2], we examined the zero-temperature magnetic properties of a uniaxial two-sublattice Néel antiferrimagnet in an external magnetic field perpendicular to the easy axis. In particular, the critical field strengths for the phase transitions were

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determined and such thermodynamical quantities as the magnetization and susceptibility were studied. The approximate ground state of the spin Hamiltonian in [2] was determined by minimizing its expectation value in a class of trial states corresponding to complete sublattice spin alignment, the directions of which were chosen as minimization parameters. Strict solutions of the minimization equations were given, and the critical field strengths were obtained from the stability conditions for the approximate ground state of the system. It was shown that, depending on the magnitude of the anisotropy and the relative magnitude of the sublattice magnetic moments, there are two or four stable magnetic phases, namely, the canted-spin (CS) and paramagnetic (P) phases found in [3], and the additional quasi-antiferrimagnetic (Q) and antiferrimagnetic (A) phases obtained in [2].

The purpose of the present paper is to derive and investigate the spin-wave energy spectra corresponding to the stable magnetic phases obtained in [2]. Similarly, as in Part I, it is shown that in some cases the energy spectra are real and positive for fields far beyond the stability region of the system's approximate ground state. This proves that the reality and positiveness of the spin-wave energy spectrum cannot serve as sole criterion for the stability of a magnetic phase.

Furthermore, we discuss the dependence of the spin-wave energy spectrum in each magnetic phase on the strength of the external magnetic field.

2. *Spin-wave energies and critical fields*

The general expression for the spin-wave energies in a two-sublattice antiferrimagnet of Néel type, in the approximation of non-interacting spin-waves, is given by Eq. (I.24), where the symbol (I.x) denotes formula x of Part. I. As we consider the case when the external magnetic field is perpendicular to the easy axis, we must put in this formula $\varphi = \pi/2$.

In the paper [2] the stable magnetic phases were obtained and the critical field strengths for the phase transitions were determined from the minimum condition for the approximate ground state energy E_{oM}. Here, we want to utilize the results of [2] in specifying the general formula (I.24) for those magnetic phases, and in determining again the critical field strengths from the standard condition for the reality and positiveness of the energy spectra. Finally, we shall compare these results with those obtained in [2].

We consider at first the more complicated case when the anisotropy and the ratio of the sublattice magnetic moments fulfil the inequality $w < w_o$ (see [2]), in which case we must distinguish four phases: the quasi-antiferrimagnetic (Q), the antiferrimagnetic (A), the canted-spin (CS) and the paramagnetic (P).

In the quasi-antiferrimagnetic and canted-spin phases, the sublattice spins form the angles θ_1 and θ_2 with the external magnetic field, which fixes the spin quantization axes in the two sublattices. These angles were determined in [2] by minimizing E_{oM} as defined by Eq. (I.10). The solutions which describe the field-dependence of the spin quantization directions (or equivalently, of the sublattice magnetization directions) in the Q and CS phases were shown in [2] to have the form

$$\sin\theta_1 = hw^{-1}(ZR_2 - \kappa X),\ \sin\theta_2 = hw^{-1}(\kappa ZR_2^{-1} - X), \tag{1}$$

where $R_2^2 = (\kappa^2 h^2 + w)/(h^2 + w)$. Accordingly, the quantities (I.14a)–(I.14c) assume the form

$$B_1 = S_2 JZR_2^{-1}\gamma_o,\ B_2 = S_1 JZR_2\gamma_o,$$

$$C_k = \tfrac{1}{2}(S_1S_2)^{1/2}J\gamma_k[R_2X - h^2w^{-1}(\kappa Z - R_2X) - 1],$$

$$D_k = \tfrac{1}{2}(S_1S_2)^{1/2}J\gamma_k[R_2X - h^2w^{-1}(\kappa Z - R_2X) + 1], \tag{2}$$

and the general formula (I.24) for the spin-wave energy spectra specifies in the Q and CS phase as follows:

$$E_{k,j} = \frac{1}{\sqrt{2}} S_2 J\gamma_o\{Z^2(R_2^{-2} + S^2R_2^2) - 2SR_2w^{-1}[X(h^2+w) - h^2ZR_2^{-1}\kappa]y_k^2 \pm$$

$$\pm Z[Z^2(R_2^{-2} - S^2R_2^2)^2 + 4Sy_k^2\{w^{-1}[X(h^2+w) - h^2ZR_2^{-1}\kappa] - SR_2\} \times$$

$$\times \{SR_2^2w^{-1}[X(h^2+w) - h^2ZR_2^{-1}\kappa] - R_2^{-1}\}]^{\frac{1}{2}}\}^{\frac{1}{2}}. \tag{3}$$

The energy spectra (3) are real and positive if $0 < h < h_a'$ and $h_a'' < h < h_n$ where

$$h_a' = (2\kappa)^{-1}\{X(1-\kappa) - [X^2(1-\kappa)^2 - 4\kappa w]^{1/2}\}, \tag{4a}$$

$$h_a'' = (2\kappa)^{-1}\{X(1-\kappa) + [X^2(1-\kappa)^2 - 4\kappa w]^{1/2}\}, \tag{4b}$$

$$h_n = (2\kappa)^{-1}\{X(1+\kappa) + [X^2(1+\kappa)^2 + 4\kappa w]^{1/2}\}. \tag{4c}$$

We see (cp. [2]) that in the Q and CS phases the region of stability of the approximate ground state coincide with those of the positiveness of the energy spectra.

If $w < w_o$, the quasi-antiferrimagnetic and the canted-spin phase are separated from each other by an antiferrimagnetic phase, in which the sublattice magnetizations are antiparallel with the larger spins pointing in the direction of the external magnetic field. Thus, we put $\theta_1 = -\pi/2$, $\theta_2 = \pi/2$. Upon specifying the expressions (I.14a)–(I.14c) we obtain from the general formula (I.24) the spin-wave energy spectrum in the A phase:

$$E_{k,j} = \frac{1}{\sqrt{2}} S_2 J\gamma_o\{(X+h)^2 + S^2(X-\kappa h)^2 - 2SZy_k^2 \pm$$

$$\pm\sqrt{[(X+h)^2 - S^2(X-\kappa h)^2]^2 + 4Sy_k^2[Z(X+h) - S(X-\kappa h)][SZ(X-\kappa h) - (X+h)]}\}^{1/2}. \tag{5}$$

The energy spectra (5) are real and positive if $h_a' < h < h_a''$ where h_a', h_a'' are given by Eqs (4a), (4b), as well as for

$$h > h_a''' = (2\kappa)^{-1}\{X(1-\kappa) + [X^2(1+\kappa)^2 - 4\kappa]^{1/2}\} > h_a''. \tag{6}$$

It was shown in [2] that the approximate ground state corresponding to the antiferrimagnetic spin configuration A is stable only for $h_a' < h < h_a''$. On the other hand, from Eq. (6) it is seen that the spin-wave energy spectrum is real and positive for fields far beyond the stability region of the system's approximate ground state.

In the paramagnetic phase, all the spins become aligned in the direction of the external field, *i.e.*, $\theta_1 = \theta_2 = \pi/2$, and (I.24) specifies as follows:

$$E_{k,j} = \frac{1}{\sqrt{2}} S_2 J \gamma_o \{(X-h)^2 + S^2(X-\kappa h)^2 + 2SZy_k^2 \pm$$

$$\pm \sqrt{[(X-h)^2 - S^2(X-\kappa h)^2]^2 + 4Sy_k^2[Z(X-h)+S(X-\kappa h)][SZ(X-\kappa h)+(X-h)]}\,\}^{1/2}. \quad (7)$$

The reality condition for the energy spectra (7) leads to the following restrictions for the external magnetic field: $h_n' < h < h_n''$ and $h > h_n$, where h_n is given by Eq. (4c) and

$$h_n' = (2\kappa)^{-1}\{X(1+\kappa) - [X^2(1-\kappa)^2 + 4\kappa]^{1/2}\}, \quad (8a)$$

$$h_n'' = (2\kappa)^{-1}\{X(1+\kappa) + [X^2(1-\kappa)^2 + 4\kappa]^{1/5}\} < h_n. \quad (8b)$$

However, we know from [2] that the approximate ground state in the paramagnetic phase is stable only for $h > h_n$. Therefore, in this case the examination of the reality of the energy spectra leads again to incorrect results.

When the anisotropy and the ratio of sublattice magnetic moments fulfil the inequality $w > w_o$, we have only two phases: the canted-spin and the paramagnetic phase. The energy spectra are given by Eqs (3) and (7), respectively. In the canted-spin phase the region of stability of the approximate ground state coincides with that of the positiveness of the energy spectra. However, as shown above in the paramagnetic phase the spin-wave energy spectrum is real and positive for fields beyond the stability region of the system's approximate ground state.

3. *Spin-wave energies as functions of the external magnetic field*

The spin-wave energies in the Q, A, CS and P phase are respectively given by Eqs (3), (5) and (7). Thus, we can analyze the behaviour of these energies in each magnetic phase under the influence of the external magnetic field perpendicular to the easy axis.

Let us start from the case $w < w_o$ and consider the quasi-antiferrimagnetic phase. From (3) results that

$$E_{k,1}(y_k = 0) = \gamma_o S_1 JZR_2, \; E_{k,2}(y_k = 0) = \gamma_o S_2 JZR_2^{-1}. \quad (9)$$

Therefore, at the point $y_k = 0$ the first spectrum branch lowers with increasing field while the second one rises. At the same time, it can be shown that $E_{k,1}(y_k = 1)$ and $E_{k,2}(y_k = 1)$ lower with increasing field. Therefore, in the Q phase the spin-wave energy spectrum changes under the influence of the field as shown in Fig. 1.

When considering the antiferrimagnetic phase, one easily obtains from Eq. (5) that

$$E_{k,1}(y_k = 0) = \begin{cases} \gamma_o S_1 J(X-\kappa h) & \text{for } h_a' < h < h_{d1} \\ \gamma_o S_2 J(X+h) & \text{for } h_{d1} < h < h_a'', \end{cases} \quad (10a)$$

$$E_{k,2}(y_k = 0) = \begin{cases} \gamma_o S_2 J(X+h) & \text{for } h_a' < h < h_{d1} \\ \gamma_o S_1 J(X-\kappa h) & \text{for } h_{d1} < h < h_a'', \end{cases} \quad (10b)$$

$$\Delta E_k = \gamma_o S_2 J\{(X+h)^2 + S^2(X-\kappa h)^2 - 2SZy_k^2 - 2S[(X+h)^2(X-\kappa h)^2 -$$

$$-(Z^2+1)(X+h)(X-\kappa h)y_k^2 + Z^2 y_k^4]^{1/2}\}^{1/2} \geqslant 0, \quad (10c)$$

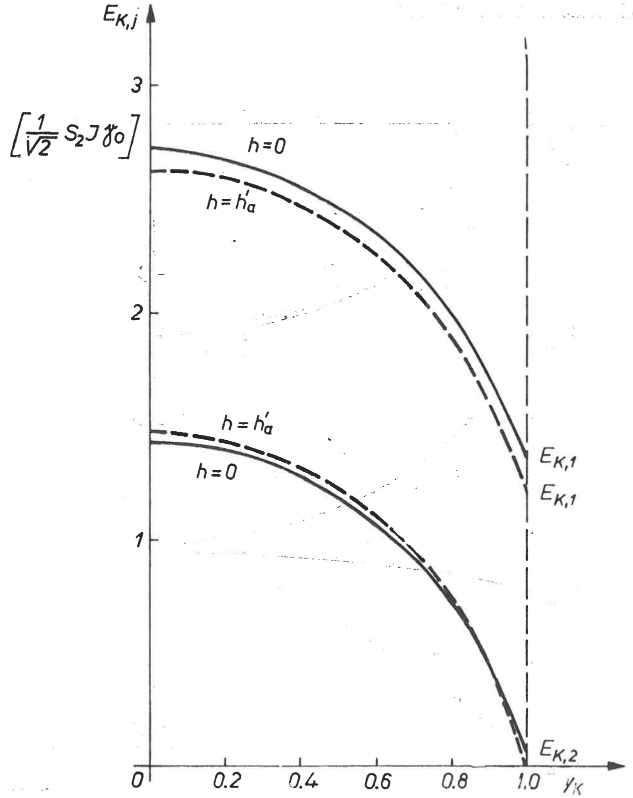

Fig. 1

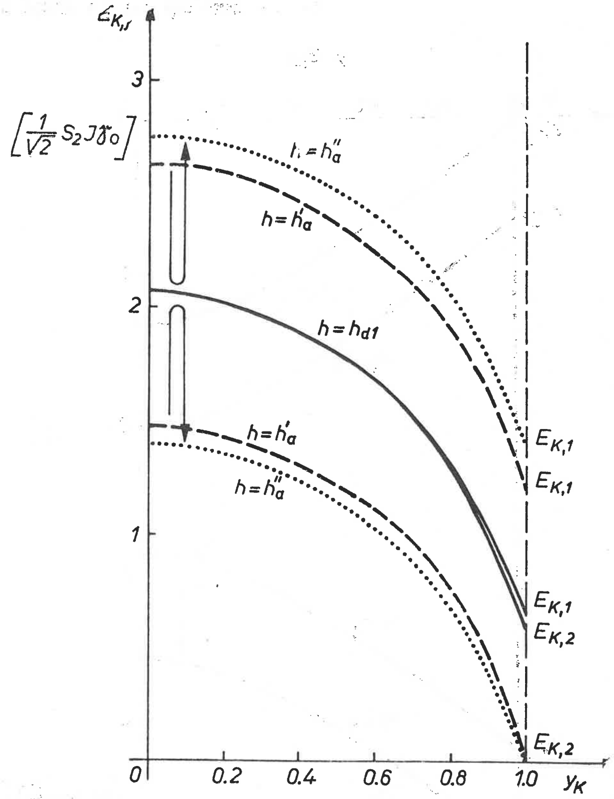

Fig. 2

Fig. 1. Free-spin-wave energy spectra in the quasi-antiferrimagnetic phase **Q** for the case $w < w_o$

Fig. 2. Free-spin-wave energy spectra in the antiferrimagnetic phase A for the case $w < w_o$

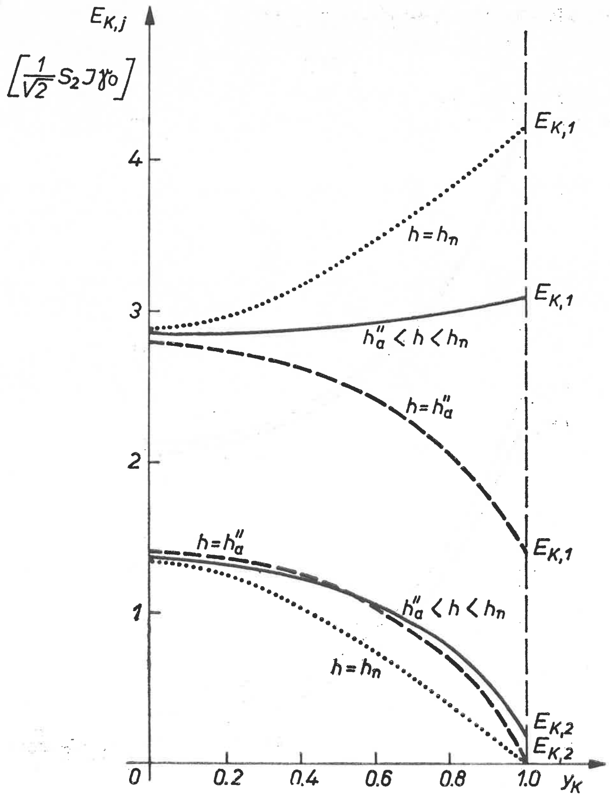

Fig. 3

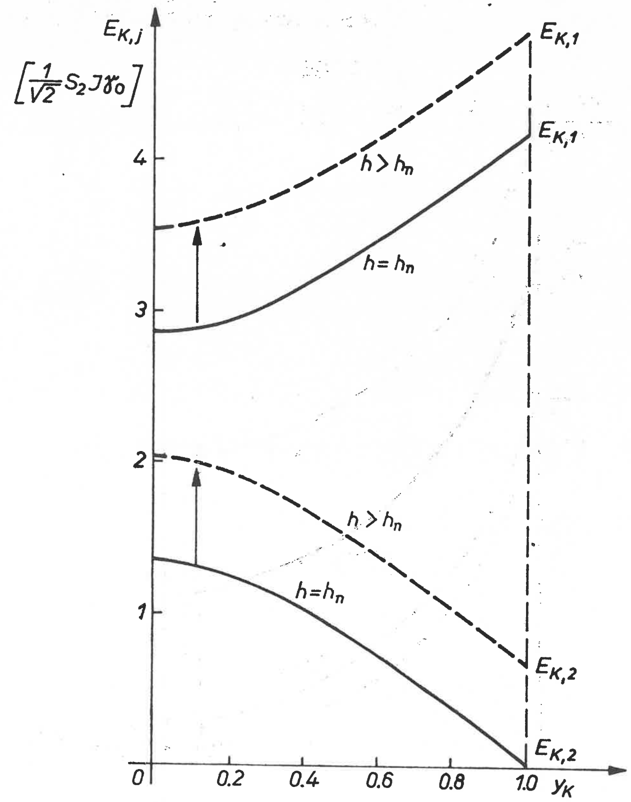

Fig. 4

Fig. 3. Free-spin-wave energy spectra in the canted-spin phase CS for the case $w < w_o$
Fig. 4. Free-spin-wave energy spectra in the paramagnetic phase P for the case $w < w_o$

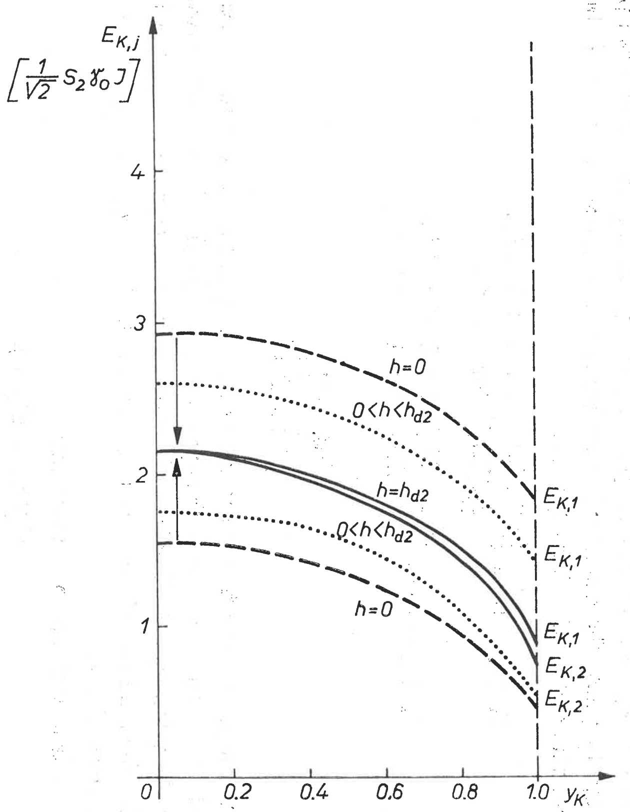

Fig. 5

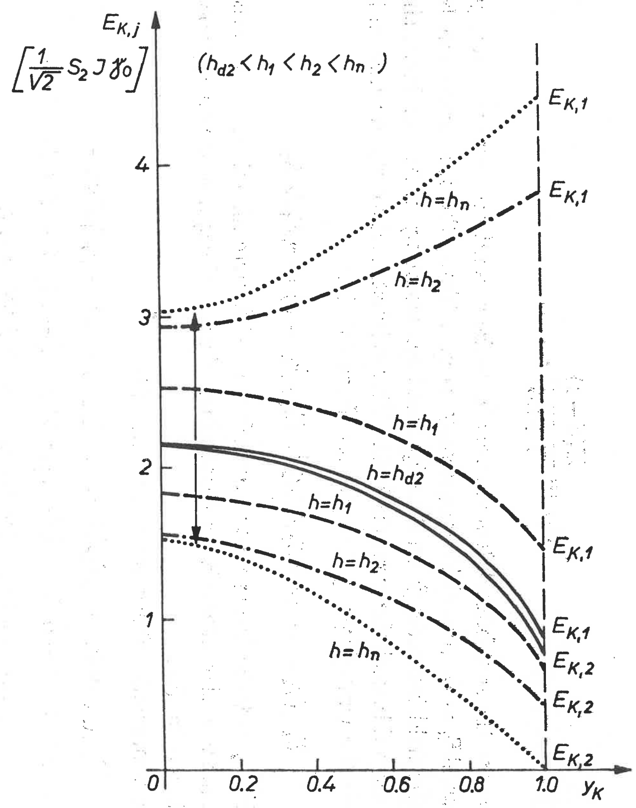

Fig. 6

Fig. 5. Free-spin-wave energy spectra in the canted-spin phase CS for the case $w > w_o$ and $0 < h < h_{d2}$
Fig. 6. Free-spin-wave energy spectra in the canted-spin phase CS for the case $w > w_o$ and $h_{d2} < h < h_n$

where

$$h_{d1} = \frac{X(S-1)}{1+\kappa S}. \tag{11}$$

Hence, in this case the first branch lies always above the second one, except when $y_k = 0$ and $h = h_{d1}$ as $\Delta E_k = 0$ (two-fold degeneracy point). Thus, the first spectrum branch lowers when the field increases from 0 to h_{d1} and rises again upon further increasing the field. For the second branch the opposite holds. This is illustrated for the A phase in Fig. 2.

As regards the canted-spin phase, we see from (3) that

$$E_{k,1}(y_k = 0) = \gamma_o S_2 JZR_2^{-1},\ E_{k,2}(y_k = 0) = \gamma_o S_1 JZR_2, \tag{12}$$

i.e., at the point $y_k = 0$ the first spectrum branch rises with increasing field while the second one lowers. A numerical analysis shows that in the CS phase the spin-wave energy spectrum changes as shown in Fig. 3.

In the paramagnetic phase we have from (7)

$$E_{k,1}(y_k = 0) = \gamma_o S_2 J(h-X),\ E_{k,2}(y_k = 0) = \gamma_o S_1 J(\kappa h - X), \tag{13a}$$

$$\Delta E_k = \gamma_o S_2 J\,\{(X-h)^2 + S^2(X-\kappa h)^2 + 2SZy_k^2 - 2S\,[(X-h)^2(X-\kappa h)^2 - -(Z^2+1)(X-h)(X-\kappa h)y_k^2 + Z^2 y_k^4]^{1/2}\}^{1/2} > 0. \tag{13b}$$

Thus, in this case both branches rise with increasing field, as illustrated in Fig. 4.

It can be seen from Eqs (3), (5) and (7) that the corresponding energy branches of the neighbouring phases are continuous at the transition points, *i.e.*,

$$E^{\mathrm{Q}}_{k,j}(h'_a) = E^{\mathrm{A}}_{k,j}(h'_a),\quad E^{\mathrm{A}}_{k,j}(h''_a) = E^{\mathrm{CS}}_{k,j}(h''_a),\quad E^{\mathrm{CS}}_{k,j}(h_n) = E^{\mathrm{P}}_{k,j}(h_n). \tag{14}$$

In the case $w > w_o$ we have only the paramagnetic phase and the canted-spin phase. In the paramagnetic phase the energy spectrum behaves analogously as in Fig. 4, but for the canted-spin phase it results from (3) that

$$E_{k,1}(y_k = 0) = \begin{cases} \gamma_o S_1 JZR_2 & \text{for } 0 < h < h_{d2} \\ \gamma_o S_2 JZR_2^{-1} & \text{for } h_{d2} < h < h_n, \end{cases} \tag{15a}$$

$$E_{k,2}(y_k = 0) = \begin{cases} \gamma_o S_2 JZR_2^{-1} & \text{for } 0 < h < h_{d2} \\ \gamma_o S_1 JZR_2 & \text{for } h_{d2} < h < h_n, \end{cases} \tag{15b}$$

where

$$h_{d2} = \left[\frac{w(S-1)}{1+\kappa^2 S}\right]^{\frac{1}{2}}. \tag{16}$$

Thus, at the point $y_k = 0$ the first spectrum branch at first lowers and then rises with increasing field. For the second branch the opposite holds. For $h = h_{d2}$ we have a two-fold degeneracy at the point $y_k = 0$. The behaviour of the spin-wave energy spectrum in the CS phase under the influence of the field is shown for $0 < h < h_{d2}$ and $h_{d2} < h < h_n$ in Figs 5 and 6, respectively.

Similarly as before it can be shown that

$$E^{\mathrm{CS}}_{k,j}(h_n) = E^{\mathrm{P}}_{k,j}(h_n). \tag{17}$$

4. *Final remarks*

In [2] the approximate ground state was determined by minimizing the expectation value of the spin Hamiltonian (I.1) in a class of trial states representing sublattice saturation states. This corresponds actually to minimizing E_{oM} and assuming complete sublattice spin alignment. It was shown that the energy of the approximate ground state E_{oM} is a continuous function of the external field. The same is true for the approximate ground state energy E_o given by Eq. (I.25). From Eqs (I.25) and (I.26) it is seen that $E_o < E_{oM}$ (as E_{oS} vanishes in all phases), but the continuity of the energy at the critical points h'_a, h''_a and h_n is preserved.

Similarly as in Part I, the magnetization is defined by the Eqs (I.46)–(I.50), except that in the formulae (I.47) and (I.50) we must interchange the indices $||$ and $\perp$. Upon specifying θ_j and the coefficients u and v in the formulae (I.47)–(I.50), we obtain the expressions for the sublattice magnetizations in each phase. From the explicit formulae for the magnetizations in all the phases it is easy to see that the magnetizations are continuous at the transition Q $\leftrightarrow$ A, A $\leftrightarrow$ CS and CS $\leftrightarrow$ P for the case $w < w_o$, and at the transition CS $\leftrightarrow$ P for the case $w > w_o$.

In this paper, the physical properties of the antiferrimagnetic system at low temperatures were examined in the limits of the linear spin-wave theory. Moreover, we re-examined again the cases when the external magnetic field is parallel or perpendicular to the anisotropy axis. The critical field for the phase transitions were here determined from the standard reality and positiveness conditions for the spin-wave energy spectra, and were compared with those obtained in [2, 4] from the stability conditions for the approximate ground state of the system. It was shown that in some cases the spin-wave energy spectra are real and positive for fields far beyond the stability region of the system's approximate ground state. This demonstrates that the analysis of the spin-wave spectra alone, when disregarding the stability of the spin-wave reference state (approximate ground state of the system), can lead to erroneous results. Thus, the reality and positiveness of the spin-wave energy spectrum cannot serve as sole criterion for the stability of a magnetic phase.

A spin-wave theory is constructed by considering the deviations of each spin from its equilibrium direction. Therefore, we have introduced local co-ordinate axes for each sublattice (see Eq. (I.5)). The equilibrium axes were obtained by minimizing the approximate ground state energy and are therefore temperature-independent. It would be interesting to find the temperature-dependent equilibrium axes and critical fields.

One way of doing it is to follow the procedure applied in [5] to antiferromagnetic systems. The temperature dependence of the phase boundaries was there obtained by introducing spin-wave interaction terms and deriving the equations of motion for the spin-wave operators. When these equations are linearized, by replacing pairs of spin-wave operators with their thermal expectation values, we obtain renormalized spin-wave spectra (in low-temperature region) and thus temperature-dependent critical fields.

In the CS phase, we have additional terms in the Hamiltonian which are linear and cubic in the spin-wave operators. The linear terms contribute a static part to the equations of motion, and the elimination of this part provides a condition from which the angles θ_j

can be determined as functions of the external field. Now, by linearizing the cubic terms in the Hamiltonian we get a temperature-dependent static part in the equations of motion, and consequently temperature-dependent equilibrium angles θ_j.

However, the same results can be obtained in another and much simpler way, without having to include spin-wave interactions. Namely, at non-zero temperatures the condition for a stable equilibrium is that the free energy have a minimum with respect to the variation of the angles θ_j. Thus, by minimizing the system's free energy with respect to θ_j one arrives at temperature-dependent equations for these angles, much like in the case of ferromagnetism [6].

Such investigations are under way and the results will be published in a separate paper.

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