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RELATIVISTIC STATISTICAL MECHANICS AND BLACKBODY RADIATION. III. COHERENCE IN PLANE BEAMS*

By J. H. Eberly**

Department of Physics and Astronomy, University of Rochester*

And A. Kujawski

Institute of Physics, Polish Academy of Sciences, Warsaw***

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Coherence properties of a plane blackbody radiation beam in an arbitrary Lorentz frame are considered. The invariant formula for the degree of coherence of such a beam is derived. A discussion showing how the coherence results are related to the problem of the transformation law of temperature and to the cosmic blackbody radiation, is presented.

1. Introduction

In our previous papers (Eberly and Kujawski 1967, 1968) devoted to relativistic statistical mechanics and blackbody radiation we have derived the manifestly Lorentz invariant thermal-equilibrium density matrix. By means of this invariant density matrix we could easily calculate space-time and spectral coherence functions describing blackbody radiation in an arbitrary Lorentz inertial frame. The detailed discussion of different correlation functions for electric and magnetic field components and its dependence on the velocity of a moving observer has been illustrated graphically. Our results have also been generalized (Brevik and Suhonen 1968, 1969, 1970) for the case of blackbody radiation within a transparent medium.

In this paper we want to complete the discussion of coherence properties of blackbody radiation by investigating the coherence properties of a plane blackbody radiation beam in an arbitrary Lorentz frame.

Our coherence results are related to two problems which recently have been widely discussed. The first one is the relativistic formulation of thermodynamics, and the second

* Research partially supported by the US Air Force Office of Scientific Research and the US Army Research Office (Durham). Address: Rochester, N.Y. 14627, USA.

** US National Academy of Science Visiting Lecturer to Eastern Europe, 1970.

*** Address: Instytut Fizyki PAN, Warszawa, Zielna 37, Poland.

is the cosmic blackbody radiation field. The thermal-equilibrium radiation field is an example of a thermodynamical system for which it is easy to investigate the relativistic transfor mation properties. As a matter of fact, the blackbody radiation thermal field served in the past for the derivation of the transformation law of temperature (Planck 1908, Pauli 1958).

Because of the known controversy (for references and discussion see Landsberg and Johns 1967, Balescu 1968, van Kampen 1968, Yuen 1970) and the many unclear points in discussions of relativistic thermodynamics, we again discuss the problem of the temperature transformation formula. One can mean by the temperature of the moving system the proper temperature T_0, *i. e.*, the temperature which is measured by a thermometer at rest with respect to the system. Since a system described in a moving frame of reference possesses momentum, an interpretation in which a momentum temperature-vector appears is necessary. In the spirit of thermodynamics based on information theory (Jaynes 1957, 1958, Ingarden and Urbanik 1961, Ingarden 1965, 1965a) one should characterize the system by the four-vector of temperature which in our case is equal to V_μ/T_0, where V_μ is the four-velocity vector.

An example of blackbody radiation for which a relativistic description may play an especially important role is the cosmic blackbody radiation (references and a review of this subject may be found in Dantcourt and Willis 1968 or Partridge 1969). A lot of attention has been paid to the problem of anisotropy of that radiation. One of the causes of the anisotropy could be the fact that the galaxy is moving. In Section 5 we discuss the anisotropy problem in terms of coherence quantities and show how they can charcterize the anisotropy of the cosmic blackbody radiation.

2. *Basic concepts of relativistic coherence theory*

At the foundation of all of our results lies the statistical character of the electromagnetic fields we deal with. It is appropriate, therefore, that we begin with a brief review of our earlier discussion (Eberly and Kujawski, 1967, 1968) of the manifestly Lorentz-invariant density operator ϱ.

We may say, first of all, that the operational significance of ϱ is one of enumeration or counting. That is, ϱ has to do only with the relative occupation probabilities associated with the eigenstates of the quantum system under consideration. For this reason it is clearly a Lorentz scalar. It is usually not easy, however, to cast ϱ into a form in which its scalar nature is manifest. For the purposes of relativistic coherence theory, though, the manifestly invariant form is practically indispensible. With its aid one is able to compute relativistic correlation functions in arbitrary Lorentz frames as simply as the corresponding rest frame quantities.

We have shown, in the first reference mentioned above, that entropy maximization can lead one to the desired manifestly scalar form for ϱ. If one assumes that the system of interest is characterized by an energy-momentum four vector operator whose average value is measured, and if the motion of the system can be labelled by its unit four-velocity,

$$V^\mu = (\gamma, \gamma \mathbf{v}), \quad V^\mu V_\mu = 1, \tag{2.1}$$

then the manifestly invariant form for ϱ is simply

$$\varrho = Z^{-1} \exp(-\alpha P \cdot V). \tag{2.2}$$

Here $P \cdot V = P^{\mu} V_{\mu}$, where P^{μ} is the system's four-vector energy-momentum operator; and Z is the partition function, defined so that trace of ϱ is unity:

$$Z = \mathrm{Tr}\,\{\exp(-\alpha P \cdot V)\}, \tag{2.3}$$

and $1/\alpha = KT_0$, Boltzmann's constant times the rest system temperature.

Of course the four-vector momentum and velocity are not the only quantities which might be needed to characterize a relativistic system. One could imagine needing to use the stress-energy tensor, for example. In such an unusual case the resulting operator ϱ would not, of course, bear any simple relation to the expression in (2.2). For the most part we will consider only density operators of the form (2.2). As should be expected, when applied to systems consisting of electromagnetic radiation, such density operators afford a description of blackbody radiation generalized to an arbitrary Lorentz frame.

In this first case of interest, that of a free electromagnetic radiation field, the energy and momentum of the system are familiar objects. We may write them in terms of the usual photon creation and annihilation operators as follows:

$$P_0 = \sum_{k\lambda} \hbar \omega_k a_{k\lambda}^{+} a_{k\lambda}^{-} \tag{2.4}$$

$$\boldsymbol{P} = \sum_{k\lambda} \hbar \boldsymbol{k} a_{k\lambda}^{+} a_{k\lambda}^{-} \tag{2.5}$$

and then the density operator can also be written explicitly in terms of $a_{k\lambda}^{-}$ and $a_{k\lambda}^{+}$ as a product over all field modes $\boldsymbol{k}$:

$$\varrho = \prod_{k\lambda} \varrho_{k\lambda}, \tag{2.6}$$

$$\varrho_{k\lambda} = (1 - \exp(-\alpha k \cdot V)) \exp(-\alpha k \cdot V a_{k\lambda}^{+} a_{k\lambda}^{-}), \tag{2.7}$$

where $k \cdot V = k_{\mu} V^{\mu}$, and k^{μ} is the momentum four-vector of a photon of mode $\boldsymbol{k}\lambda$, $k^{\mu} = (\omega_k, \boldsymbol{k}c)$.

In the remainder of this paper we will be concerned with electric and magnetic field space-time correlation functions in a moving reference frame. We define here some of the notation that we will find convenient in our investigation. As is usual in the quantum theory of coherence we will ignore vacuum fluctuations and the zero point energy (Glauber 1963). All correlation functions are then normally ordered, with positive-frequency operators. We may establish our system of units by writing here explicitly the positive and negative frequency parts of the electric field operator at space-time point $x = (t, \boldsymbol{r})$ in the volume $\mathscr{V}$:

$$E_i^{(\pm)}(x) = \pm i \left(\frac{2\pi}{\mathscr{V}}\right)^{\frac{1}{2}} \sum_{k,\lambda} \sqrt{\omega_k}\, \mathscr{E}_i^{\lambda}(k) a_{k\lambda}^{\mp} e^{\mp ik \cdot x}. \tag{2.8}$$

As before, $\boldsymbol{k}$ and λ refer to the wave vector and polarization component of the mode. We have now adopted, and will retain, the conventions that $\hbar = c = 1$, that the vector index i (and later j, k, l as well) denotes a cartesian component of the field, and that the triad $\hat{\varepsilon}^1(k)$, $\hat{\varepsilon}^2(k)$, $\hat{k}$ is right-handed and orthonormal, so that

$$\sum_{\lambda=1,2} (\hat{\varepsilon}^\lambda)_i(\hat{\varepsilon}^\lambda)_j = \delta_{ij}-\hat{k}_i\hat{k}_j. \tag{2.9}$$

We now introduce the second-order electric field space-time autocorrelation tensor with components $\mathscr{E}_{ij}$:

$$\mathscr{E}_{ij}(x_1, x_2) = \langle E_i^{(-)}(x_1)E_j^{(+)}(x_2)\rangle. \tag{2.10}$$

Because of the temporal stationarity and spatial homogeneity implied by the density operator $\varrho_{k\lambda}$ in (2.7), and by $\varrho = \prod\varrho_{k\lambda}$ as well (see, in this connection, Eberly and Kujawski 1967a), both of which properties hold in all Lorentz frames, $\mathscr{E}_{ij}$ can depend only on the space-time differences $\tau = t_1-t_2$ and $r = r_1 - r_2$. Thus, without loss of generality, we will concentrate our attention on $\mathscr{E}_{ij}(x) = \mathscr{E}_{ij}(x_1-x_2, 0)$.

The expectation in (2.10) may be evaluated if ϱ is given by (2.6), for example, in the following form:

$$\mathscr{E}_{ij}(x) = \frac{1}{4\pi^2} D_{ij} \int \frac{e^{-ik\cdot x}}{e^{\alpha k\cdot V}-1} \frac{d^3k}{\omega_k}, \tag{2.11}$$

where D_{ij} is the differential operator $\partial_i\partial_j-\nabla^2\delta_{ij}$, and $\partial_i = \partial/\partial r_i$, *etc.* We may point out the manifest Lorentz invariance of the integral in (2.11), recalling that $d^3k/2\omega_k =$ $= d^4k\ \delta(k^2)$.

In addition to the electric auto-correlation tensor defined in (2.10) there are three other second-order tensors of interest: a magnetic auto-correlation tensor and two mixed, or cross-correlation, tensors. They are defined as follows

$$\begin{aligned} \mathscr{H}_{ij}(x) &= \langle H_i^{(-)}(x)H_j^{(+)}(0)\rangle, \\ \mathscr{M}_{ij}(x) &= \langle E_i^{(-)}(x)H_j^{(+)}(0)\rangle, \\ \mathscr{N}_{ij}(x) &= \langle H_i^{(-)}(x)E_j^{(+)}(0)\rangle. \end{aligned} \tag{2.12}$$

Of course, the 36 separate correlation functions defined by (2.10) and (2.12) are equally well expressed as components of the Lorentz-covariant tensor

$$P_{\mu\nu\alpha\beta}(x) = \langle F_{\mu\nu}^{(-)}(x)F_{\alpha\beta}^{(+)}(0)\rangle, \tag{2.13}$$

where $F_{\mu\nu}$ is the usual electromagnetic field tensor. In fact, the correlation functions given in (2.10) and (2.12) are the only possible non-zero components of $P_{\mu\nu\alpha\beta}$.

It is useful to state immediately some of the symmetry properties of our electromagnetic coherence tensors, again for the especially interesting ϱ given in (2.6) (Eberly and Kujawski 1967). In any Lorentz frame one has, for such a ϱ,

$$\mathscr{E}_{ij}(x) = \mathscr{H}_{ij}(x), \text{ and } \mathscr{M}_{ij}(x) = -\mathscr{N}_{ij}(x). \tag{2.14}$$

Thus at most half of the coherence tensors are independent in this case, and it is sufficient to consider only $\mathscr{E}_{ij}$ and $\mathscr{M}_{ij}$. For this reason we now give the explicit evaluation of $\mathscr{M}_{ij}(x)$ which is analogous to (2.11) for $\mathscr{E}_{ij}(x)$:

$$\mathscr{M}_{ij}(x) = \frac{1}{4\pi^2} \varepsilon_{ijl}\partial_l\partial_0 \int \frac{e^{-ik\cdot x}}{e^{\alpha k\cdot V}-1} \frac{d^3k}{\omega_k}, \tag{2.15}$$

where ε_{ijl} is the totally anti-symmetric symbol of Levi-Civita, $\partial_0 = \partial/\partial_t$, and summation from 1 to 3 over the repeated index is understood. Note that the same Lorentz-invariant integral as in (2.11) occurs here also. Taken together, Eqs (2.11) and (2.15) provide a complete description of the second order coherence properties of blackbody radiation in any Lorentz frame.

One further point remains to be established. Coherence theory customarily deals with normalized correlations, and we want to introduce a normalization factor into our discussion, in order to speak hereafter about d e g r e e s of coherence in a sensible way. We may do this (see, for example, Mehta and Wolf 1964) by dividing the fields $E_i(x)$ and $H_i(x)$ everywhere by their amplitudes, that is, by the square roots of the corresponding electric and magnetic intensities $\mathscr{E}_{ii}(0)$ and $\mathscr{H}_{ii}(0)$. One may then show (Metha and Wolf 1964) that the d e g r e e s o f c o h e r e n c e defined thereby,

$$\gamma_{ij}(x) = \mathscr{E}_{ij}(x)/[\mathscr{E}_{ii}(0)\mathscr{E}_{jj}(0)]^{\frac{1}{2}}, \tag{2.16}$$

$$\sigma_{ij}(x) = \mathscr{M}_{ij}(x)/[\mathscr{E}_{ii}(0)\mathscr{H}_{jj}(0)]^{\frac{1}{2}} \tag{2.17}$$

are bounded in absolute value between 0 and 1.

We close this section by indicating the explicit value of the desired normalization factors in any Lorentz frame in our case of special interest, when ϱ is given by (2.6). According to the first part of (2.14) the two factors are equal to each other, so it suffices to give $\mathscr{E}_{ii}(0)$:

$$\mathscr{E}_{ii}(0) = \frac{4}{90\pi}\left(\frac{\pi}{\alpha}\right)^4 \left[\frac{1+v^2-2v_i^2}{1-v^2}\right]. \tag{2.18}$$

3. *Coherence in a blackbody field*

The formulas for correlation tensors describing the blackbody radiation field in an arbitrary inertial frame of reference, wich have been presented in Section 2, refer to the whole field existing in a sufficiently large cavity. The complete discussion of those formulas is a complicated matter. A number of curves illustrating temporal and spatial dependence of correlation tensors was discussed in our previous papers. In this section we shall discuss some of our graphs which reveal features showing essential differences from those for a plane beam which possesses a blackbody spectrum.

In our discussion we shall restrict ourselves to spatial coherence. We shall specify coherence tensors by field components taken in the plane $x = 0$ which is perpendicular

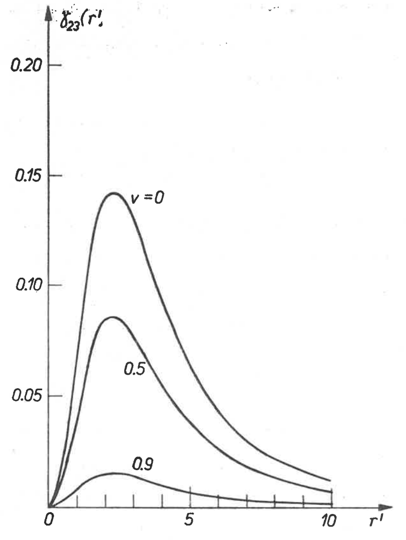

Fig. 1

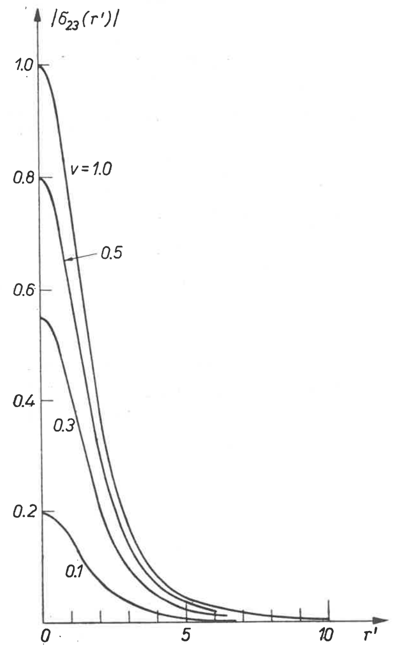

Fig. 2

to the $\mathfrak{v}$ direction. Let us consider as an example the normalized coefficients γ_{23} and σ_{23} defined in Eqs (2.16) and (2.17). These cases are illustrated by Figs 1 and 2 where $r' = r/\alpha$ and in the first case r is along the line $y = z$ in the $x = 0$ plane, and in the second case r is in the $y-z$ plane. The characteristic feature of the first curve is the fact that for $r = 0$

in each Lorentz frame the orthogonal field components which are perpendicular to the $\mathbf{v}$-direction are not correlated. From this Figure it is seen that for increasing velocity of the observer the coherence coefficient decreases. The reverse situation is illustrated by Fig. 2 where for larger values of $\mathbf{v}$ the observer finds an increasing coherence parameter.

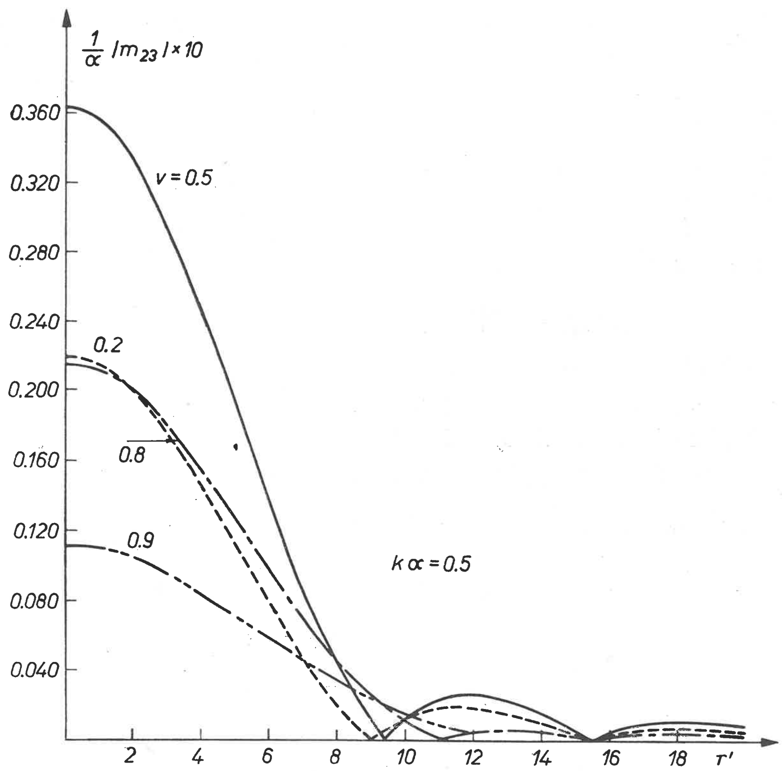

Fig. 3

For $\mathbf{r} = 0$ $\sigma_{23}(0,0)$ is proportional to the expectation value of the Poynting vector, and it is understandable that for $\mathbf{v} = 0$ $\sigma_{23} = 0$ since the average energy flux in the stationary system is equal to zero.

In the spectral domain one can find that the coefficient m_{23} defined by $m_{23}(\mathbf{r}, \omega) = \mathscr{M}_{23}(\mathbf{r}, \omega)/[\mathscr{E}_{23}(0,0)\mathscr{H}_{23}(0,0)]^{1/2}$ is not a monotonic function of $\mathbf{v}$. For $v = 0$ and $v = 1$ m_{23} vanishes. Fig. 3 illustrates this case, when $r' = r/\alpha$ and r is along the line $y = z$, $x = 0$.

It is a very difficult matter to give any explicit formulas for the coherence time and the coherence length. From the curves shown above and other graphs published previously one finds that these coherence parameters may increase as well as decrease with increasing velocity of the observer. On the other hand there is no difficulty in calculating formulas for the coherence time and the coherence length for a plane beam. This will be discussed in the next Section.

4. *Coherence aspects of a pencil of radiation*

Though coherence properties of the entire blackbody radiation field depend on the velocity of the moving observer in a way which reveals many interesting features, their practical value does not seem to be high. Most detectors are highly directional in character, so, in our opinion, from a practical point of view it is more important to investigate coherence properties associated with a pencil of radiation rather than with the entire field. This is the reason for which we shall now apply the coherence concepts developed in Section 2 to a pencil of electromagnetic radiation. With such a pencil we may associate the four-momentum operator $p^\mu = \hbar k^\mu a^+_{k\lambda} a^-_{k\lambda}$, specifying the energy and direction of propagation of the pencil. In the case of a blackbody spectrum the statistical properties of the beam are characterized by an invariant $\varrho_{k\lambda}$ defined by (2.7).

The degree of coherence for the transverse (in general non-monochromatic) field associated with the $\boldsymbol{k}$-direction is defined by

$$\mu_{lm}(\boldsymbol{r}, t) = \langle E_l^{(-)}(x_1) E_m^{(+)}(x_2)\rangle / [\mathscr{E}_{ll}(0)\mathscr{E}_{mm}(0)]^{\frac{1}{2}}. \tag{4.1}$$

This quantity is closely related to $\gamma_{ij}(x)$ defined in (2.16). The different notation will serve to emphasize hereafter our concern with a single pencil of radiation. To be precise, the only distinctions between (2.16) and (4.1) lie in the averaging. For (4.1) the relevant density operator is of course $\varrho_{k\lambda}$ given in (2.7), while $\varrho = \prod\limits_{k\lambda} \varrho_{k\lambda}$ is the appropriate density operator for the whole field. Fortunately it is not necessary to repeat all the details of earlier calculations, this time using the density operator (2.7) rather than (2.6). We may simply write the results of the earlier work, given in (2.11) and (2.15), as follows:

$$\mathscr{E}_{ij}(x) = \frac{1}{4\pi^2} \int \frac{(k^2\delta_{ij} - k_i k_j) e^{-ik\cdot x}}{e^{\alpha k\cdot V} - 1} \, k dk d\Omega_k, \tag{4.2}$$

$$\mathscr{M}_{ij}(x) = \frac{1}{4\pi^2} \int \frac{\varepsilon_{ijl} k_l e^{-ik\cdot x}}{e^{\alpha k\cdot V} - 1} \, k^2 dk d\Omega_k. \tag{4.3}$$

From the inspection of (4.2) and (4.3) one can see that the integrands

$$\tilde{\mathscr{E}}_{ij}(x) = \frac{1}{4\pi^2} \int \frac{(k^2\delta_{ij} - k_i k_j)}{e^{\alpha k\cdot V} - 1} \, e^{-ik\cdot x} k dk, \tag{4.4}$$

$$\tilde{\mathscr{M}}_{ij}(x) = \frac{1}{4\pi^2} \int \frac{\varepsilon_{ijl} k_l e^{-ik\cdot x}}{e^{\alpha k\cdot V} - 1} \, k^2 dk, \tag{4.5}$$

characterize the coherence properties of the field in a plane wave associated with the $\boldsymbol{k}$-direction.

For the sake of simplicity we consider the frame of reference in which the z-axis coincides with the $\boldsymbol{k}$-direction. From (4.4) it follows that $\tilde{\mathscr{E}}_{zz}(\boldsymbol{r}, \tau) = 0$ and $\tilde{\mathscr{E}}_{xy}(\boldsymbol{r}, \tau) = 0$. The first equality simply expresses the fact that the field is transverse with the respect to the $\boldsymbol{k}$-direction. For $\boldsymbol{r} = 0$ and $\tau = 0$ the second equality together with $\tilde{\mathscr{E}}_{xx}(0,0) = \tilde{\mathscr{E}}_{yy}(0,0)$

shows that the field associated with each $\boldsymbol{k}$-direction is completely unpolarized in any Lorentz frame. Moreover, from (4.5) $\tilde{\mathscr{M}}_{xz}(\boldsymbol{r}, \tau)$, $\tilde{\mathscr{M}}_{yz}(\boldsymbol{r}, \tau)$ and the diagonal elements all vanish, and $\tilde{\mathscr{M}}_{xy}(\boldsymbol{r}, \tau) = \tilde{\mathscr{E}}_{xx}(\boldsymbol{r}, \tau) = \tilde{\mathscr{E}}_{yy}(\boldsymbol{r}, \tau)$. The last equalities are due to the plane wave properties $|\boldsymbol{E}| = |\boldsymbol{H}|$ and $\boldsymbol{E}$ is perpendicular to $\boldsymbol{H}$.

If we specialize (4.1) to the x and y directions and use the fact that $\tilde{\mathscr{E}}_{xx} = \tilde{\mathscr{E}}_{yy}$, we obtain $\mu_{xx} = \mu_{yy} = \mu$. On introducing the four vector n_β defined by $k_\beta = n_\beta |\boldsymbol{k}|$ we may carry out the integrations in (4.4) and find:

$$\mu(\boldsymbol{r}, \tau) = \frac{\zeta\left(4, 1 + \dfrac{in \cdot x}{\alpha n \cdot V}\right)}{\zeta(4, 1)}, \tag{4.6}$$

where ζ denotes the Riemann zeta function. Since $n \cdot x$ and $n \cdot V$ are Lorentz invariants, we see from (4.6) that the value of μ depends only on $n \cdot x$ in the rest frame and on the rest-frame parameter α, which is specified by the temperature T_0 defined in the rest frame.

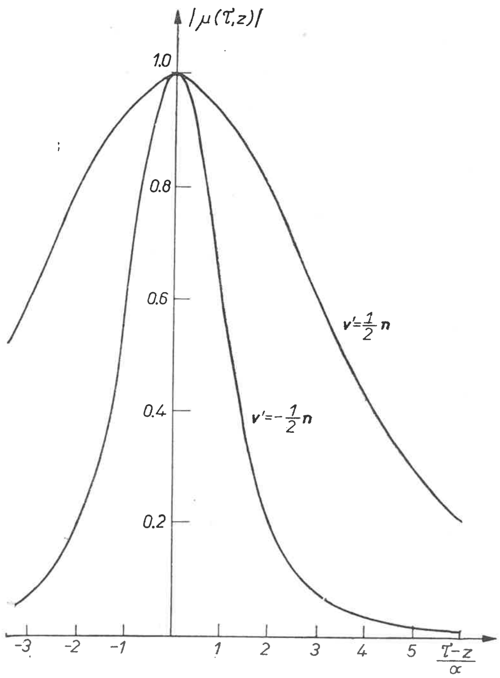

Fig. 4

The curves illustrating the dependence of $|\mu|$ on $n \cdot x = \tau - z$ for some values of v are shown in Fig. 4. As a matter of fact the family of curves could be obtained by variation of the parameter α which governs the curve illustrating temporal coherence of blackbody radiation given by Kano and Wolf 1962 (see also Mandel and Wolf 1965). Due to the fact that μ depends on $\boldsymbol{r}$ and τ through $\tau - z$, for $z = \tau$ we have $\mu = 1$. This means that, for points lying on the light cone, the components of the field are completely correlated.

We may mention that the pencil of radiation being considered has "polarization purity". This condition, analogous to that of spectral purity (the concept introduced and discussed by Mandel 1961; see also Mandel and Wolf 1961) is $\mu_{xy}(\boldsymbol{r}, \tau) = \mu_{xy}(0,0)\, \mu_{xx}(\boldsymbol{r}, \tau)$, where $\mu_{xy}(\boldsymbol{r}, \tau) = \tilde{\mathscr{E}}_{xy}(\boldsymbol{r}, \tau)/\tilde{\mathscr{E}}_{xx}(0,0)$. It is satisfied here because $\mu_{xx}(\boldsymbol{r}, \tau) = \mu_{yy}(\boldsymbol{r}, \tau)$. Of course, it means that the equality $\mu_{xx} = \mu_{yy}$ is independent of the x and y axis.

From (4.6) it is also seen that for $\boldsymbol{r}$ in the plane perpendicular to the direction of the wave, there is no dependence on the spatial variables. This is understandable since for the considered radiation field the state of the field is the same at each point of the plane perpendicular to the direction of the wave.

For $\boldsymbol{r} = 0$ and $\tau = 0$ $\tilde{\mathscr{M}}_{xy}$ (0,0) represents the z-component of the Poynting vector. Since the degree of coherence defined by $m_{xy}(\boldsymbol{r}, \tau) = \tilde{\mathscr{M}}_{xy}(\boldsymbol{r}, \tau)/[\tilde{\mathscr{E}}_{xy}\tilde{\mathscr{H}}_{xy}]^{1/2}$ is equal to μ from Fig. 4 we see that the average flux of energy is different from zero in any inertial, frame.

The Fourier transform of μ given by (4.6) yields the normalized space-dependent spectral coefficient

$$\Phi(\boldsymbol{r}, \omega) = \frac{\omega^3(n \cdot V)^4 \alpha^4 \exp(i\omega z)}{\Gamma(4)\zeta(4, 1)\,[\exp(\alpha n \cdot V) - 1]} \tag{4.7}$$

which for $\boldsymbol{r} = 0$ is the normalized Planck curve. From (4.7) it follows that for each direction $\boldsymbol{n}$ the spectral distribution is given by Planck's function, with temperature defined by $T = T_0/\gamma(1 - \boldsymbol{v} \cdot \boldsymbol{n})$, the result obtained a long time ago by Mosengeil and Planck (see Pauli 1958) by different considerations.

We have already pointed out in the previous Section that it is a very difficult matter to give the dependence of the coherence time (or the coherence length) and the spectral width on $\boldsymbol{v}$ in a closed mathematical form for the entire blackbody field. However, in the case of a plane wave, for which the coherence properties are given by (4.6), one can calculate the coherence time τ_c and the spectral width σ_ω in a way similar to that used by Mehta (1963). Let us recall that the coherence time τ_c is defined (see Born and Wolf 1966, Mandel and Wolf 1965) by

$$^{(1)}\tau_c^2 = \int_{-\infty}^{+\infty} \tau^2 |\mu(\tau)|^2 d\tau \Big/ \int_{-\infty}^{+\infty} |\mu(\tau)|^2 d\tau, \tag{4.8}$$

or by

$$^{(2)}\tau_c = \int_{-\infty}^{+\infty} |\mu(\tau)|^2 d\tau. \tag{4.9}$$

Making use of the definitions above and the fact that the spectral width is the reciprocal of the coherence time one obtains

$$\tau_c = \tau_c^{(0)}\gamma(1 - v \cos \psi), \tag{4.10}$$

$$\sigma_\omega = \sigma_\omega^{(0)}\gamma^{-1}(1 - v \cos \psi)^{-1}, \tag{4.11}$$

where ψ is the angle between $\boldsymbol{v}$ and $\boldsymbol{n}$, and where $\tau_c^{(0)}$ and $\sigma_\omega^{(0)}$ are defined in the rest frame, *i.e.*, in the frame in which the field is isotropic. Of course, (4.11) may be derived from the relativistic Doppler theory too. For $\psi = 0$ or $\psi = \pi$ we obtain $\tau_c = \tau_c^{(0)}(1-v)/(1+v)$ or $\tau_c = \tau_c^{(0)}(1+v)/(1-v)$ which show that the coherence time may get either shorter or longer when the value of v increases. Let us recall that $\boldsymbol{v}$ denotes the velocity of the moving system, *i.e.*, in the case of blackbody radiation the velocity of the frame of reference in which the radiation is isotropic. It is, however, convenient to introduce the velocity $\boldsymbol{v}'$ of the moving observer with respect to the rest frame associated with the isotropic radiation. Since $\boldsymbol{v} = -\boldsymbol{v}'$ holds, all formulas may be expressed in terms of $\boldsymbol{v}'$. The case $\boldsymbol{v}' = \frac{1}{2}\boldsymbol{n}$ corresponds to an observer travelling at velocity $v' = \frac{1}{2}$ in the same direction as the plane beam. For $\boldsymbol{v}' = -\frac{1}{2}\boldsymbol{n}$ an observer is travelling opposite to the beam direction. Fig. 4 illustrates both situations. In the case of the blackbody radiation tensors for the entire field it is very difficult to give closed formulas expressing the dependence of τ_c on v. The definitions given by (4.8) and (4.9) can be slightly generalized in order to characterize the coherence time for values of z different from zero. In the case of ${}^{(2)}\tau_c$ it easily follows that ${}^{(2)}\tau_c$ does not depend on z.

The derivation of the invariant form of μ given by (4.6) refers to blackbody radiation. One can raise the question whether this invariance property is characteristic for blackbody radiation or is a specific feature of a pencil of electromagnetic radiation.

Consider an arbitrary plane wave which, for simplicity, we shall assume to be statistically time-stationary and space-homogeneous. We do not assume anything about the polarization states in this plane. Let the l-axis be perpendicular to the z-axis which, as previously, is the direction of propagation. We want to investigate the second order coherence properties of the l-component of the $\boldsymbol{E}$ field which is labelled by E_l. One can show (Kujawski 1969) that the degree of coherence μ_{ll} defined by (4.1) is relativistically invariant. That is,

$$\mu_{ll}(x_1, x_2) = \mu'_{l'l'}(x'_1, x'_2), \tag{4.12}$$

where $\mu'_{l'l'}$ is defined like (4.1) in a new Lorentz frame. Of course, the plane determined by z' and l' is the transformed plane corresponding to the plane determined by z and l. This relativistic invariance is a consequence of the fact that for plane waves $|\boldsymbol{H}| = |\boldsymbol{E}|$ and $\boldsymbol{E}$ and $\boldsymbol{H}$ are perpendicular.

One can also show that, for normalized coefficients characterizing coherence properties of higher order, at least for the definitions given by Glauber, Mehta and Sudarshan (see Klauder and Sudarshan 1968), the invariance relations of the type (4.12) are valid. In this way we may say that the coherence "structure" of a plane wave described by normalized coherence coefficients (in general they depend on the parameter $\boldsymbol{l}$) of all orders is invariant. Here we discuss the second order coherence degree only. If in this case the plane beam is "pure" with respect to polarization states (Mandel and Wolf 1961), for each $\boldsymbol{l}$ μ_{ll} is the same, and of course μ_{ll} does not depend on $\boldsymbol{l}$. From (4.12) it follows immediately that the plane wave field which is pure with respect to polarization states in some Lorentz frame is pure in any other frame.

Finally, let us pay attention to two points. First, the electric field of an arbitrary plane wave depends on time and space through the four vector product $n \cdot x_1 = t_1 - z_1$, and in the case of time stationary and space homogeneous statistical fields the two-point second order coherence tensors depend on the difference $n \cdot x = (t_1 - t_2) - (z_1 - z_2)$. Second, though we derived (4.10) and (4.11) for a plane beam possessing a blackbody spectrum they are valid for any stationary and homogeneous electromagnetic plane wave with arbitrary spectrum.

5. *Remarks on possible applications*

The discovery that the universe is apparently permeated with blackbody radiation at a temperature of about 3 K (for references and discussion see Dantcourt and Wallis 1968, Partridge 1969) has interesting astrophysical implications. It has led to a number of difficult and ingenious experimental efforts. From the point of view of relativistic coherence theory the most striking possibility is that a measurement of the earth's velocity relative to the rest system of the radiation field might be contemplated.

We have seen that the reference frame in which the radiation field appears isotropic is the zero-average-momentum frame (or, simply, the rest frame). It is difficult to imagine that our galaxy is by chance in this frame. Our relative velocity with respect to this frame could, in principle, be very large. Even if this relative velocity is not large, two certain sources of apparent anisotropy are the revolution of the earth about the sun, and the motion of the solar system in the galaxy. A common estimate for the relative velocity of the earth with respect to the center of the galaxy is 300 km/sec.

In this section we will explore briefly the ways in which the coherence theory we have developed for blackbody radiation in an arbitrary Lorentz frame may be applied to the problem of the earth's velocity. Because most sensitive radiation detectors are highly directional, we will naturally concentrate on applications of the relations derived in Section 4 for a pencil of radiation.

The basic quantity in discussing a pencil of radiation is the normalized space-dependent spectral distribution $\Phi(\boldsymbol{r}, \omega)$ introduced in Section 4 in (4.7). It is clear from the structure of Φ that v-dependence is intimately related to ψ-dependence. Clearly, if an observer knows the direction of $\boldsymbol{v}$ but not its magnitude, a measurement of Φ at any two angles ψ will be sufficient to reveal the magnitude of $\boldsymbol{v}$.

Of course not only the spectral properties characterize the interrelation of velocity dependence and anisotropy. In principle, each of the correlation tensors discussed in detail previously (Eberly and Kujawski 1967a, 1968) may be used as a measure of velocity or anisotropy. But the degree of coherence, given in (4.6), is especially simple to use for this purpose. One measure of anisotropy is the difference $|\mu(n \cdot x;\, \psi - \pi)| - |\mu(n \cdot x;\, \psi)|$ which vanishes for $\psi = \pi/2$, and takes its maximum value when $\psi = \pi$. The ratio $|\mu(n \cdot x;\, \psi - \pi)| / |\mu(n \cdot x;\, \psi)|$ could also be used.

In a similar way, using (4.10) we see that $\tau_c(\pi - \psi) - \tau_c(\psi) \cong 2v \cos \psi$. Thus the coherence time itself may be used as device for measuring velocity and anisotropy. The same may be said for the spectral width, of course, since it is merely the reciprocal of the coherence time, apart from unimportant numerical factors which change as the precise definition of coherence time changes (Mehta 1963, and references therein).

If we assume that $\boldsymbol{v}$ has a magnitude of about 300 km/sec for the sake of illustration, we can easily estimate a maximum coherence time difference of about 2×10^{-3} times the rest system coherence time. At the peak of the 3K blackbody curve such time differences lie in the range of 10^{-10} sec., well beyond experimental capabilities. Thus the earth's relative velocity with respect to the blackbody rest system must be more than an order of magnitude greater than our assumption if these correlation techniques are to be useful.

We may mention in closing this Section the existence of a more fundamental measure of anisotropy. The inequivalence of directions in space has as one consequence the failure of the density operator to commute with the total angular momentum operator (the generator of rotations). Thus one obtains a measure of anisotropy by estimating the amount by which this commutator differs from zero. Such a measure of anisotropy is closely related to the measure of time stationary proposed by one of us (see Eberly and Singh 1970, and also Eberly and Kujawski 1968), and will not be pursued here.

Recently a lot of attention has been paid to relativistic thermodynamics (for references see Eberly and Kujawski 1967, Landsberg and Johns 1967, Balescu 1968, Yuen 1970). Our blackbody radiation considerations shed light on the problem of the ambiguous transformation formula for temperature, considered as a parameter characterizing blackbody radiation field. From all the formulas discussed in this paper one easily sees that the parameter which appears in them is the temperature four-vector $(1/T)_\mu = V_\mu/T_0$. It simply shows that the considered system of blackbody radiation is characterized by this four-vector, of which the invariant length is $1/T_0$.

If one calls the temperature of the moving system the coefficient associated with the energy, one obtains $T = T_0/\gamma$ (Eberly 1967). On the other hand, since the only thermodynamical parameter which enters into all the formulas is the temperature T_0, defined in the rest system, one can claim there is no need to introduce the transformed temperature T (Kujawski 1969a). This interpretation corresponds to the point of view that an observer who makes measurements of temperature on a moving system is forced to interpret his measurements in terms of the rest temperature of the system (Anderson 1964). To make this point of view even stronger let us pay attention to the fact that in the rest frame Eq. (4.6) takes the form

$$\mu(\boldsymbol{r}_0, \tau) = \frac{\zeta\left(4, 1 + \dfrac{i(\tau_0 - z_0)}{\alpha}\right)}{\zeta(4, 1)}. \tag{5.1}$$

This form can always be associated with (4.6) in any frame, simply by assigning the temperature T to the beam by the relation

$$T = T_0/n \cdot V. \tag{5.2}$$

We may see this point directly by writing the formula (4.6) as follows

$$\mu(\boldsymbol{r}, \tau) = \frac{\zeta\left(4, 1 + \dfrac{i(\tau - z)}{\alpha'}\right)}{\zeta(4, 1)}, \quad \alpha' = 1/KT. \tag{5.3}$$

Clearly the form is the same as that in (5.1), with the temperature defined according to (5.2).

The formula (5.2) $T = T_0/\gamma(1-v\cos\psi)$ has been previously derived in a different way and discussed in many papers in connection with the cosmic blackbody radiation (Heer and Kohl 1968, Henry and others 1968, Peebles and Wilkinson 1968). Of course if one wants to compute T_0 for the entire blackbody radiation field, in general it is necessary to measure T in three independent directions. Finally, let us point out that the interpretation leading from (4.6) to (5.3) is possible since μ is the parameter which characterizes the plane wave and, as it is known, one cannot associate in a unique way a rest frame with a plane wave (Hamity 1969).

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