SMALL ANGLE SCATTERING AND THE KAPITZA LAW

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It is shown that the condition for linear growth of the magnetoresistance of polycrystalline noble metals due to extended orbits becomes easier to satisfy owing to small angle scattering.

The linear growth of the magnetoresistance of polycrystalline noble metals at high magnetic fields (Kapitza law, Kapitza 1929, Lüthi 1960) is one of the most striking phenomena observed in galvanomagnetic measurements in metals. Thus its explanation is one of the principal aims of the theory of galvanomagnetic processes.

It is commonly believed that this effect is due to extended electronic orbits arising from cylinderlike sheets of the Fermi surface (Ziman 1958, Lifshits, Peschansky 1958, 1960, Alekseyevsky, Gaidukov 1959, Lüthi 1960). This result, however, was obtained under the assumption that the effective conductivity tensor of the polycrystalline sample can be obtained by averaging the local conductivity tensor over all crystallites. But, as has been shown in earlier papers of the author (1967), (1970), this assumption is justified only when

$$c \gg \frac{\pi}{2} \frac{n-1}{n^{1/2}} \left(\frac{\sigma_{11}}{\sigma_{33}}\right)^{1/2} \tag{1}$$

where c in the concentration of crystals with open orbits and

$$n = \frac{R(H)}{R_0} . \tag{2}$$

$R(H)$ is the magnetoresistance at field H, R_0 is the resistance at zero field and σ_{11}, σ_{33} are components of the conductivity tensor in the absence of open orbits. Extended orbits may be considered as open if the mean free path of an electron on the orbit does not allow encircling the orbit without scattering. Thus, the concentration c decreases as $1/H$.

Condition (1) can hardly be satisfied in noble metals.

The theory of small angle scattering can be found in Pippard's article (1968).

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According to the usual theory (Ziman, Lifshits, Peschansky) extended orbits in noble metals appear because in the repeated zone scheme the Fermi surface can be thought of as consisting of cylinders pointed in the (111) directions. The longitudinal cross-section of such a cylinder is shown in Fig. 1, where we see the alternating belly and neck parts of the Fermi surface. If the direction of the magnetic field forms a small angle ϑ with the cylinder axis, extended electronic orbits appear as shown on the figure. Such orbits have a definite

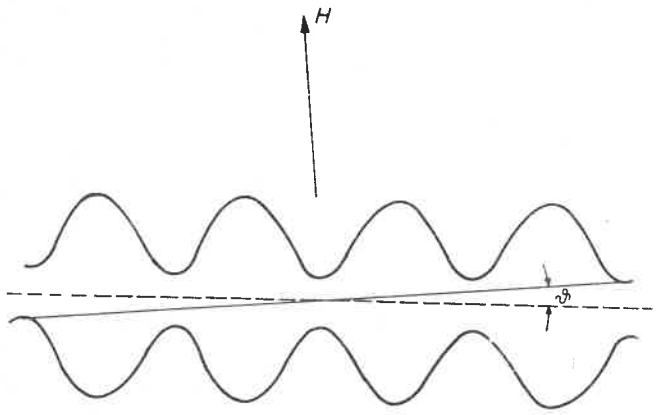

Fig. 1. Cross-section of a cylinderlike Fermi surface with indicated magnetic field direction, extended orbit plane and ϑ angle

influence on the conductivity tensor. However only electrons in a thin layer in k-space of thickness equal to the neck diameter move along such orbits. Applying Pippard's argument we conclude that small angle scattering will define the relaxation time for electronic motion along extended orbits.

We shall compute now, for a simplified model, the extended orbit contribution to the conductivity tensor. We apply the formulas

$$\sigma_{ik}^{\rightrightarrows} = -\frac{2e^2\tau}{\hbar^3}\int \varphi_{ik}(\varepsilon, p_z) f_0' d\boldsymbol{p}, \tag{3}$$

$$\varphi_{ik}(\varepsilon, p_z) = \gamma_0 \int_0^\infty e^{-\gamma_0 \xi} \langle v_i(\zeta) v_k(\zeta+\xi)\rangle \, d\xi. \tag{4}$$

(Lifshits *et al.* 1956), where integration is over momentum space. The $\boldsymbol{p}$ coordinates ε, p_z, ξ have the following meaning: ε denotes the electron kinetic energy, p_z — the momentum component in the direction of the magnetic field and ξ — the position on the orbit. f_0' denotes the derivative over ε of the distribution function. τ is the relaxation time and $\gamma_0 = \dfrac{H_0}{H}$, where H_0 is the value of the magnetic field for which the Larmor precession period T is equal to the relaxation time:

$$T = -\frac{2\pi cm}{eH} \tag{5}$$

where m — the electron mass, c — the velocity of light, e — the electric charge of the electron. The averaging in (4) is over one electronic orbit.

We shall consider for simplicity extended orbits as shown on Fig. 2: The necks are infinitely short and narrow and the electron velocity has a constant absolute value; between

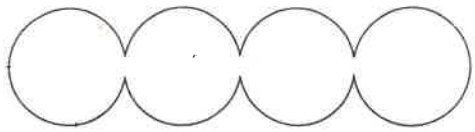

Fig. 2. An extended orbit

two necks the electron path is circular. We shall compute the σ_{yy} component of the conductivity tensor taking into account that the y component of the velocity

$$v_y(\xi) = \begin{cases} v|\sin\xi| & \text{for} \quad 0 < \xi < \pi N, \\ -v|\sin\xi| & \text{for} \quad \pi N < \xi < 2\pi N, \end{cases} \tag{6}$$

where N is the number of circles of which the extended orbit considered consists, v is constant.

After straightforward calculations we obtain the following magnetic field dependence of the extended orbit contribution to the conductivity at high magnetic fields:

$$\varphi_{yy} \propto 1 - \frac{\operatorname{th}\dfrac{N\pi\gamma_0}{2}}{\dfrac{N\pi\gamma_0}{2}}. \tag{7}$$

An extended orbit can be considered as open only if the right-hand side of (7) is of the order of 1. For simplicity we apply such units of the magnetic field that

$$\frac{1}{\gamma_0} = H\tau \tag{8}$$

where for the case of extended orbits $\tau = \tau_c$ (τ_c — relaxation time due to small angle scattering). Formula (7) corresponds to open orbits for

$$\frac{N\pi\gamma_0}{2} \geqslant 1 \tag{9}$$

and this gives for the lowest value of N at a given H

$$N_{\min} = \frac{1}{\pi} 2H\tau_c. \tag{10}$$

The above formula gives the largest angle ϑ_m (*cf.* Fig. 1) between the magnetic field and the cylinder axis, giving still open orbits:

$$\vartheta_m = \frac{1}{N_{\min}} \frac{\Delta k}{k_0} \tag{11}$$

where Δk is the neck diameter and k_0 the distance between the opposite hexagonal faces of the Brillouin zone.

In a polycrystal all directions of the crystal axes with regard to the magnetic field are, of course, equally represented. Taking into account that for noble metals cylinder axes are pointed in 4 directions we obtain the concentration c of crystals with open orbits of the cylinder type from the formula

$$c = \frac{1}{4\pi} \cdot 4\pi\vartheta_m \cdot 4 = \frac{2\pi}{H\tau_c} \frac{\Delta k}{k_0}. \tag{12}$$

As is well known from the theory of galvanomagnetic processes, the asymptotic formula for $\frac{\sigma_{11}}{\sigma_{33}}$ is

$$\frac{\sigma_{11}}{\sigma_{33}} \sim \frac{1}{H^2\tau_r^2}, \tag{13}$$

at least in the ideal case, where τ_r is the relaxation time defined from zero field resistivity. We believe that

$$\tau_r \gg \tau_c. \tag{14}$$

Inserting (12) and (13) into (1) we obtain

$$\frac{n-1}{n^{1/2}} \ll 4 \frac{\tau_r}{\tau_c} \frac{\Delta k}{k_0} \tag{15}$$

as the condition for an n-fold linear growth of the magnetoresistance caused by extended orbits.

Therefore, owing to (14), small angle scattering makes the condition (1) for linear growth of the magnetoresistance easier to fulfill.

Note that $\frac{\Delta k}{k_0}$ is equal 0.180 for copper, 0.123 for silver and 0.159 for gold (Schoenberg, Roaf 1962). The magnetoresistance of polycrystalline wires increases linearly up to ten times (silver) in Lüthi's experiments.

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