DISTRIBUTION OF MAGNETIC FIELD AND CURRENT DENSITY IN CORELESS ELECTROMAGNETS OF THE BITTER TYPE

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The distribution of current density in the disks of the winding of a Bitter type electromagnet is determined on the basis of magnetic field distribution measurements. The results show that the current density at the outer circumference of the coil is higher than that assumed in the design of the electromagnet.

1. Introduction

Coreless electromagnets of the Bitter type are at present the only devices generating constant magnetic fields of strengths greater than 150 kOe. If such electromagnets are water-cooled, they have a power input of the order of several megawatts. This causes large releases of heat at the windings to appear together with mechanical stresses, often surpassing the endurance of copper.

An important problem when designing electromagnets is the choice of the material for the windings and proper cooling holes. By selecting a material of better mechanical endurance than for pure copper, we enhance at the same time the resistivity of the winding (ϱ), what goes together with a weakening of the magnetic field at an identical power, in accord with the relation given by Fabry [1]

$$H_0 = G\left(\frac{W\lambda}{a_1\varrho}\right)^{1/2} \tag{1}$$

where G is the geometry factor, W is power, λ is the fractional volume of conductor, a_1 is the inner radius of the coil, ϱ is the resistivity of the coil's material, and H_0 is the strength of the magnetic field at the coil center.

An improper design of cooling holes decreases the geometry factor G, which depends among other things on the current density distribution. This leads to the necessity of applying higher powers and to local overheating of the winding. The heat release and mechanical stresses appearing in the winding depend on the shape of the coil, the magnitude and distribution of magnetic field H and the current density j. If the distribution $H(r, z)$ is known

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it is possible to determine the distribution $j(r)$ from the relation $\vec{j} = \text{rot}\, \vec{H}$. Heretofore, field distributions in solenoids have been calculated on the basis of an assumed current density distribution [2]. Montgomery developed simplified methods of determining $H(r, z)$ which correspond to a constant distribution of current density along any radius [3].

The purpose of this work was to determine the magnetic field distribution, and thereby the current density, in a Bitter type coil of the specifications: $a_1 = 2.5$ cm, $\alpha = 4.68$, and $\beta = 3.68$, where α is the ratio of outer radius a_2 to inner radius a_1, and β is the ratio of half the coil length b to the inner radius a_1. The cooling holes of constant diameter $d_0 = 2.5$ mm were arranged along radii so that $W_s(r) = \text{constant}$, assuming that in copper $j(r) = c/r$ and $\lambda(a_1) = 0.724$. Here, $W_s(r)$ is the heat flux in watts per cm^2, c is a constant value and $\lambda(a_1)$ is the radial space factor due to cooling hole distribution at $r = a_1$ calculated from the geometrical relation given by Montgomery *et al.* [4],

$$\lambda(a_1) = 1 - \frac{n^2 d_0^2 \cdot \sqrt{3}}{8\pi a_1} \tag{2}$$

n is the number of cooling holes on individual circles of the disk.

Advantage was taken of the cooling holes in measurements of magnetic field strength. Owing to the necessity of having free access to the coil and the impossibility of full cooling of the electromagnet during measurements, the latter were performed at a low load ($W = 4$kW and $H = 4.6$ kOe).

2. *Method of measurement*

The measurement was performed by the search coil method. The dimensions of the search coils were dictated by the diameter of the cooling holes and the sensitivity of the measuring instruments. For measuring both magnetic field components use was made of small coils wound from copper wire of 30 μm diameter. The coil for measuring the axial component H_z had $N = 550$ turns in the shape of a cylinder of outer diameter $d = 1.7$ mm and length $l = 5$ mm. For measuring the radial component H_r the coil, with $N = 250$ turns, was in the form of a flat ellipse of the axes $2a = 1.7$ mm and $2b = 5$ mm. These coil dimensions ensured the following sensitivity of measurement with a ballistic galvanometer: 6 Oe per scale division in the case of H_z, and 7 Oe per division in the case of H_r measurements.

Measurements were taken every 1 cm along the axis in each cooling hole of one row along the radius r. When measuring the H_z component the coil was lifted out of each hole to the exterior of the magnet. When measuring the H_r component the flat coil was revolved by 180°, what eliminated the influence of the H_z component.

3. *Results*

Fig. 1 shows the $H_z(r, 0)$ distribution determined in the plane at half the winding $z = 0$, and the distribution of the $H_z(r, 0)$ component calculated from the relation

$$H_z(r, 0) = H_z(a_1, 0) - \int j(r) dr + \int \frac{\partial H_r}{\partial z}(r, 0) dr \tag{3}$$

because, in accord with results of measurements, $\frac{\partial H_r}{\partial z}(r, 0) = A \simeq$ constant. Solving Eq. (3), we get

$$H_z(r, 0) = H_z(a_1, 0) + A(r - a_1) - [\Delta H + A(a_2 - a_1)] \cdot F(r) \tag{4}$$

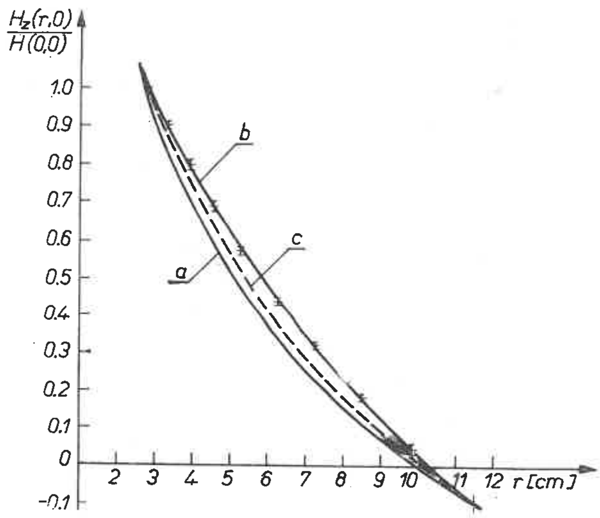

Fig. 1. Distribution of axial component of magnetic field, $H_z(r, 0)$, in winding plane $z = 0$. I — results of measurements, a — distribution of field component determined for $j(r) = c/r$, b — distribution of field component determined for $j(r) = c/r^{0.6}$, c — distribution of field component determined for $j(r) = \frac{c}{r}\lambda(r)$

where
$\Delta H = H_z(a_1, 0) - H_z(a_2, 0)$. The function $F(r)$ depends on the current density distribution:

$$F(r) = \ln\frac{r}{a_1}(\ln\alpha)^{-1} \text{ for } j(r) = \frac{c}{r}, \qquad F(r) = (r^{1-m} - a_1^{1-m})(a_2^{1-m} - a_1^{1-m})^{-1}$$

$$\text{for } j(r)\ \frac{c}{r^m},\ F(r) = \left[\ln\frac{r}{a_1} - \frac{B}{2}\left(\frac{1}{a_1^2} - \frac{1}{r^2}\right)\right]\left[\ln\alpha - \frac{B}{2}\left(\frac{1}{a_1^2} - \frac{1}{a_2^2}\right)\right]^{-1}$$

for $j(r) = \frac{c_2}{r}\lambda(r)$ with $B = n^2 d_0^2 \sqrt{3}/8\pi$, $\lambda(r)$ being the radial space factor due to the hole distribution. By the method of successive approximations the value $m = 0.6$ was found.

Fig. 2 presents the shape of the distribution of current density, $j(r) = c/r$, $j(r) = c/r^{0.6}$ and $j(r) = \frac{c}{r}\lambda(r)$, brought down to the same values $\int_{a_1}^{a_2} j(r)\,dr =$ constant. The curve c/r represents the run of current density in the winding when there are no cooling holes. The curve $\frac{c}{r}\lambda(r)$ gives the run of the current density in a magnet when the cooling holes are

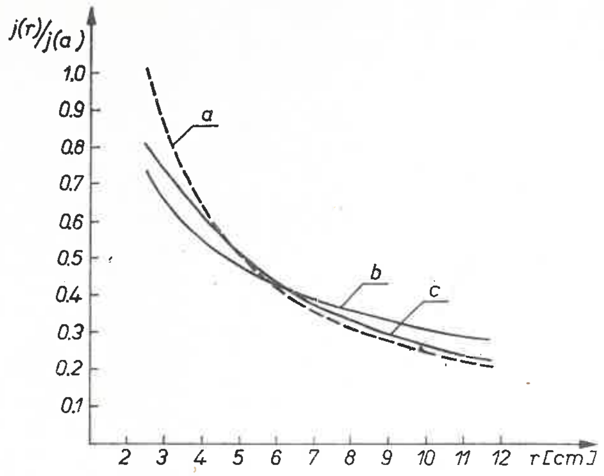

Fig. 2. Current density distribution determined from the relation: $a - j(r) = c/r$; $b - j(r) = c/r^{0.6}$; $c - j(r) = \frac{c}{r}\lambda(r)$

taken into account. The curve $c/r^{0.6}$ represents the distribution of current density in a magnet calculated on the basis of the measured magnetic field.

Figs 3 and 4 present the measured distributions of the H_z and H_r components in the axial plane of one quarter of the solenoid. In Fig. 5 we have the resultant field $\vec{H}$ at the

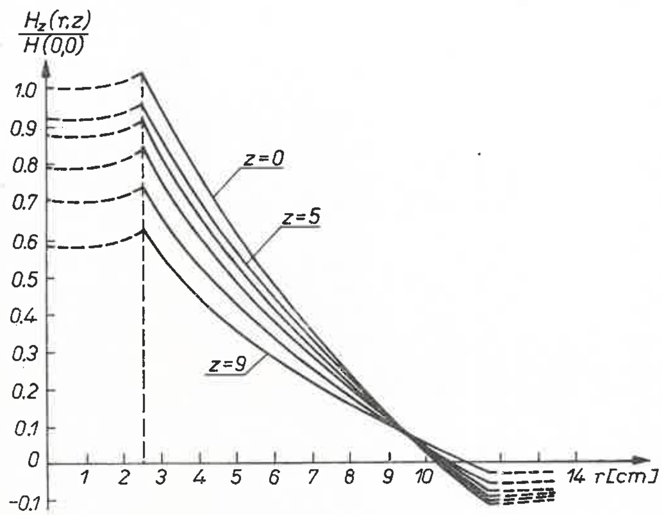

Fig. 3. Measured distribution of axial component of magnetic field, $H_z(r, z)$

measuring points; it is the sum of the $\vec{H}_z$ and $\vec{H}_r$ components of Figs 3 and 4. In Fig. 1 there is marked out the scattering of the measured results due to possible changes in the inclination of the search coil with respect to the solenoid axis.

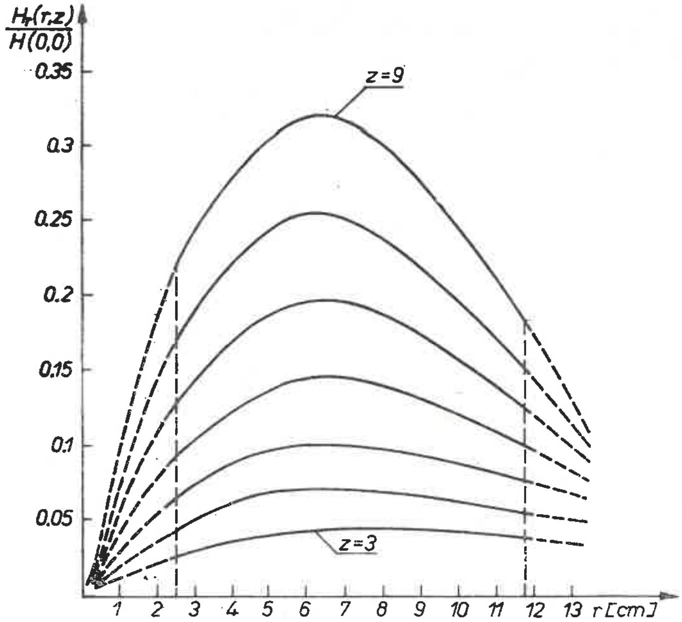

Fig. 4. Measured distribution of radial component of magnetic field, $H_r(r, z)$

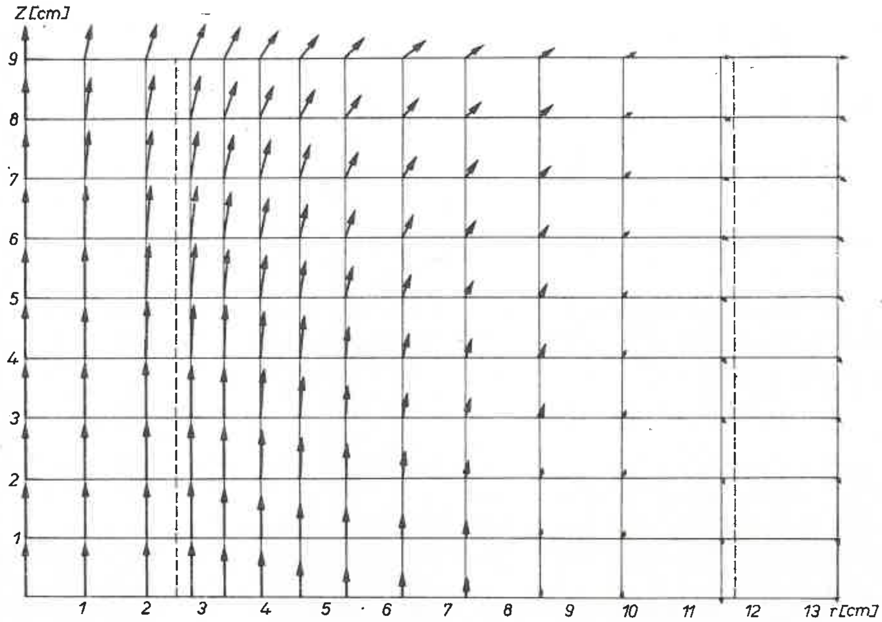

Fig. 5. Resultant magnetic field $\vec{H}$ as a sum of the $\vec{H}_z$ and $\vec{H}_r$ components

4. *Conclusions*

The magnetic field distribution determined from measurements differs from that calculated assuming a current density $j(r) = c/r$ in copper and mean distribution $j(r) = \frac{c}{r}\,\lambda(r)$ in the winding. The current density in copper at the outer circumference of the solenoid is higher by 21 per cent than the density assumed when calculating the layout of cooling holes.

As a consequence of this difference, there is a possibility of the coils becoming overheated at the outer circumference. Moreover, the change in current density distribution brings about a drop in the magnetic field accordingly with the relation

$$H_2 : H_1 = \int\limits_{a_1}^{a_2} dr \int\limits_0^b dz \frac{j_2(r)r^2}{(r^2+z^2)^{3/2}} : \int\limits_{a_1}^{a_2} dr \int\limits_0^b dz \frac{j_1(r)r^2}{(r^2+z^2)^{3/2}} \tag{5}$$

where H_2 is the magnetic field at the center of the solenoid for the distribution $j_2(r) = c_2/r^{0.6}$, and H_1 is the magnetic field at the solenoid center for $j_1(r) = c_1/r$. After solving the integrals of Eq. (5) and introducing instead of current density an identical power in both cases, we get

$$H_2 : H_1 = (ba_1)^{1/2} \cdot 1.25(a_2^{0.8} - a_1^{0.8}) \int\limits_{a_1}^{a_2} \frac{dr}{r^{0.6}(r^2+b^2)^{1/2}} :$$

$$: \ln\left[\alpha \frac{\beta+(1+\beta^2)^{1/2}}{\beta+(\alpha^2+\beta^2)^{1/2}}\right] \cdot (\beta \ln \alpha)^{-1/2}. \tag{6}$$

The drop in field calculated from these dependences comes to 5 per cent, what is in conformity with the result obtained from the approximate formula for the decrease in geometry coefficient G [4].

At full load of the electromagnet the difference between the assumed and real current density distributions may be diminished somewhat because of the temperature-increase of electric resistance at the outer circumference of the winding. None the less, when designing water-cooled electromagnets this difference should be taken into account. The obtained results enable the direction of the expected divergences to be determined when designing magnets and possibly a correction of the cooling hole layout may be introduced.

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