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MAGNETIC THIN FILM OF ARBITRARY MAGNETIC AND CRYSTALLOGRAPHIC STRUCTURE

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Spin wave theory is applied to a magnetic thin fiilm with non-translational crystallographic structure and arbitrary magnetic configuration. Equal amplitudes of the spin waves at equivalent lattice points belonging to different translational sublattices with a similar configurational environment are assumed.

1. *Introduction*

Whereas theoretical papers are mostly concerned with cubic magnetic thin films with simple surface orientation, experimental studies deal also with thin films of complicated crystallographical and magnetic structure.

In the present paper, we consider a thin magnetic film with non-translational crystallographic structure consisting of $\nu = 1, 2, ..., p$ translational sublattices.

The position vector of a magnetic ion is given by the formula

$$\vec{R}_\nu(\vec{n}) = \vec{R}_n + \vec{\varrho}_\nu, \tag{1}$$

where $\vec{\varrho}_\nu$ denotes a vector distinguishing the ν-th sublattice the components of which are not entire multiples of the basic vectors $\vec{\alpha}_i$, and

$$\vec{R}_n = \sum_{i=1}^{3} n_i \vec{\alpha}_i, \tag{2}$$

where n_i are integers, *i.e.* $n_i = 1, 2, ..., Ni$.

Our considerations are for the general case, i.e. we do not restrict ourselves to an orthogonal base of elementary vectors $\vec{\alpha}_i$ as done *e.g.* by Jelitto [1, 2].

The Valenta model [3, 4] is applied throughout. We regard the specimen as a set of N_3 parallel unlimited lattice planes denoted by indices $l = 0, 1, ..., N_3-1$. Each plane contains N_0 lattice sites, the sites from one layer l belonging either to the same translational

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sublattice ν or to different sublattices $\nu_1, \nu_2, \ldots, \nu_p$. The layers are labelled according to the formula

$$l_\nu = (n_3 + \varrho_3^\nu)m, \tag{3}$$

where m denotes the number of distinct types of layers in the considered film and ϱ_3^ν is a component of the distinctive vector of the ν-th sublattice in the $\vec{t}_3$ direction.

For specimens with translational symmetry, the latter has also to be preserved in the lattice plane, represented here by two translations $\vec{t}_1$ and $\vec{t}_2$ not parallel to one another. The set $\{\vec{t}_i\}$, where $i = 1, 2$, forms one of five planar Bravais lattices; the set of lattice planes forms a spatial Bravais lattice. The translation $\vec{t}_3$ should be chosen in such a way that the set $\{\vec{t}_i\}$ with $i = 1, 2, 3$ shall be a base of the spatial Bravais lattice. It is convenient to go over from the set of basic vectors $\{\vec{\alpha}_i\}$ connected with the crystallographical structure of the specimen to the base $\{\vec{t}_i\}$ privileged in considerations regarding thin films, *i.e.*

$$\vec{t}_i = \hat{c}_{ik}\vec{\alpha}_k, \tag{4}$$

where $\hat{c}_{ik}$ is a transformation matrix.

In many papers, the vector $\vec{t}_3$ is assumed as perpendicular to the layer, that is, to the surface of the film, even though the set $\{\vec{t}_i\}$ does not on the above assumption form a Bravais lattice.

For non-translational structures the position vector of a magnetic ion resolves into the following components:

$$\vec{R}_\nu(\vec{n}) = \sum_{i=1}^{3} (n_i + \varrho_i^\nu)\vec{t}_i = \sum_{i=1}^{3} n_i^\nu \vec{t}_i, \tag{5}$$

where n_i^ν can be integers or fractions,

or

$$\vec{R}_\nu(\vec{n}) = (\nu l \vec{j}) = \vec{j}_\nu + \vec{l}_\nu, \tag{6}$$

Above:

$\vec{j}_\nu = n_1^\nu \vec{t}_1 + n_2^\nu \vec{t}_2$ is the plane-positional vectors of a magnetic ion from the ν -th sublattice in the layer l, and $\vec{l}_\nu = n_3^\nu \vec{t}_3$; the projection of $\vec{l}_\nu$ on the direction perpendicular to the film surface is the number $l(l = 0, 1, \ldots, N_3 - 1)$ labelling the layer and given by formula (3).

2. *The Hamiltonian*

In accordance with Heisenberg's theory, the Hamiltonian of the system is of the form:

$$\mathscr{H} = -\frac{1}{2} \sum_{\nu l \vec{j}, \nu' l' \vec{j}'} J_{\nu\nu'}(l\vec{j}, l'\vec{j}')\hat{\vec{S}}_{\nu l \vec{j}} \cdot \hat{\vec{S}}_{\nu' l' \vec{j}'} + \sum_{\nu l \vec{j}} \mu_{\nu l} \vec{H}_{\nu l}^{\text{eff}} \cdot \hat{\vec{S}}_{\nu l \vec{j}}, \tag{7}$$

where

$J_{\nu\nu'}(l\vec{j}, l'\vec{j}')$ denotes the exchange integral,

$\hat{\vec{S}}_{\nu l \vec{j}}$ is the spin operator (in $\frac{1}{2}\hbar$ units) of an ion in the l-th layer and belonging to the ν-th sublattice, the position of the ion in the lattice plane being given by the vector $\vec{j}_\nu$, and

$\vec{H}^{\rm eff}_{\nu l}$ is the vector of the effective magnetic field acting on the spins of the ν-th sublattice situated in the l-th layer.

The effective field $\vec{H}^{\rm eff}_{\nu l}$ acting on an individual spin in the bulk of the specimen differs from the field acting on a spin at the surface of the film. In the surface inhomogeneity approximation [5, 6] one has:

$$\vec{H}^{\rm eff}_{\nu l} = \vec{H}^{\rm eff}_{\nu\,{\rm int}} + \vec{H}^{\rm eff}_{\nu l\,{\rm surf}}\delta_{ls}, \tag{8}$$

where $\vec{H}^{\rm eff}_{\nu\,{\rm int}}$ is thee ffective magnetic field acting on internal spins belonging to the ν-th sublattice and $\vec{H}^{\rm eff}_{\nu s\,{\rm surf}}$ is an additional field which acts in a manner to pin the surface spins of the ν-th sublattice, *i.e.* the spins belonging to the layers $s = 0$ (substrate surface) and N_3-1 (free surface layer).

We assume here the additional field $\vec{H}^{\rm eff}_{\nu s}$ to act on the spins of the ν-th sublattice belonging to the layers $s = 0, 1, \dots r-1$ and $N_3-r, \dots, N_3-1$, where r denotes the number of boundary layers in the system, *i.e.*

$$\vec{H}^{\rm eff}_{\nu l} = \vec{H}^{\rm eff}_{\nu\,{\rm int}} + \vec{H}^{\rm eff}_{\nu l}\delta_{ls}. \tag{9}$$

In order to diagonalize the Hamiltonian (7), we first transform it to bilinear form (16) by means of the Tyablikov transformation [7, 8]:

$$\hat{S}^i_{\nu l\vec{j}} = S^i_{\nu l\vec{j}}\left(1-2\,\frac{\hat{n}_{\nu l\vec{j}}}{S_{\nu l\vec{j}}}\right) + \hat{\tau}^i_{\nu l\vec{j}}, \tag{10}$$

where

$$\hat{\tau}^i_{\nu l\vec{j}} = \Omega^i_{\nu l\vec{j}} f(\hat{n}_{\nu l\vec{j}})\hat{a}_{\nu l\vec{j}} + \Omega^{i*}_{\nu l\vec{j}}\hat{a}^+_{\nu l\vec{j}} f(\hat{n}_{\nu l\vec{j}}),$$

$$\hat{n}_{\nu l\vec{j}} = \hat{a}^+_{\nu l\vec{j}}\hat{a}_{\nu l\vec{j}}.$$

The annihilation and creation operators of spin deviation, localized in the lattice point $(\nu l\vec{j})$, satisfy the Bose commutation relations

$$[\hat{a}_{\nu l\vec{j}}, \hat{a}^+_{\nu' l'\vec{j}'}] = \delta_{\nu\nu'}\delta_{ll'}\delta_{\vec{j}\vec{j}'},$$

$$[\hat{a}_{\nu l\vec{j}}, \hat{a}_{\nu' l'\vec{j}'}] = [\hat{a}^+_{\nu l\vec{j}}, \hat{a}^+_{\nu' l'\vec{j}'}] = 0. \tag{11}$$

In the approximation of magnetic quasi-saturation

$$f(\hat{n}_{\nu l\vec{j}}) = \left\{1-\frac{\hat{n}_{\nu l\vec{j}}}{S_{\nu l\vec{j}}}\right\}^{1/2} \approx 1. \tag{12}$$

By putting various values of $S^1_{\nu l\vec{j}}$, $S^2_{\nu l\vec{j}}$ and $S^3_{\nu l\vec{j}}$ into the formulae for the vectors $\Omega_{\nu l\vec{j}}$ given in [8], we can consider arbitrary magnetic configurations.

By the transformation (10), the Hamiltonian takes the form:

$$\hat{\mathscr{H}} = E_0 + \hat{\mathscr{H}}_1 + \hat{\mathscr{H}}_2 + \hat{\mathscr{H}}_3 + \hat{\mathscr{H}}_4 + \dots, \tag{13}$$

where

$$E_0 = -\frac{1}{2}\sum_{\nu l\vec{j},\nu' l'\vec{j}'} J_{\nu\nu'}(l\vec{j}, l'\vec{j}')\vec{S}_{\nu l\vec{j}}\cdot\vec{S}_{\nu' l'\vec{j}'} - \sum_{\nu l\vec{j}} \mu_{\nu l}\vec{H}^{\rm eff}_{\nu l}\cdot\vec{S}_{\nu l\vec{j}}, \tag{13a}$$

$$\hat{\mathscr{H}}_1 = -\frac{1}{2}\sum_{\nu l\vec{j},\nu' l'\vec{j}'} J_{\nu\nu'}(l\vec{j}, l\vec{j}')\,[\vec{S}_{\nu l\vec{j}}\cdot\hat{\vec{\tau}}_{\nu' l'\vec{j}'} + \vec{S}_{\nu' l'\vec{j}'}\cdot\hat{\vec{\tau}}_{\nu l\vec{j}}] - \sum_{\nu l\vec{j}} \mu_{\nu l}\vec{H}^{\rm eff}_{\nu l}\cdot\hat{\vec{\tau}}_{\nu l\vec{j}}, \tag{13b}$$

$$\hat{\mathscr{H}}_2 = \sum_{\nu l\vec{j},\nu' l'\vec{j}'} J_{\nu\nu'}(l\vec{j}, l'\vec{j}')\vec{S}_{\nu l\vec{j}}\cdot\vec{S}_{\nu' l'\vec{j}'}\left[\frac{\hat{n}_{\nu l\vec{j}}}{S_{\nu l\vec{j}}} + \frac{\hat{n}_{\nu' l'\vec{j}'}}{S_{\nu' l'\vec{j}'}}\right] -$$

$$-\frac{1}{2}\sum_{\nu l\vec{j},\nu' l'\vec{j}'} J_{\nu\nu'}(l\vec{j}, l'\vec{j}')\hat{\vec{\tau}}_{\nu l\vec{j}}\cdot\hat{\vec{\tau}}_{\nu' l'\vec{j}'} + 2\sum_{\nu l\vec{j}} \mu_{\nu l}\vec{H}^{\rm eff}_{\nu l}\cdot\vec{S}_{\nu l\vec{j}}\frac{\hat{n}_{\nu l\vec{j}}}{S_{\nu l\vec{j}}}, \tag{13c}$$

$$\hat{\mathscr{H}}_3 = \sum_{\nu l\vec{j},\nu' l'\vec{j}'} J_{\nu\nu'}(l\vec{j}, l'\vec{j}')\left[\vec{S}_{\nu l\vec{j}}\frac{\hat{n}_{\nu l\vec{j}}}{S_{\nu l\vec{j}}}\cdot\hat{\vec{\tau}}_{\nu' l'\vec{j}'} + \vec{S}_{\nu' l'\vec{j}'}\cdot\hat{\vec{\tau}}_{\nu l\vec{j}}\frac{\hat{n}_{\nu' l'\vec{j}'}}{S_{\nu' l'\vec{j}'}}\right], \tag{13d}$$

$$\hat{\mathscr{H}}_4 = -2\sum_{\nu l\vec{j},\nu' l'\vec{j}'} J_{\nu\nu'}(l\vec{j}, l'\vec{j}')\vec{S}_{\nu l\vec{j}}\cdot\vec{S}_{\nu' l'\vec{j}'}\frac{\hat{n}_{\nu l\vec{j}}\cdot\hat{n}_{\nu' l'\vec{j}'}}{S_{\nu l\vec{j}}\cdot S_{\nu' l'\vec{j}'}}. \tag{13e}$$

With respect to the condition of quasi-saturation,

$$\hat{\mathscr{H}}_3 \approx 0 \quad \text{and} \quad \hat{\mathscr{H}}_4 \approx 0, \tag{14}$$

and from the condition for minimum energy E_0,

$$\hat{\mathscr{H}}_1 \equiv 0. \tag{15}$$

We thus obtain the bilinear form of the Hamiltonian:

$$\hat{\mathscr{H}} \equiv E_0 + \hat{\mathscr{H}}_2, \tag{16}$$

that is:

$$\hat{\mathscr{H}} = E_0 + \sum_{\nu l\vec{j},\nu' l'\vec{j}'}[A_{\nu\nu'}(l\vec{j}, l'\vec{j}')\,\hat{a}^+_{\nu l\vec{j}}\hat{a}_{\nu' l'\vec{j}'} +$$

$$+\tfrac{1}{2}B_{\nu\nu'}(l\vec{j}, l'\vec{j}')\,\hat{a}^+_{\nu l\vec{j}}\hat{a}^+_{\nu' l'\vec{j}'} + \tfrac{1}{2}B^*_{\nu\nu'}(l\vec{j}, l'\vec{j}')\,\hat{a}_{\nu l\vec{j}}\hat{a}_{\nu' l'\vec{j}'}], \tag{17}$$

where

$$A_{\nu\nu'}(l\vec{j}, l'\vec{j}') = -J_{\nu\nu'}(l\vec{j}, l'\vec{j}')\vec{\Omega}^*_{\nu l\vec{j}}\cdot\vec{\Omega}_{\nu' l'\vec{j}'} + \left\{\sum_{\nu'' l''\vec{j}''} J_{\nu\nu''}(l\vec{j}, l''\vec{j}'')\left[\frac{\vec{S}_{\nu l\vec{j}}\cdot\vec{S}_{\nu'' l''\vec{j}''}}{S_{\nu l\vec{j}}} +\right.\right.$$

$$\left.\left.+\frac{\vec{S}_{\nu l\vec{j}}\cdot\vec{S}_{\nu'' l''\vec{j}''}}{S_{\nu'' l''\vec{j}''}}\right] + 2\mu_{\nu l}\vec{H}^{\rm eff}_{\nu l}\cdot\frac{\vec{S}_{\nu l\vec{j}}}{S_{\nu l\vec{j}}}\right\}\delta_{\nu\nu'}\delta_{ll'}\delta_{\vec{j}\vec{j}'}, \tag{17a}$$

$$B_{\nu\nu'}(l\vec{j}, l'\vec{j}') = -J_{\nu\nu'}(l\vec{j}, l'\vec{j}')\,\vec{\Omega}^*_{\nu l\vec{j}}\cdot\vec{\Omega}^*_{\nu' l'\vec{j}'}, \tag{17b}$$

$$B^*_{\nu\nu'}(l\vec{j}, l'\vec{j}') = -J_{\nu\nu'}(l\vec{j}, l'\vec{j}')\,\vec{\Omega}_{\nu l\vec{j}}\cdot\vec{\Omega}_{\nu' l'\vec{j}'}. \tag{17c}$$

3. *Diagonalization of the bilinear Hamiltonian*

In order to diagonalize the bilinear Hamiltonian (17), *i. e.* to obtain it in the form:

$$\hat{\mathscr{H}} = \text{const} + \sum_{\vec{\varkappa}\lambda} E(\vec{\varkappa}\lambda)\, \hat{b}^{+}_{\vec{\varkappa}\lambda} \hat{b}_{\vec{\varkappa}\lambda}, \tag{18}$$

where $\hat{b}^{+}_{\vec{\varkappa}\lambda}$ and $\hat{b}_{\vec{\varkappa}\lambda}$ are respectively creation and annihilation operators of a spin wave of energy $E(\vec{\varkappa}\lambda)$ and the wave vector

$$k = \sum_{i=1}^{2} \varkappa_i \vec{t}_i^{\,*} + \lambda \vec{t}_3^{\,*} = \vec{\varkappa} + \lambda \vec{t}_3^{\,*}, \tag{19}$$

with values $\varkappa_i$ calculated from the Born-Kármán periodic boundary conditions (provided the thin film is unlimited in directions parallel to the film surface) and values λ from the boundary conditions for thin films, we use the Tyablikov method [7] and resort to the results of Kowalewski's papers [9, 10] concerning bulk materials of arbitrary structure as well as those of Puszkarski's paper [11] on ferromagnetic films of translational structure $(p = 1)$.

Let us now transform the Hamiltonian (17) to the reciprocal lattice space by means of the modified Tyablikov-Bogolyubov transformation:

$$\begin{aligned} \hat{a}_{\nu l\vec{j}} &= \sum_{\vec{\varkappa}\lambda} \{u_{\nu l\vec{j}}(\vec{\varkappa}\lambda)\, \hat{b}_{\vec{\varkappa}\lambda} + v^{*}_{\nu l\vec{j}}(-\vec{\varkappa}\lambda)\, \hat{b}^{+}_{-\vec{\varkappa}\lambda}\}, \\ \hat{a}^{+}_{\nu l\vec{j}} &= \sum_{\vec{\varkappa}\lambda} \{u^{*}_{\nu l\vec{j}}(\vec{\varkappa}\lambda)\, \hat{b}^{+}_{\vec{\varkappa}\lambda} + v_{\nu l\vec{j}}(-\vec{\varkappa}\lambda)\, \hat{b}_{-\vec{\varkappa}\lambda}\}, \end{aligned} \tag{20}$$

which has to fulfil:

$$\begin{aligned} &\sum_{\vec{\varkappa}\lambda} \{u_{\nu l\vec{j}}(\vec{\varkappa}\lambda)\, u^{*}_{\nu' l'\vec{j}'}(\vec{\varkappa}\lambda) - v_{\nu' l'\vec{j}'}(\vec{\varkappa}\lambda)\, v^{*}_{\nu l\vec{j}}(\vec{\varkappa}\lambda)\} = \delta_{\nu\nu'}\delta_{ll'}\delta_{\vec{j}\vec{j}'}, \\ &\sum_{\vec{\varkappa}\lambda} \{v^{*}_{\nu l\vec{j}}(\vec{\varkappa}\lambda)\, u_{\nu' l'\vec{j}'}(\vec{\varkappa}\lambda) - v^{*}_{\nu' l'\vec{j}'}(\vec{\varkappa}\lambda)\, u_{\nu l\vec{j}}(\vec{\varkappa}\lambda)\} = 0, \end{aligned} \tag{21a}$$

and

$$\begin{aligned} &\sum_{\nu l\vec{j}} \{u_{\nu l\vec{j}}(\vec{\varkappa}'\lambda) u^{*}_{\nu l\vec{j}}(\vec{\varkappa}'\lambda') - v_{\nu l\vec{j}}(\vec{\varkappa}\lambda) v^{*}_{\nu l\vec{j}}(\vec{\varkappa}'\lambda')\} = \delta_{\vec{\varkappa}\vec{\varkappa}'}\delta_{\lambda\lambda'} \\ &\sum_{\nu l\vec{j}} \{u_{\nu l\vec{j}}(\vec{\varkappa}\lambda)\, v_{\nu l\vec{j}}(\vec{\varkappa}'\lambda') - v_{\nu l\vec{j}}(\vec{\varkappa}\lambda) u_{\nu l\vec{j}}(\vec{\varkappa}'\lambda')\} = 0. \end{aligned} \tag{21b}$$

The transformation (20) with the normalization conditions (21) reduces the bilinear Hamiltonian (17) to the diagonal form (18) provided the transformation functions $u_{\nu l\vec{j}}(\vec{\varkappa}\lambda)$ and $v_{\nu l\vec{j}}(\vec{\varkappa}\lambda)$ satisfy the following set of $2N$ equations (a set of two equations for each magnetic ion, denoted here by $(\nu l\vec{j})$):

$$\begin{aligned} E(\vec{\varkappa}\lambda)\, u_{\nu l\vec{j}}(\vec{\varkappa}\lambda) &= \sum_{\nu' l'\vec{j}'} \{A_{\nu\nu'}(l\vec{j}, l'\vec{j}') u_{\nu' l'\vec{j}'}(\vec{\varkappa}\lambda) + B_{\nu\nu'}(l\vec{j}, l'\vec{j}')\, v_{\nu' l'\vec{j}'}(\vec{\varkappa}\lambda)\} \\ -E(\vec{\varkappa}\lambda)\, v_{\nu l\vec{j}}(\vec{\varkappa}\lambda) &= \sum_{\nu' l'\vec{j}'} \{B^{*}_{\nu\nu'}(l\vec{j}, l'\vec{j}') u_{\nu' l'\vec{j}'}(\vec{\varkappa}\lambda) + A^{*}_{\nu\nu'}(l\vec{j}, l'\vec{j}') v_{\nu' l'\vec{j}'}(\vec{\varkappa}\lambda)\}. \end{aligned} \tag{22}$$

By translational symmetry in each layer and for each sublattice individually we can write the transformation functions as follows:

$$u_{\nu l\vec{j}}(\vec{\varkappa}\lambda) = (N_0^\nu)^{-\frac{1}{2}}\, e^{-i\vec{\varkappa}\cdot\vec{j}_\nu}\, u_\nu^l(\lambda)$$
$$v_{\nu l\vec{j}}(\vec{\varkappa}\lambda) = (N_0^\nu)^{-\frac{1}{2}}\, e^{-i\vec{\varkappa}\cdot\vec{j}_\nu}\, v_\nu^l(\lambda), \tag{23}$$

where N_0^ν is the number of sites from the ν-th sublattice in the l-th layer. The total number of sites in the film is equal to $N = \sum_\nu N_3^\nu N_0^\nu$. In the nearest-neighbours approximation, each magnetic ion $(\nu l\vec{j})$ interacts with ions $(\nu' l' \vec{j}')$ situated in layers

$$l' = l+g,$$

where $g = -r, -r+1, .., -1, 0, 1, 2, .., r$.

We denote by

$$\gamma_{\nu\nu'}^{l\,l+g}(\vec{\varkappa}) = \sum_{\vec{j}_{\nu'}}{}' e^{i\vec{\varkappa}\cdot(\vec{j}_\nu - \vec{j}_{\nu'})} \tag{24}$$

the structural factor, where Σ' is the sum over nearest neighbours of the ion $(\nu l\vec{j})$ lying in the layers $(l+g)$.

With respect to asymmetry of environment, the equations for boundary spins differ from those for internal spins.

For internal spin, they are:

$$\sum_{\nu'}\sum_{g=-r}^{r}{}' \{[A_{\nu\nu'}^{l\,l+g}(l\vec{j})\gamma_{\nu\nu'}^{l\,l+g}(\vec{\varkappa}) - E(\vec{\varkappa}\lambda)\delta_{\nu\nu'}\delta_{g0}]u_{\nu'}^{l+g}(\lambda) + B_{\nu\nu'}^{l\,l+g}(l\vec{j})\gamma_{\nu\nu'}^{l\,l+g}(\vec{\varkappa})v_{\nu'}^{l+g}(\lambda)\} = 0$$
$$\sum_{\nu'}\sum_{g=-r}^{r}{}' \{B_{\nu\nu'}^{l\,l+g*}(l\vec{j})\gamma_{\nu\nu'}^{l\,l+g}(\vec{\varkappa})u_{\nu'}^{l+g}(\lambda) + [A_{\nu\nu'}^{l\,l+g*}(l\vec{j})\gamma_{\nu\nu'}^{l\,l+g}(\vec{\varkappa}) + E(\vec{\varkappa}\lambda)\delta_{\nu\nu'}\delta_{g0}]v_{\nu'}^{l+g}(\lambda)\} = 0, \tag{25}$$

and for boundary spins:

$$\sum_{\nu'}\sum_{g}{}' \{[A_{\nu\nu'}^{s\,s+g}(s\vec{j})\gamma_{\nu\nu'}^{s\,s+g}(\vec{\varkappa}) - E(\vec{\varkappa}\lambda)\delta_{\nu\nu'}\delta_{g0}]u_{\nu'}^{s+g}(\lambda) + B_{\nu\nu'}^{s\,s+g}(s\vec{j})\gamma_{\nu\nu'}^{s\,s+g}(\vec{\varkappa})v_{\nu'}^{s+g}(\lambda)\} = 0$$
$$\sum_{\nu'}\sum_{g}{}' \{B_{\nu\nu'}^{s\,s+g*}(s\vec{j})\gamma_{\nu\nu'}^{s\,s+g}(\vec{\varkappa})u_{\nu'}^{s+g}(\lambda) + [A_{\nu\nu'}^{s\,s+g*}(s\vec{j})\gamma_{\nu\nu'}^{s\,s+g}(\vec{\varkappa}) + E(\vec{\varkappa}\lambda)\delta_{\nu\nu'}\delta_{g0}]v_{\nu'}^{s+g}(\lambda)\} = 0, \tag{26}$$

with summation in Eqs (26) extending over

$$g = 0, 1, \ldots, r \quad \text{for} \quad s = 0, 1, \ldots, r-1$$

and

$$g = -r, -r+1, \ldots, 0 \quad \text{for} \quad s = N_3-r, \ldots, N_3-1.$$

Because of the additional field pinning the spins situated in boundary layers, we have:

$$A_{\nu\nu'}^{ss}(s\vec{j}) = A_{\nu\nu'}^{ll}(l\vec{j}) - \mathscr{A}_{\nu\nu'}^{s}(s\vec{j}). \tag{27}$$

In order to reduce the set of difference equations (27) for boundary spins to the form of the set of equations for internal spins (26), Jelitto [1, 2] proposed to introduce $2r$ fictitious layers with $l = -r, -r+1, \dots, -1$ and $N_3, N_3+1, \dots, N_3-1+r$ in a manner to obtain

$$\sum_{\nu'} \sum_{g}{}' \{A^{s\,s+g}_{\nu\nu'}(s\vec{j})\gamma^{s\,s+g}_{\nu\nu'}(\vec{\varkappa})u^{s+g}_{\nu'}(\lambda) + B^{s\,s+g}_{\nu\nu'}(s\vec{j})\gamma^{s\,s+g}_{\nu\nu'}(\vec{\varkappa})v^{s+g}_{\nu'}(\lambda)\} = \sum_{\nu'} \mathscr{A}^{s}_{\nu\nu'}(s\vec{j})\gamma^{ss}_{\nu\nu'}(\vec{\varkappa})u^{s}_{\nu'}(\lambda)$$

$$\sum_{\nu'} \sum_{g}{}' \{B^{s\,s+g*}_{\nu\nu'}(s\vec{j})\gamma^{s\,s+g}_{\nu\nu'}(\vec{\varkappa})u^{s+g}_{\nu'}(\lambda) + A^{s\,s+g*}_{\nu\nu'}(s\vec{j})\gamma^{s\,s+g}_{\nu\nu'}(\vec{\varkappa})v^{s+g}_{\nu'}(\lambda)\} = \sum_{\nu'} \mathscr{A}^{s*}_{\nu\nu'}(s\vec{j})\gamma^{ss}_{\nu\nu'}(\vec{\varkappa})v^{s}_{\nu'}(\lambda). \quad (28)$$

where summation over g proceeds as follows:

$$g = -r, -r+1, \dots, -1 \quad \text{for} \quad s = 0, 1, \dots, r-1,$$

$$\text{and } g = 1, 2, \dots, r \quad \text{for} \quad s = N_3-r, N_3-r+1, \dots, N_3-1.$$

Eqs (30) form a set of $4r$ boundary equations. Accordingly, the problem resides in solving the set of homogeneous equations for the bulk material (25) with the boundary conditions (28). Owing to the introduction of fictitious layers, translational symmetry for each sublattice is achieved in the $\vec{t}_3$ direction also, and one can assume particular solutions in the form

$$u^l_\nu(\lambda) = \alpha_\nu e^{-i\lambda l}$$

$$v^l_\nu(\lambda) = \beta_\nu e^{-i\lambda l}. \quad (29)$$

The general solutions can be expressed as linear combinations of particular solutions:

$$u^l_\nu(\lambda) = \alpha_\nu \sum_{f=1}^{2r} a_f \exp\left[i\lambda_f \left(\frac{N_3-1}{2} - l\right)\right]$$

$$v^l_\nu(\lambda) = \beta_\nu \sum_{f=1}^{2r} b_f \exp\left[i\lambda_f \left(\frac{N_3-1}{2} - l\right)\right]. \quad (30)$$

In order to calculate the energy spectrum, we insert particular solutions (29) into the set of bulk material equations (25), obtaining N/p similar sets of $2p$ equations each, in unknown amplitudes α_ν and β_ν. In many structures one can assume equal spin wave amplitudes in equivalent sites belonging to different translational sublattices if their configurational environments are similar, *i. e.*

$$u^l_1(\lambda) = u^l_2(\lambda) = \dots u^l_{p_1}(\lambda);\ u^l_{p_1+1}(\lambda) = \dots u^l_{p_2}(\lambda);\ \dots;\ u^l_{p_{m-1}+1}(\lambda) = \dots u^l_{p_m}(\lambda)$$

$$v^l_1(\lambda) = v^l_2(\lambda) = \dots v^l_{p_1}(\lambda);\ v^l_{p_1+1}(\lambda) = \dots v^l_{p_2}(\lambda);\ \dots;\ v^l_{p_{m-1}+1}(\lambda) = \dots v^l_{p_m}(\lambda)$$

$$\alpha_1 = \alpha_2 = \dots \alpha_{p_1};\ \alpha_{p_1+1} = \dots \alpha_{p_2};\ \dots;\ \alpha_{p_{m-1}+1} = \dots \alpha_{p_m}$$

$$\beta_1 = \beta_2 = \dots \beta_{p_1};\ \beta_{p_1+1} = \dots \beta_{p_2};\ \dots;\ \beta_{p_{m-1}+1} = \dots \beta_{p_m}.$$

We thus obtain the following sets of homogeneous equations in the unknown amplitudes α_k and β_k ($\alpha_k = \alpha_{p_k}$ and $\beta_k = \beta_{p_k}$):

$$\sum_{k=1}^{m} \{[A_{ik}(\vec{\varkappa}\lambda) - E(\vec{\varkappa}\lambda)\ \delta_{ik}]\ \alpha_k + B_{ik}(\vec{\varkappa}\lambda)\ \beta_k\} = 0$$

$$\sum_{k=1}^{m} \{B^*_{ik}(\vec{\varkappa}\lambda)\alpha_k + [A^*_{ik}(\vec{\varkappa}\lambda) + E(\vec{\varkappa}\lambda)\ \delta_{ik}]\beta_k\} = 0, \quad (31)$$

where

$$A_{ik}(\vec{\varkappa}\lambda) = [(p_i - p_{i-1})(p_k - p_{k-1})]^{-1/2} \sum_{\nu=p_{i-1}+1}^{p_i} \sum_{\nu'=p_{k-1}+1}^{p_k} \sum_{g=-r}^{\nu}{}' A_{\nu\nu'}^{ll+g}(\vec{lj})\, \Gamma_{\nu\nu'}^{g}(\vec{\varkappa}\lambda)$$

$$B_{ik}(\vec{\varkappa}\lambda) = [(p_i - p_{i-1})(p_k - p_{k-1})]^{-1/2} \sum_{\nu=p_{i-1}+1}^{p_i} \sum_{\nu'=p_{k-1}+1}^{b_k} \sum_{g=-r}^{\nu}{}' B_{\nu\nu'}^{ll+g}(\vec{lj})\, \Gamma_{\nu\nu'}^{g}(\vec{\varkappa}\lambda). \quad (32)$$

Above, the notation used for the "spatial" structural factor is:

$$\Gamma_{\nu\nu'}^{g}(\vec{\varkappa}\lambda) = \gamma_{\nu\nu'}^{l\,l+g}(\vec{\varkappa})\, e^{-i\lambda g}. \quad (33)$$

From the set of homogeneous equations (31) which has non-trivial solutions when its determinant vanishes, we can find the energy eigenvalues of the system. Quantization of λ is obtained by substituting the general solutions (30) into the boundary conditions (28) and taking account of the normalization conditions (21) imposed on the functions of the Tyablikov-Bogolyubov transformation (20).

The simplest example of a film with non-translational symmetry is the ferromagnetic film with structure consisting of two translational sublattices ($p = 2$) which we can consider on the assumption of equal amplitudes of spin waves at equivalent sites belonging to the two translational sublattices, since in this case equations (31) reduce to a form similar to the form they have in the case of a single translational lattice ($p = 1$) discussed by Puszkarski [12]; *e. g.*, h. c. p. (Co [13]) or spinel structure with magnetic ions at tetrahedral A — sites ($CoRh_2O_4$, $NiRh_2O_4$, Co_3O_4). A case of non-translational structure consisting of several translational sublattices with different amplitudes α_i and β_i will be discussed in detail in a subsequent paper.

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