LANDAU THEORY OF THE SECOND-ORDER MAGNETIC PHASE TRANSITIONS IN UNIAXIAL FERROMAGNETS WITH EXTERNAL FIELD

By J. Sznajd

Institute for Low Temperature and Structure Research, Polish Academy of Sciences, Wrocław*

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The Landau theory of second-order magnetic phase transitions is applied to uniaxial ferromagnets with anisotropy of arbitrary origin (shape, exchange or crystal-field anisotropy) and in the presence of an external magnetic field. When the field is parallel to the easy axis, the state with magnetization antiparallel to the field direction can be realized as a metastable state within the field range $0 < h_z < 2k \left(\frac{t_c - t - k}{2b}\right)^{1/2}$. When the field is perpendicular to the easy axis, a phase transition takes place at a temperature depending on the value of the applied field [2]–[8]. Moreover, the dependence of the fluctuations correlation function on temperature and external field in the neighbourhood of the transition point is established. The correlation radius ξ_z^z of the parallel to the easy axis component of the magnetization tends to infinity as $t \to t_c(h_\perp)$.

1. Introduction

Recently, a number of papers both theoretical [1]–[8] and experimental [9]–[10] appeared dealing with the phase transition of a ferromagnet in an external magnetic field, and two different opinions about the nature of these transitions are represented by the authors. Arrott [1] and Wojtowicz and Rayl [2], *e.g.*, employ an unpublished theorem due to Griffiths, according to which the ground state of an (apparently finite) isotropic ferromagnet in the absence of an external field is a state of non-uniform magnetization. Relying upon this theorem the authors suggest that the influence of a uniform external field on such a ferromagnet may (at a certain temperature) cause it to pass from the state of non-uniform to the state of uniform magnetization. Using the molecular-field theory Wojtowicz and Rayl [2] calculated, for a torroidal model, the dependence of the transition temperature on the external field.

* Address: Instytut Niskich Temperatur i Badań Strukturalnych PAN, Wrocław, Plac Katedralny 1, Poland.

Another interpretation was given by Klamut and Durczewski [3]. They showed that, in order to explain the phase transition of a ferromagnet in an external field one does not have to invoke the assumption of the ground state to be a state of non-uniform magnetization, as such a transition takes place in a uniaxial ferromagnet regardless of the origin of the anisotropy (whether shape or exchange or crystal-field anisotropy) if the field is perpendicular to the easy axis ($H_\perp$). Calculations [4], [5] for a ferromagnet of this type, using MFA, lead to results analogous to those derived in [2] and to the following relation between the transition temperature $T_c(H_\perp)$ and the external field $H_\perp$:

$$T_c(H_\perp) = T_c\left[1 - A_s\left(\frac{g\mu_B H_\perp}{2zsK}\right)^2\right] \tag{1}$$

where T_c is the zero-field Curie temperature, g the Landé factor, μ_B — Bohr's magneton, z — coordination number, s — maximum spin eigenvalue, K — anisotropy constant, and A_s — a constant depending only on s.

The results given by Durczewski [4], [5] have also been obtained in [6]–[8] where by means of different methods the same relation (1) has been derived.

In this paper, Landau's theory of phase transitions is applied to a uniaxial ferromagnet in an external field parallel or perpendicular to the easy axis. The behaviour of the fluctuations of the magnetization components as functions of the field and temperature is also investigated.

2. *The Landau theory*

According to Landau's [11] assumption, the free energy of a uniaxial ferromagnet with the easy axis along the Oz axis and in the presence of a small magnetic field can be written in the following form:

$$F = F_0 + AM^2 + BM^4 - KM_z^2 - (\boldsymbol{M}, \boldsymbol{H}) \tag{2}$$

if the temperature T is close to T_c. Upon introducing the reduced quantities

$$f = \frac{F}{A_0 M_0^2}, \quad f_0 = \frac{F_0}{A_0 M_0^2}, \quad a = \frac{A}{A_0}, \quad b = \frac{BM_0^2}{A_0}$$

$$\boldsymbol{m} = \frac{\boldsymbol{M}}{M_0}, \quad k = \frac{K}{A_0}, \quad m_z = \frac{M_z}{M_0}, \quad \boldsymbol{h} = \frac{\boldsymbol{H}}{A_0 M_0}$$

$$\theta = \frac{A_0 M_0^2}{k_B T}, \quad A_0 = A(T=0), \quad M_0 = M(T=0, H=0). \tag{3}$$

Eq. (2) can be rewritten as

$$f = f_0 + am^2 + bm^4 - km_z^2 - (\boldsymbol{m}, \boldsymbol{h}). \tag{4}$$

Here, Landau's expression for the free energy has been supplemented by an anisotropy term (of arbitrary origin) with the effective anisotropy constant K.

The magnetization components m_z and $m_\perp$ (projection of the vector $\boldsymbol{m}$ on the easy axis and on the plane perpendicular to it, respectively) are determined by minimiziny the free energy with respect to these quantities.

The necessary conditions for the existence of a minimum are

$$\frac{\partial f}{\partial m_z} = 0, \qquad \frac{\partial f}{\partial m_\perp} = 0. \tag{5}$$

Hence, we get from (4)

$$h_z = m_z\,[2(a-k)+4bm_z^2] \tag{6a}$$

$$h_\perp = m_\perp\,[2a+4bm^2]. \tag{6b}$$

These equations we shall solve for the field-free case, and for the cases when the field is parallel or perpendicular to the easy axis.

a) $h_z = h_\perp = 0$ — field-free case, which differs from the one considered by Landau [11] only by taking into account the anisotropy, *i.e.*, by passing in the free energy from a to $(a-k)$. Therefore, we get in analogy to Landau the second-order phase transition at the point $(a-k) = 0$, where $(a-k)$ can be, after Landau, expanded in a power series with respect to $(T-T_c)$:

$$a-k = a'(T-T_c)+\ldots \equiv (t-t_c+\ldots,\quad a' > 0. \tag{7}$$

Below the transition point, *i.e.*, for $(a-k) < 0$ the following solution correspond to the minimum of the free energy:

$$m_\perp = 0, \qquad m_z = \pm\left(\frac{t_c-t}{2b}\right)^{1/2} \tag{8}$$

whereas for temperatures higher than T_c, *i.e.*, $(a-k) > 0$ the solutions are

$$m_\perp = 0,\ m_z = 0. \tag{9}$$

In the following, we assume after Landau the coefficient B in (2) to be independent of the temperature.

b) $h_\perp = 0,\ h_z > 0$ — field parallel to the easy axis. In this case, Eqs (6a) and (6b) have several solutions, of which the stable ones belong to the set

$$m_\perp = 0,\ h_z = m_z[2(a-k)+4bm_z^2] \tag{10}$$

as can be easily verified by examining the sufficient conditions for the existence of a minimum of the free energy, *i.e.*,

$$\varDelta = 4(a-k+2bm^2+4bm_z^2)(a+2bm^2+4bm_\perp^2) - 64b^2m_\perp^2m_z^2 > 0 \tag{11}$$

$$F_{zz} = 2(a-k+2bm^2+4bm_z^2) > 0.$$

The set (10) contains a cubic equation for m_z. One can therefore expect one, two or three real solutions.

If $(a-k)^3 > -\dfrac{27b}{8}h_z^2$ we have one real solution,

$$m_z = \left\{\frac{h_z}{8b} + \left[\frac{1}{8b^2}\left(\frac{h_z^2}{8} + \frac{(a-k)^3}{27b}\right)\right]^{1/2}\right\}^{1/3} + \left\{\frac{h_z}{8b} - \left[\frac{1}{8b^2}\left(\frac{h_z^2}{8} + \frac{(a-k)^3}{27b}\right)\right]^{1/2}\right\}^{1/3} \tag{12}$$

which (together with $m_{\perp} = 0$) satisfies the conditions (11). In this case the magnetization is directed along the field.

For $(a-k)^3 = -\frac{27b}{8} h_z^2$ Eq. (9) has two real solutions:

$$m_z^{(a)} = 2\left(\frac{h_z}{8b}\right)^{1/3}, \quad m_z^{(b)} = -\left(\frac{h_z}{8b}\right)^{1/3}. \tag{13}$$

One easily proves that only the solution $m_z^{(a)}$ satisfies the minimum conditions (11), whereas for $m_z^{(b)}$ we get $\Delta = 0$.

For $(a-k)^3 < -\frac{27b}{8} h_z^2$ there are three real solutions,

$$m_z^{(1)} = 2\left(\frac{|a-k|}{6b}\right)^{1/2} \cos\left(\tfrac{1}{3}\alpha\right)$$

$$m_z^{(2)} = -2\left(\frac{|a-k|}{6b}\right)^{1/2} \cos\left(60^\circ + \tfrac{1}{3}\alpha\right)$$

$$m_z^{(3)} = -2\left(\frac{|a-k|}{6b}\right)^{1/2} \cos\left(60^\circ - \tfrac{1}{3}\alpha\right) \tag{14}$$

where $\cos\alpha = h_z 6^{3/2} b^{1/2} / 8|a-k|^{3/2}$.

The solution $m_z^{(1)}$ satisfies the conditions (11). For the solutions $m_z^{(2)}$ and $m_z^{(3)}$ the magnetization is directed opposite to the field, with $m_z^{(2)}$ never satisfying the conditions (11), and $m_z^{(3)}$ fulfilling them if

$$a > 0 \text{ or } a < 0 \text{ and } h_z < 2k\left(-\frac{a}{2b}\right)^{1/2}. \tag{15}$$

From the expansion (4) it follows that the solution $m_z^{(1)}$ always corresponds to a lower free energy than the solution $m_z^{(3)}$. This means that in the temperature range here considered the

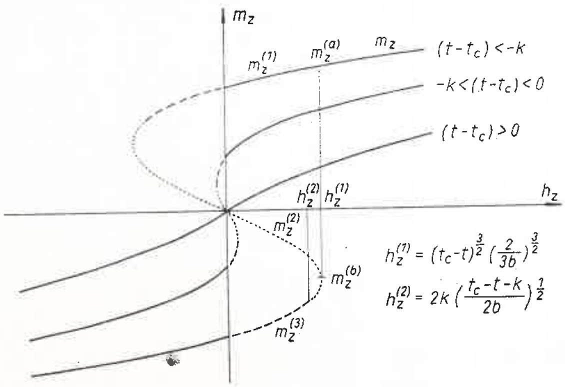

Fig. 1. Schematic dependence of the parallel component of the magnetization m_z on the field parallel to the easy axis, for three values of the temperature. (Solid, broken and dotted lines represent respectively the stable, metastable, and unstable state)

state $m_z^{(1)}$ is the stable one, while the solution $m_z^{(3)}$ can be realized as a metastable state for fields from the interval $0 < h_z < 2k\left(\frac{t_c - t - k}{2b}\right)^{1/2}$ (Fig. 1).

c) $h_z = 0$, $h_\perp > 0$ — field perpendicular to the easy axis. In this case Eqs (6a) and (6b) yield the solutions

$$m_\perp = \frac{h_\perp}{2k}, \quad m_z = \pm\left[-\frac{a-k}{2b}\left(\frac{h_\perp}{2k}\right)^2\right]^{1/2} \tag{16}$$

and

$$h_\perp = m_\perp(2a + 4bm_\perp^2), \; m_z = 0 \tag{17}$$

of which the solution (16) meets the conditions (11) for $\left(a - k + 2b\left(\frac{h}{2k}\right)^2\right) < 0$. In contrast to Eq. (10) the cubic equation in (16) has only one stable real solution which satisfies the conditions (11) if $\left(a - k + 2b\left(\frac{h}{2k}\right)^2\right) > 0$. This solution has the form:

$$m_\perp = \left\{\frac{h_\perp}{8b} + \left[\frac{1}{8b^2}\left(\frac{h_\perp^2}{8} + \frac{a^3}{27b}\right)\right]^{1/2}\right\}^{1/3} + \left\{\frac{h_\perp}{8b} - \left[\frac{1}{8b^2}\left(\frac{h_\perp^2}{8} + \frac{a^3}{27b}\right)\right]^{1/2}\right\}^{1/3} \tag{17a}$$

$$\text{if } a^3 > -\frac{27b}{8}h_\perp^2,$$

$$m_\perp = 2\left(\frac{h_\perp}{8b}\right)^{1/3} \tag{17b}$$

$$\text{if } a^3 = -\frac{27b}{8}h_\perp^2,$$

and

$$m_\perp = 2\left(\frac{|a|}{6b}\right)^{1/2}\cos\left(\tfrac{1}{3}\alpha\right) \tag{17c}$$

$$\text{if } a^3 < -\frac{27b}{8}h_\perp^2, \text{ where } \cos\alpha = h_\perp\frac{6^{3/2}b^{1/2}}{8|a|^{3/2}}.$$

It is readily seen that for

$$a - k + 2b\left(\frac{h_\perp}{2k}\right)^2 = 0 \tag{18}$$

the solution (16) coincides with the respective form (depending on the field strength and temperature) of the solution (17a)–(17c). By utilizing the Landau expansion (7), condition (18) leads to a relationship between the temperature and the external field strength for which (18) holds:

$$t_c(h_\perp) = t_c - \left(\frac{h_\perp}{2k}\right)^2 \tag{19}$$

It is readily seen that the solution (16) for which $m_z \neq 0$ corresponds to a ferromagnetic state (F), whereas (17a)–(17c) represents a paramagnetic state (P).

Upon inserting the solutions (16) and (17) for the magnetization into the expansion (4) and calculating from the resulting free energy the specific heat at constant field, we obtain for the difference of the respective specific heats when approaching the temperature $t_c(h_\perp)$ the result

$$\Delta C_{h_\perp} = C_{h_\perp}^{(F)} - C_{h_\perp}^{(P)} = \frac{1}{2b}\left[t_c - \left(\frac{h_\perp}{2k}\right)^2\right]. \tag{20}$$

Thus, the specific heat has a jump at the temperature (19), and the magnitude of this jump $\Delta C_{h_\perp}$ decreases with increasing field $h_\perp$. One easily proves that the entropy (at constant field) is continuous at $t_c(h_\perp)$. Therefore we obtained, in accordance with [1]–[8], that in a uniaxial ferromagnet placed in an external field perpendicular to the easy axis a phase transition of second order takes place. The temperature of this transition, $t_c(h_\perp)$, is lower than the ordinary (field-free) Curie temperature t_c as is seen from Eq. (19).

3. Susceptibility

Differentiating both sides of Eqs (6a) and (6b) with respect to h_z and $h_\perp$ we get a set of four equations for the components of the susceptibility tensor as functions of the temperature and magnetization. The solutions are

$$\chi_z^z = \frac{\partial m_z}{\partial h_z} = \frac{1}{2}\,\frac{W+4bm_\perp^2+k}{4Wbm_\perp^2+(W+k)(W+4bm_z^2)} \tag{21}$$

$$\chi_\perp^z = \frac{\partial m_z}{\partial h_\perp} = \frac{1}{2}\,\frac{-2bm_z m_\perp}{4Wbm_\perp^2+(W+k)\,(W+4bm_z^2)} \tag{22}$$

$$\chi_\perp^\perp = \frac{\partial m_\perp}{\partial h_\perp} = \frac{1}{2}\,\frac{W+4bm_z^2}{4Wbm_\perp^2+(W+k)\,(W+4bm_z^2)} \tag{23}$$

$$\chi_z^\perp = \frac{\partial m_\perp}{\partial h_z} = \chi_\perp^z \tag{24}$$

where $W = a-k+2bm^2$. Henceforth we shall confine ourselves to the case of the field perpendicular to the easy axis.

The insertion of the solutions (16) and (17) into Eqs (22) and (23) gives us the components of the susceptibility tensor for a uniaxial ferromagnet in a field perpendicular to the easy axis, and on both sides of the transition curve (19). In the low-temperature (F) phase, $W = 0$, $m_\perp = \dfrac{h_\perp}{2k}$, $m_z = \left(\dfrac{t_c(h_\perp)-t}{2b}\right)^{1/2}$; hence, we have from (22) and (23)

$$\chi_\perp^z = -\frac{h_\perp}{4k^2\left[\frac{1}{2b}\,(t_c(h_\perp)-t)\right]^{1/2}} \tag{25a}$$

$$\chi_\perp^\perp = \frac{1}{2k}. \tag{25b}$$

In the high-temperature (P) phase, $m_z = 0$; thus,

$$\chi^z_\perp = 0 \tag{26a}$$

$$\chi^\perp_\perp = \frac{1}{2(a+6bm^2_\perp)}. \tag{26b}$$

The change of the magnetization vector length m due to a change of the external field strength is given by the susceptibility $\chi_\perp = \frac{\partial m}{\partial h_\perp} = \frac{m_\perp}{m}\chi^\perp_\perp + \frac{m_z}{m}\chi^z_\perp$. In the ferromagnetic phase we have $\chi_\perp = 0$; thus, m is field-independent. In the paramagnetic phase $\chi_\perp = \chi^\perp_\perp$; in this case m increases with increasing field and, the larger the value of the applied field the slower the increment of m. The initial susceptibility $\chi_\perp$ increases with increasing temperature, reaching a maximum at $t = t_c + 6bm^2_\perp - k$ which decreases and moves to higher temperatures with increasing field.

4. *Fluctuations*

All the above results have been derived under the assumption that no fluctuations of the magnetization components occur. In other words, we have assumed that $m_i(\boldsymbol{r}) = \langle m_i(\boldsymbol{r})\rangle$ ($i = z, \perp$), *i. e.*, the local magnetization is equal to its spatial average.

Kadanoff *et al.* [12] showed how, within the Landau theory, the influence of the fluctuations of m_i at the point $\boldsymbol{r}_i$ on the neighbouring sites can be taken into account. The quantity describing this fluctuations correlation has the form

$$g^i_j(\boldsymbol{r}_1, \boldsymbol{r}_2) = \langle[m_i(\boldsymbol{r}_1) - \langle m_i(\boldsymbol{r}_1)\rangle]\,[m_j(\boldsymbol{r}_2) - \langle m_j(\boldsymbol{r}_2)\rangle]\rangle \tag{27}$$

In the classical statistical mechanics there exists a general method permitting to connect the functions (27) with the change of the averages of $m_z(\boldsymbol{r})$ and $m_\perp(\boldsymbol{r})$. If the quantities $m_z(\boldsymbol{r})$ and $m_\perp(\boldsymbol{r})$ enter the Hamiltonian in the form

$$\mathscr{H} = \mathscr{H}_0 - \int (m_z(\boldsymbol{r})h_z(\boldsymbol{r}) + m_\perp(\boldsymbol{r})h_\perp(\boldsymbol{r}))\,d\boldsymbol{r} \tag{28}$$

then the variation $h_\perp(\boldsymbol{r}) \to h_\perp(\boldsymbol{r}) + \delta h_\perp(\boldsymbol{r})$ induces the following change of the components of the average magnetization:

$$\delta\langle m_\perp(\boldsymbol{r}_1)\rangle = \theta \int d\boldsymbol{r}_2 g^\perp_\perp(\boldsymbol{r}_1, \boldsymbol{r}_2)\,\delta h_\perp(\boldsymbol{r}_2) \tag{29}$$

$$\delta\langle m_z(\boldsymbol{r}_1)\rangle = \theta \int d\boldsymbol{r}_2 g^z_\perp(\boldsymbol{r}_1, \boldsymbol{r}_2)\,\delta h_\perp(\boldsymbol{r}_2) \tag{30}$$

whereas the variation $h_z(\boldsymbol{r}) \to h_z(\boldsymbol{r}) + \delta h_z(\boldsymbol{r})$ gives

$$\delta\langle m_\perp(\boldsymbol{r}_1)\rangle = \theta \int d\boldsymbol{r}_2 g^\perp_z(\boldsymbol{r}_1, \boldsymbol{r}_2)\,\delta h_z(\boldsymbol{r}_2) \tag{31}$$

$$\delta\langle m_z(\boldsymbol{r}_1)\rangle = \theta \int d\boldsymbol{r}_2 g^z_z(\boldsymbol{r}_1, \boldsymbol{r}_2)\,\delta h_z(\boldsymbol{r}_2). \tag{32}$$

In the Landau theory, to take account of the fluctuations we have to augment the free energy (4) by the terms $c(\nabla m_z(\boldsymbol{r}))^2 + c(\nabla m_\perp(\boldsymbol{r}))^2$ describing the inhomogeneity of the magnetization distribution. We get, therefore,

$$f = f_0 + (a-k)\,[m_z(\boldsymbol{r})]^2 + a[m_\perp(\boldsymbol{r})]^2 + b[m_z(\boldsymbol{r})]^4 + 2b[m_z(\boldsymbol{r})]^2[m_\perp(\boldsymbol{r})]^2 +$$
$$+ b[m_\perp(\boldsymbol{r})]^4 - (\boldsymbol{m}(\boldsymbol{r}), \boldsymbol{h}(\boldsymbol{r})) + c\{[\Delta m_z(\boldsymbol{r})]^2 + [\Delta m_\perp(\boldsymbol{r})]^2\}. \tag{33}$$

Varying the free energy (33) according to $h_\perp(\boldsymbol{r}) \to h_\perp(\boldsymbol{r})+\delta h_\perp(\boldsymbol{r})$ and minimizing it with respect to $m_z(\boldsymbol{r})$ and $m_\perp(\boldsymbol{r})$ we get

$$\{2(a-k)+4b[m_\perp(\boldsymbol{r})]^2+12b[m_z(\boldsymbol{r})]^2-2c\nabla^2\}\,\delta m_z(\boldsymbol{r})+8bm_\perp(\boldsymbol{r})m_z(\boldsymbol{r})\delta m_\perp(\boldsymbol{r})=0 \quad (34)$$

$$\{2a+4b[m_z(\boldsymbol{r})]^2+12b[m_\perp(\boldsymbol{r})]^2-2c\nabla^2\}\,\delta m_\perp(\boldsymbol{r})+8bm_\perp(\boldsymbol{r})m_z(\boldsymbol{r})\,\delta m_z(\boldsymbol{r})=\delta h_\perp(\boldsymbol{r}). \quad (35)$$

Similarly, the variation $h_z(\boldsymbol{r})\to h_z(\boldsymbol{r})+\delta h_z(\boldsymbol{r})$ leads to

$$\{2(a-k)+4b[m_\perp(\boldsymbol{r})]^2+12b[m_z(\boldsymbol{r})]^2-2c\nabla^2\}\,\delta m_z(\boldsymbol{r})+8bm_\perp(\boldsymbol{r})\,m_z(\boldsymbol{r})\,\delta m_\perp(\boldsymbol{r})=\delta h_z(\boldsymbol{r}) \quad (36)$$

$$\{2a+4b[m_z(\boldsymbol{r})]^2+12b\,[m_\perp(\boldsymbol{r})]^2-2c\nabla^2\}\,\delta m_\perp(\boldsymbol{r})+8bm_\perp(\boldsymbol{r})\,m_z(\boldsymbol{r})\,\delta m_z(\boldsymbol{r})=0. \quad (37)$$

Following Kadanoff [12] we make now a somewhat inconsistent step, by inserting into Eqs (34)–(37) instead of $m_z(\boldsymbol{r})$ and $m_\perp(\boldsymbol{r})$ their spatial averages $\langle m_z(\boldsymbol{r})\rangle=m_z$, $\langle m_\perp(\boldsymbol{r})\rangle=m_\perp$ determined with the aid of Eqs (6a) and (6b). Taking into account in Eqs (34)–(37) the relations (29)–(32) and $h_i(\boldsymbol{r}_1)=\int d\boldsymbol{r}_2\,\delta h_i(\boldsymbol{r}_2)\,\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$ we have

$$(a-k+2bm_\perp^2+6bm_z^2-c\nabla^2)\,g_\perp^z(\boldsymbol{r}_1,\boldsymbol{r}_2)+4bm_\perp m_z g_\perp^\perp(\boldsymbol{r}_1,\boldsymbol{r}_2)=0 \quad (38)$$

$$(a+2bm_z^2+6bm_\perp^2-c\nabla^2)\,g_\perp^\perp(\boldsymbol{r}_1,\boldsymbol{r}_2)+4bm_\perp m_z g_\perp^z(\boldsymbol{r}_1,\boldsymbol{r}_2)=\tfrac{1}{2}\theta\,\delta(\boldsymbol{r}_1-\boldsymbol{r}_2) \quad (39)$$

$$(a-k+2bm_\perp^2+6bm_z^2-c\nabla^2)\,g_z^z(\boldsymbol{r}_1,\boldsymbol{r}_2)+4bm_\perp m_z g_z^\perp(\boldsymbol{r}_1,\boldsymbol{r}_2)=\tfrac{1}{2}\theta\,\delta(\boldsymbol{r}_1-\boldsymbol{r}_2) \quad (40)$$

$$(a+2bm_z^2+6bm_\perp^2-c\nabla^2)\,g_z^\perp(\boldsymbol{r}_1,\boldsymbol{r}_2)+4bm_\perp m_z g_z^z(\boldsymbol{r}_1,\boldsymbol{r}_2)=0. \quad (41)$$

We shall solve these equations for the field perpendicular to the easy axis, *i. e.*, using the solutions (16) and (17). In the paramagnetic phase $t>t_c(h_\perp)$, where the solutions (17) are valid, Eqs (38)–(41) reduce to

$$(a-k+2bm_\perp^2-c\nabla^2)\,g_\perp^z(\boldsymbol{r}_1,\boldsymbol{r}_2)=0$$
$$(a+6bm_\perp^2-c\nabla^2)\,g_\perp^\perp(\boldsymbol{r}_1,\boldsymbol{r}_2)=\tfrac{1}{2}\theta\,\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$$
$$(a-k+2bm_\perp^2-c\nabla^2)\,g_z^z(\boldsymbol{r}_1,\boldsymbol{r}_2)=\tfrac{1}{2}\theta\,\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$$
$$(a+6bm_\perp^2-c\nabla^2)\,g_z^\perp(\boldsymbol{r}_1,\boldsymbol{r}_2)=0. \quad (42)$$

Solving these equations we get

$$g_i^i(r)=\frac{\theta}{8\pi c}\frac{1}{r}e^{\frac{-r}{\xi_i^i}},\qquad r=|\boldsymbol{r}_1-\boldsymbol{r}_2|$$
$$g_j^i(r)=0\qquad i\neq j \quad (43)$$

where ξ_i^i is called the correlation radius which in our case has the form

$$\xi_z^z=\left(\frac{c}{a-k+2bm_\perp^2}\right)^{1/2} \quad (44)$$

$$\xi_\perp^\perp=\left(\frac{c}{a+6bm_\perp^2}\right)^{1/2}. \quad (45)$$

The expressions (26b) and (45) give us a simple relation between the correlation radius and the susceptibility in the paramagnetic phase, namely, $\xi_\perp^\perp=(2c\chi_\perp)^{1/2}$.

It follows from Eqs (43) and (18) that the correlation radius ξ_z^z in the vicinity of the transition curve $(a-k+2bm_\perp^2=0)$ tends to infinity; hence, in this region the correlation function $g_z^z(\boldsymbol{r}_1, \boldsymbol{r}_2)$ decreases very slowly with increasing distance (like $1/r$). The correlation radius $\xi_\perp^\perp$ at the transition curve (19) has a finite value depending on the field strength $h_\perp$ and the anisotropy k. The correlation radius increases with increasing temperature, has

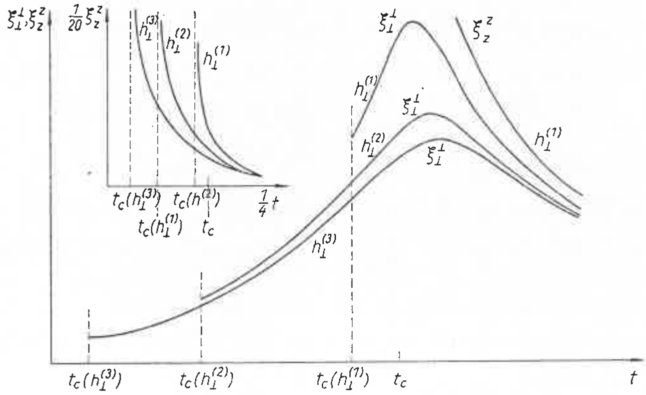

Fig. 2. Schematic temperature dependence of the correlation radius ξ_z^z and $\xi_\perp^\perp$ in the paramagnetic region $t > t_c(h_\perp)$, for three strengths $h_\perp^{(3)} > h_\perp^{(2)} > h_\perp^{(1)}$ of the magnetic field perpendicular to the easy axis

a maximum at the point $t = t_c + 6bm_\perp^2 - k$, and decreases above it. The maximum itself decreases and moves to higher temperatures with increasing external field (Fig. 2). A similar temperature dependence of the maximum value of the longitudinal correlation radius for $T > T_c$ and a magnetic field parallel to the easy axis was obtained in [13].

In the ferromagnetic phase, taking into account (16), we get the following form of Eqs (38)–(41)

$$\begin{aligned}
(4bm_z^2 - c\nabla^2)\, g_\perp^z(r) + 4bm_z m_\perp g_\perp^\perp(r) &= 0 \\
(k + 4bm_\perp^2 - c\nabla^2)\, g_\perp^\perp(r) + 4bm_z m_\perp g_\perp^z(r) &= \tfrac{1}{2}\theta\delta(r) \\
(4bm_z^2 - c\nabla^2)\, g_z^z(r) + 4bm_z m_\perp g_z^\perp(r) &= \tfrac{1}{2}\theta\delta(r) \\
(k + 4bm_\perp^2 - c\nabla^2)\, g_z^\perp(r) + 4bm_z m_\perp g_z^z(r) &= 0.
\end{aligned} \tag{46}$$

Solving this set we get

$$g_z^z(r) = N\{D_2 - D_1 e^{-r(x_1 - x_2)}\} \frac{e^{-rx_2}}{r} \tag{47}$$

for $t \to t_c(h_\perp)$, $\quad g_z^z(r) \to \dfrac{\theta}{8\pi c} \dfrac{1}{r}$

$$g_\perp^\perp(r) = N\{B_2 e^{-r(x_2 - x_1)} - B_1\} \frac{e^{-rx_1}}{r} \tag{48}$$

for $t \to t_c(h_\perp)$, $\quad g_\perp^\perp(r) \to \dfrac{\theta}{8\pi c} \dfrac{1}{r}\, e^{-r\left(\frac{k + b\left(\frac{h_\perp}{k}\right)^2}{c}\right)^{1/2}}$

$$g_\perp^z(r) = g_z^\perp(r) = N \cdot L\, \{1 - e^{-r(x_1 - x_2)}\} \frac{e^{-rx_2}}{r} \tag{49}$$

for $t \to t_c(h_\perp)$, $g_\perp^z \to 0$

where

$$N = \frac{\theta}{16\pi c^2 (x_1^2 - x_2^2)}, \quad L = 4bm_z m_\perp$$

$$x_{1,2}^2 = \frac{2(k+4bm^2) \pm [4(k+4bm^2)^2 - 64 \cdot kbm_z^2]^{1/2}}{4c}$$

$$D_1 = 8bm_\perp^2 - 2cx_1^2 + 2k, \quad D_2 = 8bm_\perp^2 - 2cx_2^2 + 2k$$

$$B_1 = 8bm_z^2 - 2cx_1^2; \; B_2 = 8bm_z^2 - 2cx_2^2. \tag{50}$$

Like in the paramagnetic phase, it follows from Eqs (47) and (50) that in the ferromagnetic phase the correlation function $g_z^z(r)$ decreases very slowly with increasing distance. From the results derived above one can conclude that in the case considered here, *i. e.*, for the phase transition of a uniaxial ferromagnet in a field perpendicular to the easy axis, the component of the magnetization parallel to the easy axis fulfils all the conditions required of an order parameter, namely:

(*i*) it vanishes on one side of the transition point, cp. Eq. (17);

(*ii*) it approaches zero continuously from the other side of the transition point and is not uniquely determined, cp. Eq. (16).

Moreover, the correlation radius of the parallel to the easy axis component of the magnetization tends to infinity in the vicinity of the transition point.

5. *Conclusions*

The Landau theory of second-order phase transitions is applied to a uniaxial ferromagnet in an external magnetic field.

The following results are obtained:

1. In the case of the field parallel to the easy axis, the magnetization components as functions of the field strength and temperature are determined.

2. When the field is parallel to the easy axis, it turns out that for temperatures $t < t_c$ there exists a field range $0 < h_z < 2k(t_c - t - k)^{\frac{1}{2}}/2b$ where the state with magnetization antiparallel to the field direction can be realized as a metastable state.

3. The results for the field perpendicular to the easy axis are in complete agreement with the results obtained in [2]–[8], *i. e.*, there is a magnetic ferro-para phase transition when the field is perpendicular to the easy axis, and the temperature of this transition depends on the field strength, cp. Eq. (19).

4. It is shown that this transition is of the second order, as the magnetic specific heat has a jump at the transition point. The amount of this jump is calculated, Eq. (20).

5. The dependence of the transversal susceptibility $\chi_\perp$ on the temperature above the transition point is also investigated. It turns out that it has a maximum at a certain temperature, and that the increase of the field smears out this maximum and shifts it toward higher temperatures.

6. In the last part of this paper the influence of the temperature and field on the correlation radius of the fluctuations of the components m_z and $m_\perp$ in the neighbourhood of the

transition point is determined and discussed. The correlation radius ξ_z^z of the component parallel to the easy axis m_z tends to infinity when approaching the transition temperature $t_c(h_\perp)$ from the paramagnetic phase.

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