RELAXATION OF MAGNONS ON DISLOCATIONS IN UNIAXIAL FERROMAGNETS

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The relaxation time of uniform magnons in ferromagnets having uniaxial type of anisotropy is calculated. The relaxation mechanism consists in two-magnon scattering processes on the deformation field of dislocations. The coupling between magnons and dislocation strains results from the magnetoelastic energy.

1. *Introduction*

Experimental facts suggest that the ferromagnetic resonance line-width depends to a large extent on the crystallographic perfectness of the material investigated. The presence of any defects results in an increase of the line-width. Of many kinds of defects, dislocations seem to be particularly effective in damping the uniform precession mode excited in ferromagnetic resonance.

In previous papers [1, 2] relaxation of magnons by the two-magnon scattering mechanism on dislocations in cubic ferromagnets was discussed. Here we present similar calculations of the relaxation time of uniform magnons for ferromagnets having uniaxial type anisotropy. Some results are given for hexagonal ferromagnetics with full account for anisotropy of the magnetic and elastic properties. An estimate of the contribution to the relaxation time from the dislocation core is also presented.

2. *Hamiltonian*

Similarly as in [1] we use the phenomenological theory of magnons (see *e.g.* [3]). The interaction of magnons with dislocations is due to the magnetoelastic energy. Let e_{ij} denote the strains of the deformation field induced by the dislocation and M_i the components of the local magnetization. The general expression for the magnetoelastic

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energy density, linear with respect to strains e_{ij} is of the form

$$F_{me} = \sum_{n,i,j,l=x,y,z} V_{nlij} M_n M_l e_{ij} + \sum_{i,j,l,m=x,y,z} \gamma_{ijlm} \frac{\partial \boldsymbol{M}}{\partial x_i} \cdot \frac{\partial \boldsymbol{M}}{\partial x_j} e_{lm}. \tag{1}$$

For crystals of uniaxial symmetry the explicit expression for the first term in (1) is (see *e.g.* [4])

$$\begin{aligned}(1/M_0^2)\{M_x^2[L_1 e_{xx} + L_2 e_{yy} + M_1 e_{zz}] + M_y^2[L_2 e_{xx} + L_1 e_{yy} + M_1 e_{zz}] + \\ + (M_x M_y + M_y M_x)(L_1 - L_2) e_{xy} + (M_x M_z + M_z M_x) N_0 e_{xz} + \\ + (M_y M_z + M_z M_y) N_0 e_{yz}\}.\end{aligned} \tag{2}$$

Here the new parameters L_1, L_2, M_1, N_0 denote the usual magnetoelastic constants. Symmetry considerations allow one to reduce the matrix of coefficients γ_{ijlm} in (1) to several independent phenomenological parameters. It will appear, however, that the terms in (1) proportional to the gradients of the local magnetization will not contribute to relaxation of uniform magnons considered here so it is of no use to specify the coefficients γ_{ijlm}.

The components of the local magnetization $\boldsymbol{M}(\boldsymbol{r})$ at the point $\boldsymbol{r}$ are expanded into plane waves, whose amplitudes are determined by the magnon operators. In the lowest order approximation, correct up to the terms quadratic in the magnon operators, the components of the local magnetization are (see *e.g.* [5] for details)

$$\begin{aligned} M_+(\boldsymbol{r}) &= M_x(\boldsymbol{r}) + iM_y(\boldsymbol{r}) = (4\mu_B M_0/V)^{1/2} \sum_k a_k e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \ldots, \\ M_z(\boldsymbol{r}) &= M_0 - (2\mu_B/V) \sum_{k,k'} a_{k'}^+ a_{k'+k} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}. \end{aligned} \tag{3}$$

M_0 is the saturation magnetization, V is the volume of the sample and μ_B denotes the Bohr magneton. a_k^+ (a_k) denote the creation (annihilation) operator for magnon of the wave vector $\boldsymbol{k}$. (As discussed in [1] the above simple interpretation is exact only for applied magnetic field strength H_0 much larger than $2\pi M_0$.)

Now we insert (3) into the expression (1) for the magnetoelastic energy density and retain only terms proportional to $a_k^+ a_{k'}$, *i.e.* terms describing the lowest order (two-magnon) scattering processes on strains e_{ij} of dislocation. After integration over the volume of the crystal we obtain the leading term in the magnon interaction energy due to strains e_{ij}

$$\begin{aligned}\mathscr{H}_{md} = -\frac{\mu_B}{M_0 V} \sum_{k,k'} \int_V d\boldsymbol{r} \exp[i(\boldsymbol{k}' - \boldsymbol{k})\cdot\boldsymbol{r}] \times \\ \times \{(L_1 + L_2)(e_{xx} + e_{yy}) + 2M_1 e_{zz} - 2M_0^2 \sum_{l,j,m,n} \gamma_{ljmn} k_l k_j' e_{mn}\} a_k^+ a_{k'} + \text{h.c.}\end{aligned} \tag{4}$$

3. *Relaxation time*

The relaxation time τ of the uniform magnons $k = 0$ determined by the two-magnon processes is given by the formula (*cf.* [1])

$$1/\tau = (2\pi/\hbar) \sum_k |W_k|^2 \delta(\varepsilon_k - \varepsilon_0), \tag{5}$$

where

$$W_k = (2\mu_B/M_0V) \int d\boldsymbol{r} \exp(-i\boldsymbol{k}\cdot\boldsymbol{r}) \{(L_1+L_2)(e_{xx}+e_{yy})+2M_1e_{zz}\}. \tag{6}$$

ε_k and ε_0 denote the energy of the magnon $\boldsymbol{k}$ and the uniform magnon ($k=0$), respectively. For a uniaxial ferromagnetic crystal we have (see *e.g.* [5])

$$\varepsilon_0 = 2\mu_B\{[H_0+(2K_1/M_0)+4\pi(N_x-N_z)M_0][H_0+(2K_1/M_0)+ +4\pi(N_y-N_z)M_0]\}^{1/2}, \tag{7}$$

and

$$\varepsilon_k = 2\mu_B\{[H_0+(2K_1/M_0)-4\pi N_zM_0+C_1M_0(k_x^2+k_y^2)+C_2M_0k_z^2]\times \times[H_0+(2K_1/M_0)-4\pi N_zM_0+C_1M_0(k_x^2+k_y^2)+C_2M_0k_z^2+4\pi M_0\sin^2\vartheta_k]\}^{1/2}. \tag{8}$$

H_0 denotes the external magnetic field, applied along the z-axis of the coordinate system taken parallel to the anisotropy direction. N_x, N_y, N_z are the demagnetizing factors (we assume that the crystal is of ellipsoidal shape and that the principal directions coincide with the coordinate axes). K_1 is the anisotropy constant defined by the following expression for the density of magnetocrystalline free energy of uniaxial crystals

$$F_k = (K_1/M_0^2)(M_x^2+M_y^2). \tag{9}$$

We assume $K_1>0$ *i.e.* easy axis in the z-direction. C_1 and C_2 denote the exchange constants for the direction of $\boldsymbol{k}$ perpendicular and parallel to the anisotropy axis, respectively. The angle between the magnon wave vector $\boldsymbol{k}$ and the easy axis is denoted by ϑ_k.

For calculation of the matrix element W_k in the formula for the relaxation time τ due to a dislocation it is convenient to introduce a new coordinate system (x_1, x_2, x_3) defined by the dislocation. Let x_3 be directed along the dislocation line and let ϑ and η denote the Euler angles for the transformation from the (x, y, z) coordinate system to the new one, (x_1, x_2, x_3) (ϑ is the angle between the z nad x_3 axes, and η is the angle between x and x_1). The transformation from (x, y, z) to (x_1, x_2, x_3) is provided by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\eta & -\sin\eta & 0 \\ \cos\vartheta\sin\eta & \cos\vartheta\cos\eta & -\sin\vartheta \\ \sin\vartheta\sin\eta & \sin\vartheta\cos\eta & \cos\vartheta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}. \tag{10}$$

The components of the strain tensor appearing in W_k, Eq. (6) can be expressed in terms of the strains e_{11}, e_{12} *etc.* calculated in the new coordinate system (x_1, x_2, x_3) as follows

$$e_{xx}+e_{yy} = (1-\sin^2\vartheta\sin^2\eta)e_{11}+(1-\sin^2\vartheta\cos^2\eta)e_{22}+\sin^2\vartheta\, e_{33}- -\sin^2\vartheta\sin 2\eta\, e_{12}-\sin 2\vartheta\sin\eta\, e_{13}-\sin 2\vartheta\cos\eta\, e_{23}, \tag{11}$$

$$e_{zz} = \sin^2\vartheta\sin^2\eta\, e_{11}+\sin^2\vartheta\cos^2\eta\, e_{22}+\cos^2\vartheta\, e_{33}+ +\sin^2\vartheta\sin 2\eta\, e_{12}+\sin 2\vartheta\sin\eta\, e_{13}+\sin 2\vartheta\cos\eta\, e_{23}. \tag{12}$$

4. *Elastically isotropic medium*

In the present Section we assume that, as far as elastic properties are concerned, the ferromagnet can be considered isotropic. We take the strains of a dislocation calculated for an isotropic elastic continuum [6]. Some special results for the case of elastically anisotropic medium will be discussed in the next Section.

a) Screw dislocation

The only non-vanishing components e_{ij} of the strain tensor for a screw dislocation parallel to the x_3 axis are [6]

$$e_{13} = -\frac{b}{4\pi}\frac{\sin\varphi}{\varrho},$$

$$e_{23} = \frac{b}{4\pi}\frac{\cos\varphi}{\varrho}. \tag{13}$$

ϱ and φ are the polar coordinates on the (x_1, x_2) — plane, $x_1 = \varrho\cos\varphi$, $x_2 = \varrho\sin\varphi$. The length of the Burgers vector is denoted by b.

The matrix element (6) is given by

$$W_k^s = -\frac{\mu_B b(L_1+L_2-2M_1)\sin 2\vartheta}{2\pi M_0 V}\int d\boldsymbol{r}\exp(-i\boldsymbol{k}\cdot\boldsymbol{r})\frac{\cos(\varphi+\eta)}{\varrho}. \tag{14}$$

The integration domain in Eq. (14) is restricted to a hollow cylinder coaxial with the dislocation line, whose lenght is L (the length of the dislocation line) and the internal and external radii are r_0 and r_1, respectively, *i.e.*, we assume that the deformation field of the dislocation vanishes for ϱ exceeding r_1 and we ignore the contribution to W_k from the core of dislocation $\varrho < r_0$. It is justified to put $r_0 = 0$ in final results with negligible error (*cf.* [1]). The contribution to the inverse relaxation time from the dislocation core is discussed in the Appendix A for a special model. The result of integration is

$$W_k^s = i\,\frac{2\mu_B b(L_1+L_2-2M_1)}{M_0 V}\sin 2\vartheta\cos(\Phi+\eta)\frac{\sin(k_3L/2)}{k_3}\times$$

$$\times\frac{J_0(r_0k_0)-J_0(r_1k_0)}{k_0}. \tag{15}$$

Here k_0, k_3, Φ denote the cylindrical coordinates of the wave vector $\boldsymbol{k}$ (they are defined by the components k_1, k_2, k_3 of $\boldsymbol{k}$ in the (x_1, x_2, x_3) coordinate system: $k_1 = k_0\cos\Phi$, $k_2 = k_0\sin\Phi$). J_0 is the Bessel function.

In order to calculate the relaxation time from Eq. (5) we replace the sum $\Sigma_k(\ldots)$ by the integral $V/(2\pi)^3\int d\boldsymbol{k}(\ldots)$ and we approximate the factor $[\sin(k_3L/2)/k_3]^2$ appearing in W_k by $(\pi/2)L\delta(k_3)$ (see [1]). Therefore only magnons scattered perpendicularly to the dislocation line ($k_3 = 0$) contribute to the inverse relaxation time τ_s. For $k_3 = 0$ we have

$$\delta(\varepsilon_k-\varepsilon_0) = [4\mu_B u(\Phi+\eta)]^{-1}[2\pi M_0\alpha(\Phi+\eta)]^{-1/2}\times \tag{16}$$

$$\times\{1+\chi^2[1-\sin^2\vartheta\sin^2(\Phi+\eta)]^2\}^{-1/2}\times$$

$$\times\theta\{u^2(\Phi+\eta)\}\delta\{k_0-u(\Phi+\eta)\sqrt{2\pi M_0/\alpha(\Phi+\eta)}\}$$

where

$$\alpha(\Phi+\eta) = C_1 M_0 + (C_2 - C_1) M_0 \sin^2\vartheta \sin^2(\Phi+\eta), \tag{17}$$

$$u(\Phi+\eta) = \{\sin^2\vartheta \sin^2(\Phi+\eta) + [1/\chi^2 + (1-\sin^2\vartheta\sin^2(\Phi+\eta))^2]^{1/2} - -N_z - [(N_x - N_y)^2 + 1/\chi^2]^{1/2}\}^{1/2}, \tag{18}$$

$$\chi = 2\pi M_0/(\varepsilon_0/2\mu_B), \tag{19}$$

and $\theta(x)$ is defined as equal 1 for $x \geqslant 0$ and 0 for $x < 0$.

Finally, introducing a new integration variable $\psi = \Phi + \eta$ we arrive at the formula for the relaxation time τ_s of uniform magnons scattered on a screw dislocation

$$1/\tau_s = \frac{\mu_B b^2 L (L_1 + L_2 - 2M_1)^2 \sin^2 2\vartheta}{4\pi^2 \hbar V M_0^3} \int_0^{\pi/2} d\psi \frac{\theta(u^2)\,[1 - \mathrm{J}_0(\lambda u)]^2 \cos^2\psi}{u^2\sqrt{1+\chi^2[1-\sin^2\vartheta\sin^2\psi]^2}}, \tag{20}$$

where $u = u(\psi)$, $\lambda = r_1\sqrt{2\pi M_0/\alpha(\psi)}$.

b) Edge dislocation

The non-vanishing components of the strain tensor for an edge dislocation parallel to the x_3-axis, whose the Burgers vector b is parallel to the x_1-axis are [6]

$$e_{11} = -\frac{b}{4\pi(1-\nu)}\frac{1}{\varrho}\sin\varphi\,(\cos^2\varphi - \sin^2\varphi) - \frac{b}{2\pi}\frac{1}{\varrho}\sin\varphi,$$

$$e_{22} = \frac{b}{4\pi(1-\nu)}\frac{1}{\varrho}\sin\varphi\,(3\cos^2\varphi + \sin^2\varphi) - \frac{b}{2\pi}\frac{1}{\varrho}\sin\varphi,$$

$$e_{12} = \frac{b}{4\pi(1-\nu)}\frac{1}{\varrho}\cos\varphi\,(\cos^2\varphi - \sin^2\varphi) \tag{21}$$

(ν is the Poisson constant).

Calculations of the relaxation time from Eqs (5), (6) and (21) proceed in the same way as for the case of a screw dislocation discussed above. The relaxation time τ_e due to the edge dislocation is finally given by

$$1/\tau_e = \frac{\mu_B b^2 L}{16\pi^2\hbar(1-\nu)^2 M_0^3 V}\int_0^{2\pi} d\psi \frac{\theta(u^2)}{u^2\sqrt{1+\chi^2[1-\sin^2\vartheta\sin^2\psi]^2}} \times \times\Big\{(1-2\nu)\,[2L_1 + 2L_2 - (L_1+L_2-2M_1)\sin^2\vartheta]\,[1-\mathrm{J}_0(\lambda u)]\sin(\psi-\eta) + \frac{L_1+L_2-2M_1}{2}\times \times[1-\mathrm{J}_0(\lambda u)]\sin^2\vartheta\sin(\psi+\eta) - \frac{L_1+L_2-2M_1}{2}[1-\mathrm{J}_0(\lambda u) - 2\mathrm{J}_2(\lambda u)]\sin^2\vartheta\sin(3\psi-\eta)\Big\}^2. \tag{22}$$

$\mathrm{J}_0, \mathrm{J}_2, \ldots$ denote the Bessel functions.

5. *Hexagonal ferromagnet*

For real ferromagnetic crystals the anisotropy of elastic properties may have significant influence on the relaxation time. Now we present an estimate of the role of elastic anisotropy.

We shall apply our results to hexagonal cobalt. We shall ignore the secondary effects of magnetic state of the crystal on its elastic properties *i.e.* we shall neglect stresses induced by magnetization and the dependence of elastic moduli on the magnetization.

Because of the complexity of calculations we consider a special case of an edge dislocation lying in the hexagonal plane and having the Burgers vector in the same plane. Let us take the x-axis of the coordinate system as parallel to the dislocation line, the y-axis parallel to the Burgers vector, b, and the z-axis, as previously, as the hexagonal axis. Now it is convenient to express the scattering matrix element W_k, Eq. (6), in terms of components of the stress tensor

$$W_k = \frac{2\mu_B}{M_0 V(\bar{C}_{13}^2 - C_{13}^2)} \{[(L_1+L_2) C_{33} - 2M_1 C_{13}] \int d\boldsymbol{r} \sigma_{yy} \exp (i\boldsymbol{k} \cdot \boldsymbol{r}) + \\ + [-(L_1+L_2) C_{13} + 2M_1 C_{11}] \int d\boldsymbol{r} \sigma_{zz} \exp (i\boldsymbol{k} \cdot \boldsymbol{r})\} \tag{23}$$

where C_{ij} are the elastic stiffness constants.

The stress tensor for the above specified dislocation is (see [7])

$$\sigma_{xx} = \frac{C_{12}C_{33} - C_{13}^2}{\bar{C}_{13}^2 - C_{13}^2} \sigma_{yy} + \frac{C_{11}C_{13} - C_{12}C_{13}}{\bar{C}_{13}^2 - C_{13}^2} \sigma_{zz},$$

$$\sigma_{yy} = -\frac{Kb\gamma}{2\pi} \frac{(\delta - 1)y^2 z + \gamma z^3}{(y^2 - \gamma z^2)^2 + \delta\gamma y^2 z^2}, \tag{24}$$

$$\sigma_{zz} = \frac{Kb}{2\pi} \frac{y^2 z - \gamma z^3}{(y^2 - \gamma z^2)^2 + \delta\gamma y^2 z^2},$$

$$\sigma_{yz} = \frac{Kb}{2\pi} \frac{y^3 - \gamma y z^2}{(y^2 - \gamma z^2)^2 + \delta\gamma y^2 z^2}.$$

The following notation is used:

$$K = (\bar{C}_{13} + C_{13})[C_{44}(\bar{C}_{13} - C_{13})/C_{33}(\bar{C}_{13} + C_{13} + 2C_{44})]^{1/2},$$

$$\bar{C}_{13} = (C_{11}\, C_{33})^{1/2},$$

$$\gamma = C_{11}/C_{33},$$

$$\delta = (\bar{C}_{13} - C_{13})(\bar{C}_{13} + C_{13} + 2C_{44})/(\bar{C}_{13} C_{44}). \tag{25}$$

Now we assume small departures from elastic isotropy. We expand the factor $\{(y^2 - \gamma z^2)^2 + \delta\gamma y^2 z^2\}^{-1}$ in Eqs (24) into powers of the anisotropy parameters $(\gamma^2 - 1)$ and $(\gamma\delta - 2\gamma - 2)$, retaining only the lowest order terms.

The matrix element W_k, calculated from Eq. (23) in the same way as that described in the preceding Section, takes the following form

$$W_k = \frac{-2iKb\mu_B}{M_0 V(C_{13}^2 - C_{13}^2)} \frac{\sin(k_x L/2)}{k_0 k_x} \times$$
$$\times\{A_1 \sin 3\Phi \, J_2 + (A_1 \sin 3\Phi + A_2 \sin \Phi)\times \tag{26}$$
$$\times(J_0 - 1)/2 + A_3 \sin 7\Phi \, J_6 + (A_3 \sin 7\Phi + A_4 \sin 5\Phi) J_4 +$$
$$+ (A_3 \sin 7\Phi + A_4 \sin 5\Phi + A_5 \sin 3\Phi) J_2 +$$
$$+ (A_3 \sin 7\Phi + A_4 \sin 5\Phi + A_5 \sin 3\Phi + A_6 \sin \Phi)(J_0 - 1)/2\}$$

k_x, k_0, Φ are the cylindrical coordinates of the wave vector $\boldsymbol{k}$ ($k_z = k_0 \sin \Phi$, $k_y = k_0 \cos \Phi$). The parameters A_i are defined by

$$A_1 = [(L_1+L_2)C_{33} - 2M_1 C_{13}][-\gamma+\delta-1]\gamma +$$
$$+ [-(L_1+L_2)C_{13} + 2M_1 C_{11}][-\gamma-1],$$
$$A_2 = [(L_1+L_2)C_{33} - 2M_1 C_{13}][-3\gamma-\delta+1]\gamma +$$
$$+ [-(L_1+L_2)C_{13} + 2M_1 C_{11}][-3\gamma+1],$$
$$A_3 = (1/16)\{[(L_1+L_2)C_{33} - 2M_1 C_{13}]\times$$
$$\times[\gamma^3 + \gamma^2(-2\delta+3) + \gamma(\delta^2 - 3\delta + 3) - \delta + 1]\gamma +$$
$$+ [-(L_1+L_2)C_{13} + 2M_1 C_{11}][\gamma^3 + \gamma^2(-\delta+3) + \gamma(-\delta+3) + 1]\},$$
$$A_4 = (1/16)\{[(L_1+L_2)C_{33} - 2M_1 C_{13}]\times$$
$$\times[7\gamma^3 + \gamma^2(-6\delta+9) + \gamma(-\delta^2 + 3\delta - 3) + 5\delta - 5]\gamma +$$
$$+ [-(L_1+L_2)C_{13} + 2M_1 C_{11}][7\gamma^3 + \gamma^2(-3\delta+9) + \gamma(\delta-3) - 5]\},$$
$$A_5 = (1/16)\{[(L_1+L_2)C_{33} - 2M_1 C_{13}]\times$$
$$\times[21\gamma^3 + \gamma^2(-2\delta+3) + \gamma(-3\delta^2 + 9\delta - 25) + 7\delta - 7]\gamma +$$
$$+ [-(L_1+L_2)C_{13} + 2M_1 C_{11}][21\gamma^3 + \gamma^2(-\delta+3) + \gamma(3\delta-25) - 7]\},$$
$$A_6 = (1/16)\{[(L_1+L_2)C_{33} - 2M_1 C_{13}]\times$$
$$\times[35\gamma^3 + \gamma^2(10\delta-15) + \gamma(3\delta^2 - 9\delta - 39) - 11\delta + 11]\gamma +$$
$$+ [-(L_1+L_2)C_{13} + 2M_1 C_{11}][35\gamma^3 + \gamma^2(5\delta-15) + \gamma(-3\delta-39) + 11]\}. \tag{27}$$

The relaxation time can be calculated by the method explained above

$$1/\tau = \frac{\mu_B L K^2 b^2}{4\pi^2 \hbar V M_0^3 (C_{13}^2 - C_{13}^2)^2} \int_0^{\pi/2} d\Phi \frac{\theta(u^2)}{u^2 \sqrt{1+\chi^2 \cos^4 \Phi}} \times$$

$$\times\{A_1 \sin 3\Phi\, J_2+(A_1 \sin 3\Phi+A_2 \sin \Phi)(J_0-1)/2+A_3 \sin 7\Phi\, J_6+$$
$$+(A_3 \sin 7\Phi+A_4 \sin 5\Phi)J_4+$$
$$+(A_3 \sin 7\Phi+A_4 \sin 5\Phi+A_5 \sin 3\Phi)J_2+$$
$$+(A_3 \sin 7\Phi+A_4 \sin 5\Phi+A_5 \sin 3\Phi+A_6 \sin \Phi)(J_0-1)/2\}^2. \quad (28)$$

The Bessel functions $J_n = J_n(\lambda u)$ depend here on the argument λu, where

$$\lambda = r_1 \sqrt{2\pi/(C_1 \cos^2 \Phi+C_2 \sin^2 \Phi)}, \quad (29)$$

$$u = \{-\cos^2 \Phi+[1/\chi^2+\cos^4 \Phi]^{1/2}+N_x+N_y-$$
$$-[(N_x-N_y)^2+1/\chi^2]^{1/2}\}^{1/2} \quad (30)$$

and χ is defined by Eq. (19).

Under the assumption of independent scattering from single dislocations we can estimate from the relaxation time τ the contribution to the ferromagnetic resonance line-width due to dislocations. For the special case of a system of parallel dislocations distributed with density n, the contribution to the ferromagnetic resonance line-width due to dislocations is given by (see [1])

$$\Delta H = (\hbar/2\mu_B\tau)(V/L)n. \quad (31)$$

It is interesting to compare relaxation time (or the corresponding line-width) calculated for elastically isotropic medium, Eq. (22), with the result obtained with elastic anisotropy taken into account. For definiteness we consider a crystal of cobalt cut in the form of a very long circular cylinder ($N_z = 0$, $N_x = N_y$). We take the following values of the material parameters (see [4] and [5]):

$$C_{11} = 3.071\times10^{12}\ \text{dyn/cm}^2, \qquad C_{12} = 1.650\times10^{12}\ \text{dyn/cm}^2,$$
$$C_{13} = 1.027\times10^{12}\ \text{dyn/cm}^2, \qquad C_{33} = 3.581\times10^{12}\ \text{dyn/cm}^2,$$
$$C_{44} = 0.755\times10^{12}\ \text{dyn/cm}^2, \qquad L_1 = 1.821\times10^{8}\ \text{dyn/cm}^2,$$
$$L_2 = 2.531\times10^{8}\ \text{dyn/cm}^2, \qquad M_1 = -2.500\times10^{8}\ \text{dyn/cm}^2,$$
$$M_0 = 1400\ \text{gauss};\ C_1 \approx C_2 = 8.507\times10^{-13}\ \text{cm}^2\ (C_1 = 2JSa^2/\mu_B M_0,$$

where for cobalt $JS = 160°k_B$ (k_B is the Boltzmann constant) and the distance between nearest neighbours $a = 2.5\times10^{-8}$ cm). For the value of the Burgers vector we take $b = 2.5\times10^{-8}$ cm. As a representative value of the parameter r_1 defining the range of the deformation field of the dislocation we take 0.64 μm; this gives $\lambda = 110$. We choose the value $\chi = 0.35$, corresponding to the ferromagnetic resonance frequency equal to 66 kMc/sec.

The results of calculations are conveniently expressed in terms of the line-width using Eq. (31). For the isotropic case we find from Eq. (22) $\Delta H = 0.239\times10^{-8}\, n$ (cm²Oe) whereas if the anisotropy of elastic properties is taken into account as in Eq. (28) we obtain $\Delta H = 0.157\times10^{-8}\, n$(cm²Oe).

APPENDIX A

Estimate of the influence of disclocation core on the line-width

In the present calculations as well as in [1] we neglected the contribution to the inverse relaxation time from the scattering on the region of high deformation along the dislocation line, the so-called dislocation core. The dislocation core is expected to be ineffective in scattering magnons because its diameter is much smaller than the typical wavelengths of magnons involved in the scattering.

In the present Appendix we shall justify this guess. We shall estimate the upper bound on the contribution of dislocation core to the inverse relaxation time. Inside the dislocation core the material is highly deformed. We take the simplest model into considerations: we assume that strains inside the core are so high that the material is no longer ferromagnetic. A region of vanishing magnetization inside the ferromagnetic material will scatter magnons because of the appearance of local demagnetizing fields by the mechanism proposed by Sparks, Loudon and Kittel [8] in another connection. Let the dislocation core be a circular cylinder of the length L and the radius r_0. We assume that the local magnetization is zero inside the cylinder. The stray magnetic field $\boldsymbol{H}(\boldsymbol{r})$ induced by the boundary by the local magnetization $\boldsymbol{M}(\boldsymbol{r})$ will be calculated by methods of magnetostatics. Let us expand tho local magnetization $\boldsymbol{M}(\boldsymbol{r})$ into the Fourier components $\boldsymbol{M}(\boldsymbol{r}) = \sum_k \boldsymbol{M}_k(\boldsymbol{r})$ where $\boldsymbol{M}_k(\boldsymbol{r}) = \boldsymbol{m}_k e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ and the components of the amplitude $\boldsymbol{m}_k$ can be inferred from Eq. (3), *e.g.*

$$m_k^x = (\mu_B M_0/V)^{1/2}(a_{-k}^+ + a_k), \ldots \tag{A.1}$$

The magnetic field $\boldsymbol{H}_k(\boldsymbol{r})$ induced by the mode $\boldsymbol{M}_k(\boldsymbol{r})$ can be calculated from the appropriate magnetostatic potential $\psi_k(\boldsymbol{r})$

$$\boldsymbol{H}_k(\boldsymbol{r}) = -\mathrm{grad}\ \psi_k(\boldsymbol{r}) \tag{A.2}$$

where

$$\psi_k(\boldsymbol{r}) = -\int_V d\boldsymbol{r}\, \frac{\mathrm{div}\,\boldsymbol{M}_k(\boldsymbol{r})}{|\boldsymbol{r}-\boldsymbol{r}'|} + \oint_\Sigma d\sigma\, \frac{\boldsymbol{M}_k(\boldsymbol{r})\cdot\boldsymbol{n}(\boldsymbol{r})}{|\boldsymbol{r}-\boldsymbol{r}'|}\,. \tag{A.3}$$

The first integral extends over the volume of the ferromagnetic sample, the second one over the surface of the dislocation core, $\boldsymbol{n}(\boldsymbol{r})$ is a unit vector normal to the surface.

The energy of the internal demagnetizing field $\boldsymbol{H}(\boldsymbol{r}) = \sum_k \boldsymbol{H}_k(\boldsymbol{r})$ is

$$\mathscr{H}_d \equiv -\tfrac{1}{2}\int_V d\boldsymbol{r}\boldsymbol{M}(\boldsymbol{r})\cdot\boldsymbol{H}(\boldsymbol{r}) = -\tfrac{1}{2}\sum_{kk'}\int_V d\boldsymbol{r}\boldsymbol{M}_k(\boldsymbol{r})\cdot\boldsymbol{H}_{k'}(\boldsymbol{r}). \tag{A.4}$$

From (A.3) it is evident that the energy $\mathscr{H}_d$ is quadratic in the magnetization mode amplitudes $\boldsymbol{m}_k$. Expressing $\boldsymbol{m}_k$ in terms of magnon operators we shall obtain from (A.4) explicit formula for the energy of magnon interaction due to the dislocation core.

The magnetostatic potential (A.3) can be easily calculated in the coordinate system (x_1, x_2, x_3) connected with the dislocation. Let the x_3-axis be directed along the dislocation line and x_1 and x-axes be parallel to each other. As usually, the (x, y, z) coordinate system is determined by the principal axes of the ellipsoidal sample, the z-axis being parallel to the applied magnetic field H_0. The angle between the dislocation line (x_3-axis) and the applied magnetic field (z-axis) is denoted by ϑ. We decompose the local magnetization $\boldsymbol{M}(\boldsymbol{r})$ in

Eq. (A.3) into plane waves and use the expansion

$$\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} = \sum_{m=-\infty}^{\infty} \int_0^{\infty} d\xi e^{im(\varphi-\varphi')} J_m(\xi\varrho) J_m(\xi\varrho') e^{-\xi|x_3-x_3'|} \tag{A.5}$$

(ϱ, φ, x_3 are the cylindrical coordinates of $\boldsymbol{r}$). After integration we obtain

$$\begin{aligned}\psi_k(\boldsymbol{r}) = & -2\pi r_0 m_k^{(1)} \sum_{n=-\infty}^{\infty} \{i^{n-1} J_{n-1}(k_0 r_0) e^{-i(n-1)\Phi} + \\ & + i^{n+1} J_{n+1}(k_0 r_0) e^{-i(n+1)\Phi}\} I_n(k_3 r_0) K_n(k_3 \varrho) e^{in\varphi} e^{ik_3 x_3} + \\ & + 2\pi i r_0 m_k^{(2)} \sum_{n=-\infty}^{\infty} \{i^{n-1} J_{n-1}(k_0 r_0) e^{-i(n-1)\Phi} - i^{n+1} J_{n+1}(k_0 r_0) e^{-i(n+1)\Phi}\} \times \\ & \times I_n(k_3 r_0) \dot{K}_n(k_3 \varrho) e^{in\varphi} e^{ik_3 x_3} + (1-\delta_{k,0}) \boldsymbol{m}_k \cdot \boldsymbol{k} \{-4\pi i e^{i\boldsymbol{k}\cdot\boldsymbol{r}}/k^2 + \\ & + 4\pi i r_0^2 \sum_{n=-\infty}^{\infty} i^n e^{-in\Phi} [\int_0^1 J_n(k_0 r_0 z) I_n(k_3 r_0 z) z dz] K_n(k_3 \varrho) e^{in\varphi} e^{ik_3 x_3}\}.\end{aligned} \tag{A.6}$$

Using (A.6) together with (A.2) in (A.4) and replacing $\boldsymbol{m}_k$ by appropriate combinations of magnon operators we obtain the magnon interaction Hamiltonian $\mathscr{H}_d$. From the complete expression for $\mathscr{H}_d$ we choose only those terms which describe direct scattering of the uniform magnon ($k = 0$) into the degenerate manifold of $k \neq 0$ magnons, *i.e.* terms of the form

$$\mathscr{H}_d = \sum_{k\neq 0} W_k^{\text{core}} a_k^+ a_0 + \text{h.c.} \tag{A.7}$$

These terms lead to the inverse relaxation time given by the formula (5).

In the present case, using the approximation $[\sin(k_3 L/2)/k_3]^2 \approx (\pi/2) L\delta(k_3)$ we have

$$\begin{aligned}|W_k^{\text{core}}|^2 = & \frac{8\pi^5 \mu_B^2 M_0^2 r_0^4 L}{V^2} \left\{\frac{3\sin^2\vartheta \cos 2\Phi}{r_0^2 k_0^2} \times \left[k_0 r_0 \left(\frac{r_0}{R} J_3(k_0 R) - J_3(k_0 r_0)\right) - \right.\right. \\ & \left.\left. -4\left(\frac{r_0^2}{R^2} J_2(k_0 R) - J_2(k_0 r_0)\right)\right] + (2-6\sin^2\vartheta \sin^2\Phi) \int_0^1 J_0(k_0 r_0 z) z dz\right\}^2 \delta(k_3).\end{aligned} \tag{A.8}$$

Here R is a parameter of the order of magnitude of linear dimensions of the sample (for a spherical shape of the sample R is the radius of the sample), we shall replace R by infinity in the final results.

The relaxation time due to the dislocation core is given by

$$\begin{aligned}1/\tau_{\text{core}} = & \frac{2\pi^3 \mu_B M_0 r_0^4 L}{\hbar V C_1} \int_0^{\pi/2} d\Phi \frac{\theta(u^2)}{\sqrt{1+\chi^2[1-\sin^2\vartheta \sin^2\Phi]^2}} \times \left\{\frac{3\sin^2\vartheta \cos 2\Phi}{\lambda_0^2 u^2} \times \right. \\ & \times \left[\lambda_0 u \left(\frac{r_0}{R} J_3(\Lambda u) - J_3(\lambda_0 u)\right) - 4\left(\frac{r_0^2}{R^2} J_2(\Lambda_0 u) - J_2(\lambda_0 u)\right)\right] + \\ & \left. + (2-6\sin^2\vartheta \sin^2\Phi) \int_0^1 J_0(\lambda_0 u z) z dz\right\}^2\end{aligned} \tag{A.9}$$

where

$$\Lambda = R\sqrt{2\pi/C_1},\ \lambda_0 = r_0\sqrt{2\pi/C_1}$$

and $u = u(\Phi)$ is defined by Eq. (18).

In the limit $R \to \infty$, $\lambda_0 u \ll 1$ and $\chi \ll 1$ the inverse relaxation time $1/\tau_{core}$ or the corresponding contribution to the resonance line-width ($\Delta H_{core} = (\hbar/2\mu_B\tau)(V/L)n$ is given by the simple expression

$$(\Delta H)_{core} = \frac{\pi^4 M_0 r_0^4 n}{8C_1}(4-12\sin^2\vartheta+27\sin^4\vartheta) \qquad \text{(A.10a)}$$

for $\Delta = N_z - \pi(1-N_z)^2\ M_0/(H_0-4\pi N_z M_0) < 0$

or

$$(\Delta H)_{core} = \frac{\pi^3 M_0 r_0^4 n}{4C_1}\,\theta\,(\vartheta-\vartheta_0)\{(4-12\sin^2\vartheta+27\sin^4\vartheta)\times$$

$$\times[\pi/2 - \arcsin(\sqrt{\Delta}/\sin\vartheta)] - 2\sqrt{\Delta}\sqrt{\sin^2\vartheta-\Delta}\,(12-18\Delta-9\sin^2\vartheta)\} \qquad \text{(A.10b)}$$

for $\Delta > 0$, where $\vartheta_0 = \arcsin\sqrt{\Delta}$.

$(\Delta H)_{core}$ strongly depends on the radius r_0 of the dislocation core. Taking the values of parameters appropriate for cobalt and taking r_0 of the order of magnitude of a few lattice constants, say $r_0 = 10^{-7}$ cm, we obtain from (A.10a), for the case $N_z = 0$, $\vartheta = \pi/2$, $(\Delta H)_{core} = 0.38\times10^{-10}n$ (cm²Oe) which is completely negligible as compared with the contribution from the extended deformation field of the dislocation.

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