INFLUENCE OF ELECTRON-SPIN EXCHANGE INTERACTION IN FERROMAGNETIC SEMICONDUCTORS ON DYNAMICS AND TRANSPORT OF CHARGE CARRIERS

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The problem of the dynamics and transport of narrow-band charge carriers in ferromagnetic (ferrimagnetic) semiconductors at low temperatures ($\ll T_C$) is considered.

It is shown that electron-magnon coupling is here not strong and does not form small magnetic polarons due to prevailing "static" $s-d$ exchange interaction.

The influence of electron-magnon coupling on the small polaron transport is discussed in details. The contribution of the magnon scattering to the transport is considered as well as its competition with other scattering mechanisms.

The specific effect for high-frequency or non-ohmic small polaron conduction is obtained which is associated with one-magnon spin-flip hops. The conditions of observation of the effect are discussed. The observation of the effect provides a fairly precise determination of the parameter $|A_{sd}|$ of $s-d$ exchange coupling.

Some additional questions are discussed in the concluding remarks.

Introduction

In the last decade, a consistent detailed transport theory of the small polaron was developed for low-mobility (nonmagnetic) crystalline semiconductors [1, 2, 3]. The fundamental characteristics of the small polaron dynamics and transport are as follows:

1) occurrence of radically different transport mechanisms — aither band type conduction at low temperature T (lower than a characteristic T_{xx}) and frequencies $\omega(<\omega_{xx})$ of the electric field (the band regime) or hopping at high $T\ (>T_{xx})$ and/or $\omega\ (>\omega_{xx})$ (the hopping regime), the T- and ω-dependences of the transport coefficients being essentially different for the two types of transport;

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2) strong narrowing of the small polaron conduction band (in the band regime) as compared with the initial electron one, what is due to the strong electron-phonon (*EP*) coupling.

Little attention has hitherto been paid to the effect of exchange interactions on the small polaron conduction in magnetic (ferro-, antiferro-, ferri- and paramagnetic) crystals (see [2], [7]). On the other hand, the basic concepts and methods of the small polaron theory [1, 2, 3] were applied and extended in some recent papers [4, 5] to the study of the dynamics and transport of electrons in ferromagnetic crystalline semiconductors[1] at low $T \ll T_C$ (T_C — is Curie point); assuming that electron-magnon (*EM*) coupling can be strong, the two above-mentioned basic features of the small polaron were found for a new quasi-particle named *magnetic polaron* (an electron "dressed" by a magnon "cloud"), though *EP* coupling was neglected. The $s-d$ exchange model [6] used and strong *EM* coupling was treated as caused by large values of $|A_{sd}|/I$, with A_{sd} and $I\,(>0)$ denoting the integral of "$s-d$" and "$d-d$" exchange integrals. At the same time it appears that some factors should be additionally taken into account in the analysis of the problem, such as the basic "static" part of the $s-d$ exchange field which can effectively "reduce" the *EM* coupling.

In the present paper, two problems are in fact studied:

a) Is the *EM* coupling sufficiently strong to cause the autolocalization of the electron (*i. e.* to create the strong coupling "magnetic polaron" of the small polaron type) in a ferromagnetic semiconductor at $T \ll T_C$ and $|A_{sd}|/I \gg 1$?

b) What is the effect of *EM* coupling on the dynamics and transport (and some optical properties) of the small polarons in such crystals at $T \ll T_C$?

It is shown (Sec. 1), for $2S > 1$ at least (S is the magnitude of the localized spin in the Heisenberg model), that the "static" part of the $s-d$ exchange field leads to *EM* coupling and to the respective band narrowing; these effects are not strong. It can be expected, therefore, that the transport of such "magnetic polarons" is of a standard band type (rather than of small polaron type). On the other hand (Sec. 2), the *EM* coupling causes an additional relaxation mechanism for small polarons in the band regime which can dominate with characteristic T-dependences of the transport coefficient under some conditions. Finally (Sec. 3), the high-frequency small polaron conduction (which is due to hopping processes at low T as well) shows a rather narrow infrared absorption peak in (practice, for frequencies lower than the basic small polaron Gaussian infrared absorption band, see [1c]). This peak is a characteristic of both the small polaron and the $s-d$ exchange and has a certain structure — its experimental observation leads to the determination of the parameter $|A_{sd}|$.

1. Motion of the electron and small polaron in a ferromagnetic semiconductor (spin-wave region)

The situation is considered when the charge carrier electrons or holes of a crystalline ferromagnetic semiconductor constitute a nondegenerate gas with Boltzmann equilibrium distribution and sufficiently low concentration $N_c \equiv N_c(T)$, the interaction between those

[1] Actually, it is expected that the consideration and basic results of the paper concern the case of ferrimagnetic semiconductors as well.

charge carriers being neglected. Then our study deals with some aspects of the dynamics of an electron moving in a narrow conduction band of width $D_e \approx 2z\Delta_e$ (z is the number of nearest neighbours; Δ_e the typical Bloch resonance integral) and interacting with boson elementary excitations: phonons (ph) (optical, acoustical) and magnons. The spin-wave region, at low $T \ll T_C$ and relatively small (average) magnon number $N_m/N \ll 1$, is implied here. N being the number of elementary cells in the basic crystal volume[2].

The hamiltonian of the electron-boson system[3] takes into account the conservation of the z-component of the total spin and can be presented, using the $s-d$ model [6], as follows (see [1, 2])[4]:

$$\mathscr{H} = \mathscr{H}_e + \mathscr{H}_{eb} + \mathscr{H}_b \tag{1}$$

with the operators (in what follows, $\hbar \equiv 1$)

$$\mathscr{H}_e = \frac{\boldsymbol{p}^2}{2m} + V(\boldsymbol{r}) + A_1(\boldsymbol{r})\sigma^z, \qquad \mathscr{H}_b = \sum_{l=0,1,2\ldots} \sum_f \omega_{fl} b_{fl}^+ b_{fl},$$

$$\mathscr{H}_{eb} = \frac{1}{\sqrt{N}} \sum_{l=0,1,2\ldots} \sum_f \{\hat{V}_{fl}(\boldsymbol{r}) e^{i\boldsymbol{f}\cdot\boldsymbol{r}} b_{fl} + \text{h.c.}\} \equiv \mathscr{H}_{ep} + \mathscr{H}_{EM}. \tag{2}$$

describing, respectively, the electron in the static lattice field and the "static" part $A_1(\boldsymbol{r})$ of the $s-d$ exchange field ($A_1(\boldsymbol{r})\ \sigma^z$), the "free" bosons - phonons (with $l \equiv j$ ($= 1, 2, 3, \ldots$) being the branch number) and magnons ($l \equiv 0$) — with eigen-frequencies ω_{fj} and $\Omega_f \equiv \omega_{f,l\equiv 0}$, and the linear electron-boson coupling, *i. e.* the electron-phonon (EP), $\mathscr{H}_{ep}$, and electron-magnon (EM), $\mathscr{H}_{EM}$, ones with "coupling" coefficients $V_{fj}(\boldsymbol{r})$ (c — numbers) and spin operators $\hat{V}_{f,l\equiv 0} \equiv A_f(\boldsymbol{r})\sigma^+$. In (1) and (2) the following notation (apart from standard symbols), is introduced: electron spin operators $\sigma^+ = \sigma^x + i\sigma^y = (\sigma^-)^+$ with $\sigma^z\ |\sigma\rangle = \sigma|\sigma\rangle$ and spin number $\sigma = \frac{1}{2}$, $\sigma = -\frac{1}{2}$ for $|\sigma\rangle \equiv |\uparrow\rangle$ and $|\sigma\rangle \equiv |\downarrow\rangle$ states, respectively; b_{fl}^+ and b_{fl}—Bose operators for phonons (b_{fj}^+; b_{fj}) and magnons ($\beta_f^+ \equiv \beta_{f,l\equiv 0}^+$; β_f). Here, by definition, in the ground state all localized spins are directed along the negative z-axis

$$A_1(\boldsymbol{r}) = 2S \sum_{\boldsymbol{s}} A(\boldsymbol{r}-\boldsymbol{s}); \; A_f(\boldsymbol{r}) = -\sqrt{2S} \sum_{\boldsymbol{s}} A(\boldsymbol{r}-\boldsymbol{s})\, e^{-i\boldsymbol{f}\cdot(\boldsymbol{r}-\boldsymbol{s})}. \tag{3}$$

where $A\ (\boldsymbol{r}-\boldsymbol{s})$ is the integral of $s-d$ exchange of the electron ($\boldsymbol{r}$) with $\boldsymbol{s}$ denoting the vector of the crystal cell or of the site of appropriate type.

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[2] In what follows we can neglect the relatively small effect of the effective anisotropic integral magnetic field and spin-orbital coupling.

[3] In fact, s-conduction electron stands for an electron (hole) in a narrow band of the d-type. The orbital degeneracy and possible overlapping of such bands, for simplicity, are neglected. This does not change results qualitatively and only simplify the estimates. In the concrete estimates, the case of cubic crystals is mainly implied.

[4] Operator $\mathscr{H}_{EM}^{(1)}$ of the EM coupling is the first linear term of the expansion of the operator H_{EM} in small N_m/N; the next bilinear term has the form

$$\mathscr{H}_{\text{int}}^{(2)} = \left(\frac{2}{NS}\right)^{1/2} \sum A_{f-f'}(\boldsymbol{r}) e^{i(\boldsymbol{f}-\boldsymbol{f}')\cdot\boldsymbol{r}} \beta_f^+ \beta_{f'} \sigma^z$$

and leads to renormalization of the magnon frequencies.

Let us now take into account a certain similarity of the structure of the operators $\mathscr{H}_{eb}$ and $\mathscr{H}_{EM}$ in (1) and (2). Then, being interested in the problems *a*) and *b*) as formulated in the *Introduction*, we may apply to the study of eigenvalues and eigenstates of $\mathscr{H}$ the well-known procedure of the small polaron theory [1, 2]. New "renormalized" quasi-particles are basically decoupled by the appropriate unitary transformation of the hamiltonian $\mathscr{H}$ and constitute the unperturbed system ($\mathscr{H}_0$) — the localized (in the $\mathbf{s}^{\text{th}}$ cell) small polaron (including the magnon "cloud" as well) and the "new" bosons (phonons and magnons) with displaced oscillator centres (and the same frequencies, see footnote 4), the perturbation $\mathscr{H}_1$ being the "residual" relatively weak polaron-boson coupling which includes inter-site transitions ($\mathbf{s} \to \mathbf{s}' \neq \mathbf{s}$) of the polaron.

The appropriate unitary operator T^s, transforming $\mathscr{H} \to \tilde{\mathscr{H}} = T^s \mathscr{H} T^{s+}$, can be chosen (see *e. g.* [1]) as follows (the structure of $\mathscr{H}_{EM}$ in (2) is taken into account):

$$T^s = T^s_{ph} T^s_M = \exp(U^s) \text{ with}$$

$$U^s \equiv \sum_{l=0,1\dots} U^s_l \equiv \sum_l \sum_f \frac{1}{\sqrt{N}} (X^s_{fl} b_{fl} - \text{h. c.}), \tag{4}$$

where

$$T^s_{ph} = \prod_j \exp(U^s_j) \text{ and } T^s_M = \exp(U^s_{l=0});$$

$$X^s_{fl} = X_{fl} e^{if\cdot s} \equiv X^{s=0}_{fl} e^{if\cdot s};$$

$$X_{fj} \equiv X_{f,l\equiv j} = -\frac{1}{\omega_{fj}} \langle \mathbf{s}=0 | V_{fj}(\mathbf{r})\, e^{if\cdot r} | \mathbf{s}=0 \rangle,$$

$$X_{f(M)} \equiv X_{f,l\equiv 0} = \Phi_f \sigma^-, \quad \Phi_f \approx \frac{A}{2A_{sd}S - \Omega_f} \tag{5}$$

with $A_f = -\sqrt{2S}\rangle\ \mathbf{s}=0|A^*_f(\mathbf{r})\ e^{if\cdot r}|0\rangle$. In the long-wave approximation ($|f|\,a \ll 1$) the following estimates are used:

$$X_{f(ac)} \equiv X(\omega^{ac}_f) \approx X^{ac}_0 \left(\frac{\omega_D}{\omega_f}\right)^{1/2}$$

$$\text{with } \omega^{ac}_f \approx u_0 |\mathbf{f}| \text{ and } \omega_D \approx u_0 f_D, \tag{6}$$

for (longitudinal) acoustic phonons with ω_D and f_D the Debye frequency and wave number;

$$\Omega_f = 2zIS \left(1 - \frac{1}{z} \sum_{\delta} e^{if\cdot\delta}\right) \approx \frac{f^2}{2\mu} \text{ with } \mu^{-1} = zISa^2 \tag{7}$$

$$A_f \approx -\sqrt{2S}\, A_{sd} \text{ with } A_{sd} \equiv \left\langle \mathbf{s}=0 \left| \frac{A_1(\mathbf{r})}{2S} \right| \mathbf{s}=0 \right\rangle (\gtrless 0),$$

and $\boldsymbol{\delta}$ is the vector of the nearest neighbour. In (4)–(7): A_{sd} is the effective $s-d$ exchange parameter; $|\mathbf{s}\rangle \equiv \varphi_s(\mathbf{r}) = \varphi(\mathbf{r}-\mathbf{s})$ is the orthonormalized, localized (in the $\mathbf{s}^{\text{th}}$ cell), Wannier function for the conduction band. The relation (4) for phonons is exact, whereas (5) (for magnons) is only approximate, with accuracy to correction of the order $\sim N_m/N$

($\ll 1$) and $(2S)^{-1}$ (< 1), for $2S > 1$. In other words, the diagonalization of the basic "one-site" operator $h^s \equiv \langle \mathbf{s}|\mathcal{H}|\mathbf{s}\rangle$ is achieved exactly for *EP* coupling, but approximately (with the accuracy noted) for *EM* coupling. This difference is associated with the operator structure of the *EM* coupling "coefficient" $V_{f,l=0}(\mathbf{r})$ in (2), which corresponds to a conservation of the z-component of the total spin as noted above. Therefore, $T^s_M T^{s'+}_M \neq T^{s-s'}_M$ while $T^s_{ph} \times T^{s'+}_{ph} = T^{s-s'}_{ph}$ (see, *e. g.*, [1]). The relation (5) corresponds to the following simultaneous transformations:

$$\beta_f^+ \to \tilde{\beta}_f^+ = T^s_f \beta_f^+ T^{s+} \approx \beta_f^+ - [\hat{U}^s,\ \beta_f^+] = \beta_f^+ + \frac{1}{\sqrt{N}} \Phi_f^* e^{-if\cdot s}\sigma^+,$$

$$\sigma^+ \to \tilde{\sigma}^+ \approx \sigma^+ + \frac{2}{\sqrt{N}} \sum_f \Phi_f e^{if\cdot s}\beta_f^+ \sigma^z + \frac{1}{N}\sum_f |\Phi_f|^2 \sigma^z\sigma^+,$$

$$\sigma^z \to \tilde{\sigma}^z \approx \sigma^z - \frac{1}{\sqrt{N}} \sum_f \{\Phi_f e^{if\cdot s}\beta_f^+ \sigma^- + \text{h. c.}\} - \frac{1}{N}\sum_f |\Phi_f|^2 \sigma^+\sigma^-, \tag{8}$$

and (4) to the exact transformation $b_{fj} \to \tilde{b}_{fj} = b_{fj} - N^{-\frac{1}{2}} X^*_{fj} e^{-if\cdot s}$ (displacement of boson oscillator centres)[5].

The eigenvalues $\varepsilon_{sn\sigma}$ and states $|sn\sigma\rangle$ of the transformed basic operator

$$\tilde{h}^s = T^s \langle \mathbf{s}|\mathcal{H}|\mathbf{s}\rangle T^{s+} \equiv \langle \mathbf{s}|\hat{\mathcal{H}}|\mathbf{s}\rangle$$

$$= h^s_p + h^s_b \approx 2A_{sd} S\sigma^z + \varepsilon_s - \delta\varepsilon + Q\sigma^+\sigma^- + \sum_{f,l} \omega_{fl} b^+_{fl} b_{fl} \tag{9}$$

can be given in the form:

$$\varepsilon_{sn\sigma} = \bar{\varepsilon}_s + \varepsilon_\sigma + \varepsilon_n \text{ and } |sn\sigma\rangle = |\mathbf{s}\rangle\, |n^{(s)}\sigma\rangle \equiv |\mathbf{s}\rangle\, T^{s+} |n\sigma)$$

where

$$|n\sigma) = \prod_{fl} \frac{(b^+_{fl} e^{-if\cdot s})^{N_{fl}}}{\sqrt{N_{fl}!}} |0\rangle\, |\sigma\rangle \text{ and } |0\rangle \text{ is boson vacuum;} \tag{10}$$

$\bar{\varepsilon}_s = \varepsilon_s - \delta\varepsilon$ and $\delta\varepsilon = \frac{1}{N}\sum |X_{fj}|^2\, \omega_{fj}$ is the standard polaron shift of the electron "site" level $\varepsilon_s = \left\langle \mathbf{s} \left| \frac{\mathbf{p}^2}{2m} + V(\mathbf{r}) \right| \mathbf{s} \right\rangle$, so the "site" polaron level $\bar{\varepsilon}_s$ can be chosen $\varepsilon_s = \bar{\varepsilon} = 0$ in an ideal lattice; $\varepsilon_n = \sum_{fl} \omega_{fl} N_{fl}$ is the boson energy with $n \equiv (n_{ph};\, n_M) = (\ldots N_{fl} \ldots)$, $n_{ph} \equiv (\ldots N_{fj} \ldots)$ and $n_M \equiv (\ldots . N_{f,l=0} \ldots .)$. In (9) and (10),

$$Q = \frac{1}{N}\sum_f \frac{|A_f|^2}{2A_{sd}S - \Omega_f} \approx \frac{1}{N}\sum_f \frac{|A_f|^2}{2A_{sd}S} \approx A_{sd} \tag{11}$$

[5] Practically the *EM* coupling diagonalization by means of T^s_M is an expansion in $N_m/N \ll 1$ and $(2S)^{-1} < 1$. Terms omitted in (8) contain products of 2, 3 *etc.* operators β_f^+ and β_f or additional factors $\sim (2S)^{-1}$. The diagonalization in the next approximation lead to a correction in (5) of the form $\delta\Phi\tilde{\sigma}^z$ at $\delta\Phi \approx \sum |\Phi_{f'}|^2 \frac{A_f}{2A_{sd}S - \Omega_f}$, this being small, $|\delta\Phi/\Phi| \sim (2S)^{-1} < 1$ for $2S > 1$.

and

$$\varepsilon_\sigma = A_{sd}(S+1)\,\delta_{\sigma,\uparrow} - A_{sd}S\delta_{\sigma,\downarrow} \tag{12}$$

describes the split of the "site" energy level of the polaron, including both the contribution of the "static" part of the $s-d$ exchange field ($A_{sd}S$) and the asymmetric shift[6] $A_{sd}\delta_{\sigma,\uparrow}(\neq 0$ for $\sigma = +\frac{1}{2})$ due to EM coupling. This asymmetry can be interpreted in accordance with the study of the ground state of the system in [8, 9] since only the electron with spin $\sigma = +\frac{1}{2}$ can excite (at $T = 0°\mathrm{K}$) a magnon and become coupled with it due to the EM coupling conserving z-component of the total spin. The split of the $\sigma = +\frac{1}{2}$ and $\sigma = -\frac{1}{2}$ levels is

$$G = |\varepsilon_{\sigma=-\frac{1}{2}} - \varepsilon_{\sigma=\frac{1}{2}}| = |A_{sd}|(2S+1) \tag{13}$$

so that

$$\frac{G}{kT} \gg \frac{G}{I(2S+1)} \approx \frac{G}{kT_C} > 1 \text{ for } T \ll T_C \text{ and } \frac{|A_{sd}|}{I} \gg 1. \tag{14}$$

The parameters

$$\Phi_{EM} = \frac{1}{N}\sum |\Phi_f|^2 \simeq (2S)^{-1} \text{ and } |\Phi_f|^2 \approx (2S)^{-1} \tag{15}$$

describing EM coupling in this model are small for $2S < 1$, $\Phi_{EM} < 1$ and $|\Phi_f| < 1$, and in this meaning EM coupling is not strong. From this fact it follows that for $2S > 1$ the EM coupling is not an important factor of electron "localization" and the creation of a small magnetic polaron, as is the case for strong EP coupling. The magnitude of the EM shift $|A_{sd}|$ can be sufficiently large. In contrast to the case of small polarons, the simple connection between the shift of the level and the coupling parameter does not appear here even if $|A_{sd}| > \Delta_e$. We will see this also when considering the narrowing of the polaron bandwidth (as compared with that of the initial electron band). The reason for this difference seems to be clear. The contribution of the static part of the $s-d$ exchange coupling $A_1(\mathbf{r})\,\sigma^z$ is considerably larger than the dynamical one in the spin-wave regime (for $2S > 1$ at least). When other factors of electron "localization" (strong EP coupling or statistical dispersion of the local levels, ε_s, in disordered systems) are not operating, the dynamics and kinetics of the charge carriers ("magnetic polarons") should be rather described by the standard band theory and Bloch transport equation with their "weak" boson scattering (see *e. g.* [12]), and not by the procedure discussed above, in which charge carriers are considered (in basic approximation) to be localized (9) and the perturbation is determined by the "inter-site" operator[7]

$$\hat{h}_{ss'} \equiv \langle \mathbf{s}|T^s\mathscr{H}T^{s'+}|\mathbf{s}'\rangle_{s\neq s'} \approx \Delta_e(\delta)T^sT^{(s+\delta)+}$$
$$\text{at } \mathbf{s}' = \mathbf{s}+\boldsymbol{\delta}, \tag{16}$$

Assuming further that EP coupling is sufficiently strong so that small-polaron criteria are fulfilled [1, 2] we will consider this case only) and investigating the influence of EM coupling on small polaron transport at low T ($\ll T_C$) in an ideal crystal, we apply the pro-

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[6] Temperature-dependent corrections to the ground EM shift are small $\sim N_m/N \ll 1$ or $\sim(N_m/N)S^{-1} \ll 1$,

[7] As in [1, 2] we neglect the comparatively small correction from $s-d$ exchange coupling to this operator as compared with that of the lattice field $\Delta_e(\boldsymbol{\delta}) \equiv \langle \mathbf{s}|V(\mathbf{r})|\mathbf{s}+\boldsymbol{\delta}\rangle$.

cedure discussed above and separate the unperturbed ($\mathscr{H}_0$) and perturbation ($\mathscr{H}_1$) hamiltonians in $\mathscr{H}$ in the basis of orthonormalized band-type (non-localized) functions [1, 2]

$$|\boldsymbol{k}n\sigma\rangle = \frac{1}{\sqrt{N}} \sum_{s} e^{-i\boldsymbol{k}\cdot\boldsymbol{s}} |\boldsymbol{s}n\sigma\rangle = \frac{1}{\sqrt{N}} \sum_{s} e^{-i\boldsymbol{k}\cdot\boldsymbol{s}} |\boldsymbol{s}\rangle\, T^{s+} |n\sigma\rangle, \tag{17}$$

Here (in connection with the choice of functions $|n\sigma)$ in (10)), $\boldsymbol{k}$ is the wave number of the small polaron in the reduced-zone scheme, which changes when n varies (the small contribution of the U-processes is neglected here for $T \ll T_C$). In this basis (see [1])

$$\mathscr{H}_0 = \hat{D}\mathscr{H} \text{ and } \mathscr{H}_1 = \hat{Y}\mathscr{H}, \tag{18}$$

where operators $\hat{D}$ and $\hat{Y}$ describe the nondissipative ($\hat{D}$) and dissipative ($\hat{Y}$) parts of the polaron motion, without and with a change of phonon numbers (N_{fj}) and/or spin and magnon numbers, respectively[8].

In Eq. (16):

$$\langle \boldsymbol{k}n\sigma|\hat{D}\mathscr{H}|\boldsymbol{k}'n'\sigma'\rangle \equiv \langle \boldsymbol{k}n\sigma|\mathscr{H}|\boldsymbol{k}n\sigma\rangle \delta_{\boldsymbol{k}\boldsymbol{k}'}\delta_{nn'}\delta_{\sigma\sigma'} \text{ and}$$

$$\langle \boldsymbol{k}n\sigma|\mathscr{H}|\boldsymbol{k}n\sigma\rangle \equiv \varepsilon_{n\sigma} + \mathscr{E}_{n\sigma}(\boldsymbol{k}). \tag{19}$$

$$\langle \boldsymbol{k}n\sigma|\hat{Y}\mathscr{H}|\boldsymbol{k}'n'\sigma'\rangle \equiv \langle \boldsymbol{k}n\sigma|\mathscr{H}|\boldsymbol{k}'n'\sigma'\rangle (1-\delta_{nn'}\delta_{\sigma\sigma'}) \text{ (see (16))}$$

and

$$\langle \boldsymbol{k}n\sigma|\mathscr{H}|\boldsymbol{k}'n'\sigma'\rangle = \frac{1}{N} \sum_{s} \sum_{\boldsymbol{\delta}} e^{-i(\boldsymbol{k}-\boldsymbol{k}')\boldsymbol{s}} \Delta_e(\boldsymbol{\delta}) e^{-i\boldsymbol{k}'\cdot\boldsymbol{\delta}} (n\sigma|T^{s}T^{s'+}|n'\sigma'),$$

$$\delta_{nn'} \equiv \prod_{f} \delta_{N_f, N_f'}.$$

For an equilibrium system of bosons,

$$\mathscr{E}_{n\sigma}(\boldsymbol{k}) \rightarrow \mathscr{E}_{\sigma}(\boldsymbol{k}) = \sum_{n} \varrho_b(n)\, \mathscr{E}_{n\sigma}(\boldsymbol{k}) = \sum_{\boldsymbol{\delta}} e^{-i\boldsymbol{k}\cdot\boldsymbol{\delta}} \Delta_e(\delta) Q_\sigma(\delta), \tag{20}$$

so that the width of the "averaged" polaron subbands ($\sigma = \pm\frac{1}{2}$)

$$D_p^\sigma \approx 2z\, \Delta_p^\sigma \approx 2z\Delta_e Q_\sigma \approx D_e Q_\sigma \tag{21}$$

with the narrowing factor

$$Q_\sigma(\delta) = \sum_{n} \varrho_b(n) \langle n^{(\delta)}\sigma|n^{(0)}\sigma\rangle = \text{tr}\, [\hat{\varrho}_b T^{(\delta)} T^{(0)+}] < 1. \tag{22}$$

Here,

$$\hat{\varrho}_b = \exp(\beta\Omega_0 - \beta \sum_{f,l} \omega_{fl} b_{fl}^{+} b_{fl}),$$

$$\varrho_b(n) \equiv (n|\hat{\varrho}_b|n) = \exp(\beta F_0 - \beta \sum \omega_{fl} N_{fl})$$

the trace is taken over boson variables; $\beta = (kT)^{-1}$. The matrix elements $(n\sigma|T^sT^{s'+}|n'\sigma')$ in (19) are calculated using the relation

$$T^sT^{s'+} = T_{ph}^{s-s'} T_M^s T_M^{s'+} \text{ with } T_M^{s-s'} = \exp(\sum X_{fj}^{ss'} b_{fj} - \text{h. c.}) \tag{23}$$

[8] The change of definition of the operator $\hat{Y}$ (and, consequently of $\hat{D}$) as compared with "pure" EP coupling case is associated with the fact that the z-component of the total spin is conserved.

and expanding $T^s_M T^{s'+}_M$ into powers of N_m/N ($\ll 1$) and $(2S)^{-1}$ (< 1) (with the accuracy corresponding to (8)):

$$T^s_M T^{s'+}_M \approx D^{\uparrow\uparrow}_{ss'}\sigma^+\sigma^- + D^{\downarrow\downarrow}_{ss'}\sigma^-\sigma^+ + D^{\downarrow\uparrow}_{ss'}\sigma^- + D^{\uparrow\downarrow}_{ss'}\sigma^+ \,. \tag{24}$$

In (24) the identity $\sigma^+\sigma^- + \sigma^-\sigma^+ = 1$ is used and (*cf.* [5]) operators $D^{\uparrow\uparrow}_{ss'}(D^{\downarrow\downarrow}_{ss'})$ and $D^{\downarrow\uparrow}_{ss'}(D^{\uparrow\downarrow}_{ss'})$ describe transitions without and with spin flip, respectively:

$$D^{\uparrow\uparrow}_{ss'} = \tfrac{1}{2}\{\mathrm{ch} \sum_f (\Phi^{*ss'}_f \beta_f + \Phi^{s'}_f \beta^+_f) + \mathrm{ch} \sum_f (\Phi^{s's}_f \beta^+_f + \Phi^{*s}_f \beta_f)\} \times$$
$$\times \exp\{-\tfrac{1}{2} \sum_f [|\Phi_f|^2 - \Phi^{s'}_f \Phi^{*s}_f]\} = (D^{\downarrow\downarrow}_{ss'})^+ \exp\{-\sum_f [|\Phi_f|^2 - \Phi^{s'}_f \Phi^{*s}_f]\} \tag{25}$$

(with an accuracy up to x^4 for the expansion of ch x in x) so that, with (15) and the accuracy to corrections $\sim (2S)^{-1} < 1$,

$$D^{\uparrow\uparrow}_{ss'} \approx -(D^{\downarrow\downarrow}_{ss'})^+ \approx \tfrac{1}{2}\{\mathrm{ch} \sum_f (\Phi^{*ss'}_f \beta_f + \Phi^{s'}_f \beta^+_f) + \mathrm{ch} \sum_f (\Phi^{s's}_f \beta^+_f + \Phi^{*s}_f \beta_f)\}, \tag{25'}$$

and

$$D^{\downarrow\uparrow}_{ss'} \approx -(D^{\uparrow\downarrow}_{ss'})^+ \approx \sum_f (\Phi^{ss'}_f - \Phi^s_f)\beta^+_f \tag{26}$$

with accuracy to terms $\sim (2S)^{-1}$, $\tfrac{1}{2} \sum_{f'} [|\Phi_{f'}|^2 - \Phi^{*s}_{f'} \Phi^{s'}_{f'}] \Phi^{*s}_f \beta_f$ and terms $\sim (2S)^{-1} N_m/N$;

$$X^{s's}_{fj} \equiv X_{fj} e^{if\cdot s}(e^{if\cdot\delta} - 1) \text{ and } \Phi^{s's}_f \equiv \Phi^{s'}_f - \Phi^s_f = \Phi_f e^{if\cdot s}(e^{if\cdot\delta} - 1) \tag{27}$$

with $\boldsymbol{\delta} \equiv \mathbf{s}' - \mathbf{s}$.

Average quantities, tr $\varrho_b \Lambda_1 \Lambda_2 \ldots$ (like $\Phi_\sigma(\delta)$) with Λ denoting unitary operators T^s or T^{s+}, are calculated using expansions of the type

$$\Lambda_1 \equiv \exp\left(\frac{1}{\sqrt{N}} \hat{U}_1\right) \approx 1 + \frac{\hat{U}_1}{\sqrt{N}} + \frac{\hat{U}_1^2}{2N} + \ldots$$

and retaining the finite (at $N \to \infty$) contribution of the form $N^{-1} \sum_f \chi(f)$. Then (exactly for *EP* coupling but approximately for *EM* coupling in accordance with (8), (24)–(26))

$$\langle \Lambda_1 \Lambda_2 \rangle_b \equiv \mathrm{tr}\, \varrho_b \Lambda_1 \Lambda_2 \approx 1 + \frac{1}{2N} \langle U_1^2 + U_2^2 + 2U_1 U_2 \rangle_b$$
$$\approx \exp\left\{\frac{1}{2N} \langle U_1^2 + U_2^2 + 2U_1 U_2 \rangle_b\right\} \tag{28}$$

and similarly for $\langle \Lambda_1 \Lambda_2 \Lambda_3 \rangle_b$ *etc.* In particular,

$$Q^\sigma(\boldsymbol{\delta}) = \frac{\Delta^\sigma_p(\boldsymbol{\delta})}{\Delta_e(\boldsymbol{\delta})} = e^{-\Phi^\sigma_\delta} = Q_{ph}(\boldsymbol{\delta}) Q^\sigma_M(\boldsymbol{\delta}) \tag{29}$$

with

$$Q_{ph}(\delta) = e^{-\Phi_\delta} \text{ and } Q_M(\delta) = e^{-\Phi^{M\sigma}_\delta}$$

Here

$$\Phi^\sigma_\delta = \frac{1}{2N} \langle (U^{s=0})^2 + ((U^{s=\delta})^+)^2 + 2U^{s=0}(U^{s=\delta})^+ \rangle_b = \Phi_\delta + \Phi^{M\sigma}_\delta, \tag{30a}$$

and

$$\Phi_\delta = \frac{1}{N}\sum |X_{fj}|^2 \operatorname{cth}\frac{\beta\omega_{fj}}{2}(1-\cos \boldsymbol{f}\cdot\boldsymbol{\delta}) \equiv \sum_j \Phi_\delta(j) \equiv \Phi_\delta(T)$$

$$\Phi_\delta^{M\sigma} = \frac{1}{2N}\sum |\Phi_f|^2(1-\cos \boldsymbol{f}\cdot\boldsymbol{\delta})\left(\operatorname{cth}\frac{\beta\Omega_f}{2}+2\sigma\right) \equiv \Omega_\delta^{M\sigma}(T)$$

can be treated as the electron-boson, *EP* and *EM*, coupling parameters (see [1] for Φ_δ) For $2S > 1$, with (15) and (7),

$$\Phi_\delta^{M\sigma} \approx \frac{1}{4S}\left\{1+2\sigma+C_1\left(\frac{kT}{2IS}\right)^{5/2}+0\left(\frac{1}{2S}\right)\right\}\Bigg|_{C_1<1} \quad i.\ e.\ \Phi_\delta^{M\sigma} < \Phi_{EM} < 1,$$

and the "magnon" narrowing $Q_M^\sigma(\delta)$ is insignificant. $\Phi_\delta^{M\sigma}$ is finite for $\sigma = \frac{1}{2}$ giving $Q_M^\uparrow(\delta)$ $\approx \left(1-\frac{1}{2S}\right)^{-1} \approx (1-\Phi_{EM})^{-1}$, but vanishes for $\sigma = -\frac{1}{2}$, giving $Q_M^\downarrow(\delta) = 1$ at $T = 0°$K. Let us note that the subband $\sigma = \frac{1}{2}$ is the lowest one at $A_{sd} < 0$, its "magnon" narrowing $Q_M^\uparrow(\delta)$ being in accordance with the respective estimates at $T = 0°$K made in [8] (and in [9] if the formula (23) is applied for $2S > 1$, see the note added in the proof in [9]) for $|A_{sd}| \gg \Delta_e$ neglecting *EP* coupling[9]. (For $A_{sd} > 0$ the lowest subband corresponds to $\sigma = -\frac{1}{2}$). Thus, *EM* coupling does not cause significant band narrowing $Q_\sigma(\delta) \approx Q_{ph}(\delta) = e^{-\Phi_\delta}$ and $D_p^\sigma \approx D_p \approx 2z\Delta_p \approx 2z\Delta_e e^{-\Phi}$, so that the width of the two subbands ($\sigma = \pm\frac{1}{2}$) are nearly equal. For $\Phi \gg 1$ (strong *EP* coupling) and $\Delta_e/\mathscr{E} \ll 1$ the case

$$D_p \ll kT \text{ and } D_p \ll G \tag{30}$$

is realized for not too low T (see [1]). Here

$$\mathscr{E} = \frac{1}{N}\sum_{fj} |X_{fj}|^2 \sin^2(\tfrac{1}{2}\boldsymbol{f}\cdot\boldsymbol{\delta})\omega_{fj} \tag{31}$$

is the characteristic parameter of the small polaron (activation energy in its high-temperature hopping)[10].

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[9] Here, the polaron resonance integral $\Delta_p = \Delta_e e^{-\Phi}$ plays the role of Δ_e, so that $\Delta_p \ll |A_{sd}|$ at (30).

[10] Actually, the strong *EP* coupling can be due to the contribution of one(j^{th}) phonon branch ($\Phi_{ph} \approx \Phi_{ph}(j)$) mainly of the longitudinal polar phonons, $\Phi \approx \Phi^{\text{pol}}$, or acoustic ones, $\Phi \approx \Phi^{ac}$, when respectively,

$$\mathscr{E} \approx \mathscr{E}^{(\text{pol})} \approx \frac{1}{2}\omega_{\text{pol}}\Phi_0^{\text{pol}} \text{ and } \mathscr{E} \approx \mathscr{E}^{(ac)} \approx \frac{\pi}{9}\omega_D\Phi_0^{ac} \tag{32}$$

with $\Phi_0 \equiv \Phi(T=0)$. The Einstein model for narrow branch optical phonons is used

$$\omega_f^{(\text{pol})} \approx \omega_{\text{pol}} \gg \delta\omega \tag{33}$$

with $\delta\omega$ being the width of the branch. Let us note that the characteristic temperature T_{xx} (see *Introduction*) is estimated (for (*ac*) with (9)) as follows

$$T_{xx} \sim T_1 \quad \text{with} \quad T_1^{\text{pol}} \approx \frac{\omega_{\text{pol}}}{2k\ln 4\Phi_0} \quad \text{when } \Phi \approx \Phi^{\text{pol}} \text{ [1] or}$$

$$T_1^{ac} \approx \frac{\omega_D}{2k\pi^{3/4}\Phi_0^{1/4}} \text{ when } \Phi \approx \Phi^{ac} \text{ [10].} \tag{34}$$

2. Low-temperature small polaron conduction in ferromagnetic samiconductors

In this section, the low-temperature ($T \ll T_C$) conductivity $\bar{\sigma}_{\mu\nu}(\omega) \equiv \mathrm{Re}\,\sigma_{\mu\nu}(\omega)$ of non-degenerate small polarons in a ferromagnetic crystalline semiconductor is considered at sufficiently low frequencies $\omega(<\omega_{xx}<D_P\ll\{\omega_{\mathrm{pol}};\ \omega_D;\ kT_C\})$ when the band regime can be actually expected (*i. e.* when $T_C \lesssim T_{xx}$)[11]. In this region of T and ω, *EM* coupling gives rise to additional scattering mechanisms, apart from those due to acoustic and polar phonons. The contribution of the latter is discussed in detail in several papers (see, *e. g.* [1, 2]) and can be neglected at $\omega_{\mathrm{pol}} \gg kT_C \gg kT$. As in [1], the study is based on Kubo formulas for $\sigma_{\mu\nu}(\omega)$, *e. g.*

$$\bar{\sigma}_{xx}(\omega) \equiv \mathrm{Re}\,\sigma_{xx}(\omega) = |e|\,N_c\mu_{xx}(\omega)$$

with

$$\mu_{xx}(\omega) = \frac{|e|}{E_\beta(\omega)}\,\mathrm{Re}\int_0^\infty dt\, e^{-\eta t}\cos\omega t\langle v_x(t)v_x\rangle$$

$$E_\beta(\omega) = \tfrac{1}{2}\omega\,\mathrm{cth}\,\frac{\beta\omega}{2}. \tag{35}$$

$\mu_{xx}(\omega)$ being the frequency dependent (longitudinal) mobility (other notation is standard, [1]) The matrices of the current operators, in particular of $j_\mu = ev_\mu$, in the $|\boldsymbol{k}n\sigma\rangle$ basis as determined for small polarons [1, 2] are used (by neglecteing the small corrections due to *EM* coupling). The explicit calculation of (35) is performed in $|\boldsymbol{k}n\sigma\rangle$ basis by applying the method developed in [1] (see also [11]). The idea was to calculate separately the nondissipative $(\hat{D}\hat{\varrho}_x(E_\pm))$ and dissipative $(\hat{Y}\hat{\varrho}_x(E_\pm))$ parts (see (18)) of the effective density matrices $\hat{\varrho}_x(E_\pm)$ determining mobility

$$\mu_{xx}(\omega) = \mu^b_{xx}(\omega)+\mu^h_{xx}(\omega) \equiv \frac{|e|}{2E_\beta(\omega)}\sum_\pm\{\mathrm{tr}\,v_x\hat{D}\hat{\varrho}_x(E_\pm)+\mathrm{tr}\,v_x\hat{Y}\hat{\varrho}(E_\pm)\}; \tag{36}$$

here

$$(E-iL_0-iL_1)\,\hat{\varrho}_x(E_\pm) = \hat{v}_x \tag{37}$$

and

$$L_q\hat{A} \equiv [\mathscr{H}_q, \hat{A}]|_{q=0,1} \text{ with } \mathscr{H}_0 = \hat{D}\mathscr{H},\ \mathscr{H}_1 = \hat{Y}\mathscr{H},\ E \equiv E_\pm = (\eta\mp i\omega)|_{\eta\to+0};$$

$$\hat{v}_x = \tfrac{1}{2}(\hat{\varrho}v_x+v_x\hat{\varrho}) \text{ with } \hat{\varrho} = \exp(\beta F-\beta\mathscr{H}) \text{ and } v_x = i[\mathscr{H}, x].$$

In (36), $\mu^b_{xx}(\omega)$ and $\mu^h_{xx}(\omega)$ determine the contributions of band-type and hopping transport with equations for $\hat{D}\hat{\varrho}_x(E)$ and $\hat{Y}\hat{\varrho}_x(E)$:

$$\hat{I}(E)\,\hat{D}\hat{\varrho}_x(E) = \hat{Q}_x(E) \text{ with}$$

$$\hat{I}(E) = E-iL_0-i\hat{D}L_1+\hat{D}L_1\hat{G}(E)\,\hat{Y}L_1 \tag{38}$$

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[11] *EM* coupling can give only small corrections to the transport coefficients in the hopping region at $\omega = 0$ $(T > T_1)$ when this region overlaps the considered one, *i.e.*, when $T_C > T_1$.

and $\hat{Y}\hat{\varrho}_x(E) = \hat{\varrho}_x^h(E) + \delta\hat{\varrho}_x(E)$ with $\hat{\varrho}_x^h(E) \equiv \hat{G}(E)\,\hat{Y}\hat{v}_x$, and

$$\delta\hat{\varrho}_x(E) \equiv i\hat{G}(E)\,\hat{Y}L_1\hat{D}\hat{\varrho}_x(E), \tag{39}$$

where
$\hat{Q}_x(E) \equiv \hat{D}\,\{\hat{v}_x + iL_1\hat{\varrho}_x^h(E)\}$, *i. e.* $\hat{Q}_x(E) \approx \hat{D}\hat{v}_x$ in the band regime; $\hat{G}(E) \equiv (E - iL_0 - i\hat{Y}L_1)^{-1}$. Then, the appropriate regular perturbation theory is applied as given in [1] and based on a regular expansion of $\hat{G}(E)$ in L_1 (*i. e.* in the perturbation $\mathscr{H}_1$), the basic (∘) terms being determined by neglecting corrections of the order of the small parameters of the theory as discussed in [1, 2].

As in [1, 2], the situation is considered (at $T \ll T_C$) when the characteristic transport time $\tau_{tr} \gg t_R$, t_R being the typical largest relaxation time of bosons (phonons, magnons), *i. e.* of the deformation of the atomic and magnetic structure of the crystal, so that individual scattering acts (or hops) can be basically treated as noncorrelated ones in a Markovian process. In this approximation (neglecting the polaron-boson drag effect in band transport and small equilibrium fluctuations of boson numbers), the ω - and T-dependent nonequilibrium "distribution function" $\varphi_x(\alpha; \pm\omega)$ of small polarons over $\alpha \equiv (\boldsymbol{k}, \sigma)$ can be introduced:

$$\langle \boldsymbol{k}n\sigma|\hat{D}\,\hat{\varrho}_x(E_\pm)|\boldsymbol{k}'n\sigma\rangle \equiv \delta_{\boldsymbol{kk}'}\varrho_0(\alpha n)\,\varphi_x(\boldsymbol{k}n\sigma; \pm\omega) \to \delta_{\boldsymbol{kk}'}\varrho_0(\alpha n)\,\varphi_x(\alpha; \pm\omega),$$

$$\langle \alpha n|\hat{\varphi}_x(E_\pm)|\alpha' n'\rangle \equiv \delta_{nn'}\delta_{\alpha\alpha'}\varphi_x(\alpha; \pm\omega) \tag{40}$$

where $\varphi_x(\alpha; \omega) \equiv \varphi_x(\alpha; \omega, T)$; $\varrho_0(\alpha n) = \langle \alpha n|\varrho_0|\alpha n\rangle$ with $\hat{\varrho}_0 \equiv \exp(\beta F_0 - \beta\mathscr{H}_0)$ so that $\varrho_0(\alpha, n) = \varrho_b(n)\,\varrho_0(k\sigma) \approx \varrho_b(n)\,\varrho_0(\sigma)$ and $\varrho_0(\sigma) = e^{-\beta\varepsilon_\sigma}(e^{-\beta\varepsilon_\downarrow} + e^{-\beta\varepsilon_\uparrow})^{-1}$ as $\varrho_0(\boldsymbol{k}\sigma) = e^{\beta F_0 - \beta\mathscr{E}_\sigma(\boldsymbol{k})}$ $\times\varrho_0(\sigma) \approx \varrho_0(\sigma)$ where $\mathscr{E}_\sigma(\boldsymbol{k}) \approx \mathscr{E}(\boldsymbol{k})$ and $|\mathscr{E}(\boldsymbol{k})| \ll kT$ see (30) (the operator $\hat{\varphi}_x(E_\pm)$ is defined as acting in the α variable "space").

Then, the basic (∘) mobility which is here determined by band type transport is as follows:

$$\mu_{xx}(\omega) \approx \overset{b}{\underset{\circ}{u}}_{xx}(\omega) = \frac{|e|}{2E_\beta(\omega)} \sum_\pm \sum_\alpha v_x(\alpha)\varphi_x(\alpha; \pm\omega)$$

$$\approx \frac{|e|\beta}{2} \sum_\pm \sum_{k\sigma} v_x(\alpha)\varphi_x(\alpha; \pm\omega)\,\varrho_0(\sigma) \tag{41}$$

the approximate equality holds for $\omega \ll kT$, when $E_\beta(\omega) \approx kT\left[1 + 0\left(\frac{\omega}{kT}\right)^2\right]$. In (41):

$$v_x(\alpha) = \sum \varrho_b(n)\langle \alpha n|v_x|\alpha n\rangle \approx v_x(\boldsymbol{k}) = \frac{\partial\mathscr{E}(\boldsymbol{k})}{\partial k_x} = \sum_\delta e^{i\boldsymbol{k}\cdot\boldsymbol{\delta}}v_x(\boldsymbol{\delta})$$

with

$$v_x(\delta) = i\delta_x\Delta_e(\delta) \tag{42}$$

The transport equation determining $\varphi_x(\alpha;\omega)$ can be derived from (38) and (39), (42) by summing (38), in $|\boldsymbol{k}n\sigma\rangle$ basis, over boson numbers (n) and by taking into account that, basically,

$$\sum_n \langle\alpha n|\hat{v}_x|\alpha n\rangle \equiv \tfrac{1}{2}\sum_n \varrho_0(\alpha n)\,\langle\alpha n|v_x\hat{\varrho}\hat{\varrho}_0^{-1}+\text{h. c.}|\alpha n\rangle$$

$$\approx \varrho_0(\alpha)\sum_n \varrho_b(n)\,\langle\alpha n|v_x|\alpha n\rangle \equiv \varrho_0(\alpha)\,v_x(\alpha).$$

For sufficiently low external magnetic fields $\boldsymbol{H}\approx 0$ (see (65)), the following can be written

$$[\pm i\omega+\hat{R}]\,\varphi_x(\boldsymbol{k};\pm\omega)\approx v_x(\boldsymbol{k}) \tag{43}$$

with the scattering operator $\hat{R}$ defined by the relation

$$R\varphi_x(\alpha;\omega)=\sum\varrho_b(n)\,\langle\alpha n|L_1\hat{G}(E)\,L_1\hat{\varphi}_x|\alpha n\rangle$$
$$\simeq \sum_{\alpha'}[\underset{\circ}{W}_{\alpha\alpha'}\varphi_x(\alpha;\omega)-\underset{\circ}{W}_{\alpha'\alpha}\varphi_x(\alpha';\omega)] \tag{44}$$

and the basic scattering probability $\underset{\circ}{W}_{\alpha\alpha'}$ independent of ω at low ω ($\langle\omega_{xx}\ll(\omega_D;\,k\,T_C)$). The basic scattering probability $\underset{\circ}{W}_{\alpha\alpha'}$ in (44) can be calculated (see (18–(19)) as the basic contribution of

$$Y_{\alpha\alpha'}=\text{Re}\int_0^\infty dt\,e^{-\eta t}\,\langle(\mathscr{H}_1)_{\alpha\alpha'}(\mathscr{H}_1(t))_{\alpha'\alpha}\rangle_b \tag{45}$$

$$i.\ e.\ \underset{\circ}{W}_{\alpha\alpha'}=\underset{\circ}{Y}_{\alpha\alpha'}$$

where $A(t)\equiv e^{i\mathscr{H}t}A\,e^{-i\mathscr{H}t}$ and $A_{\alpha\alpha'}\equiv\langle\alpha|A|\alpha'\rangle$. The general formal solution of (43) and (38) can now be expressed through the eigenvalues of the operator $\hat{R}$, and transport relaxation times $\tau_r^\sigma(r=1,2\,\ldots)$, so that [1c]

$$\mu_{xx}(\omega)\approx\overset{b}{\underset{\circ}{\mu}}_{xx}(\omega)\approx\tfrac{1}{2}|e|\beta\sum_\sigma\sum_\pm\sum_{r=1,2,\ldots}\frac{|v_{x,r}|^2\tau_r^\sigma(\pm\omega)}{1+\omega^2(\tau_r^\sigma(\pm\omega))^2}\,\varrho_0(\sigma)$$

$$\approx\tfrac{1}{2}|e|\beta\sum_\pm\sum_r\frac{|v_{x,r}|^2\tau_r(\pm\omega)}{1+\omega^2\tau_r^2(\pm\omega)} \tag{46}$$

where $\tau_r\equiv\tau_r^{\sigma_0}$ and $\sigma\equiv\sigma_0$ for the lowest subband ($\sigma_0=\frac{1}{2}$ at $A_{sd}<0$, but $\sigma=-\frac{1}{2}$ at $A_{sd}>0$). In the last equality the basic contribution of the conduction in the lowest subband is only accounted for at $G\gg kT$, $\varrho_0(\sigma_0)\approx 1$ and $\varrho_0(\sigma\neq\sigma_0)\simeq e^{-\beta G}\ll 1$, and the "interband" (with spin flip) scattering ($\sigma_0\to\sigma\neq\sigma_0$) can also be neglected when calculating the respective basic scattering probabilities in (44) and τ_r in (46) (with the same accuracy of up to $\sim\exp(-\beta G)$) because the average Planck number of magnons taking part in such processes is $\sim\overline{N}_f=\overline{N}(\Omega_f)|_{\Omega_f\sim G}\approx e^{-\beta G}$ (see also (71) and (74)). Thus, in what follows

$$\underset{\circ}{W}_{\alpha\alpha'}\equiv\underset{\circ}{W}{}^{\sigma\sigma'}_{kk'}=\underset{\circ}{W}_{kk'}(\sigma)\,\delta_{\sigma\sigma'}+\underset{\circ}{W}_{kk'}(\sigma\sigma')\,(1-\delta_{\sigma\sigma'})\to\underset{\circ}{W}_{kk'}(\sigma_0)\,\delta_{\sigma\sigma'}\delta_{\sigma\sigma_0} \tag{47}$$

and $\alpha\equiv(\boldsymbol{k}\sigma_0)=\boldsymbol{k}$.

The analysis of the expansions of $Y_{\alpha\alpha'}$ (39) in the perturbation $\mathscr{H}_1$ (similar to that for polar phonons in [1, 2]) shows that under conditions (30) we have (neglecting corrections of the order of the small expansion parameters in the theory [1, 2]): 1) The basic contribution $\underset{\circ}{W}_{\alpha\alpha'}$ in $W_{\alpha\alpha'}$ is given as a sum

$$\underset{\circ}{W}_{\alpha\alpha'} = \sum_{q=0,2} W^{(q)}_{\alpha\alpha'} \tag{48}$$

of the Born contribution ($W^{(0)}_{\alpha\alpha'} \sim \Delta_e^2 e^{-2\Phi}$) and non-Born one ($W^{(2)}_{\alpha\alpha'} \sim \Delta_e^4$) at $\Phi \gg 1$:

$$\begin{aligned} W^{(0)}_{\alpha\alpha'} &= \operatorname{Re} \int_0^\infty dt\, e^{-\eta t} \langle (\mathscr{H}_1)_{\alpha\alpha'} (\mathscr{H}_1(t)_0)_{\alpha'\alpha} \rangle_b \\ &= 2\pi \sum_{nn'} \varrho_0(n) |\langle \alpha n | \mathscr{H}_1 | \alpha' n' \rangle|^2 \, \delta(\varepsilon_{\alpha n} - \varepsilon_{\alpha' n'}); \end{aligned} \tag{49}$$

$$W^{(2)}_{\alpha\alpha'} = \operatorname{Re} \int_0^\infty dt\, e^{-\eta t} \int_0^t dt_1 \langle (\mathscr{H}_1)_{\alpha\alpha'} ([[\mathscr{H}_1(t)_0, \mathscr{H}_1(t)_0], \mathscr{H}_1(t_1)_0])_{\alpha'\alpha} \rangle_b, \tag{50}$$

where

$$A(t)_0 \equiv e^{i\mathscr{H}_0 t} A e^{i\mathscr{H}_0 t}.$$

2) The basic terms in $W^{(0)}_{kk'}$ and $W^{(2)}_{kk'}$, at the T under consideration ($T < T_1$, $T \ll T_C$), describe, two-boson scattering processes of absorption — emission ($\sim \delta(\omega_{fl} - \omega_{f'l'})$), as one-boson scattering (absorption or emission) processes are practically forbidden in a very narrow band ($D_p \ll kT$). The scattering is due either to two-phonon or two-magnon processes because the z component of the total spin should be conserved. Thus,

$$\underset{\circ}{W}_{kk'} = \underset{\circ}{W}_{kk'(ph)} + \underset{\circ}{W}_{kk'(M)} = \sum_{q=0,2} [W^{(q)}_{kk'(ph)} + W^{(q)}_{kk'(M)}]. \tag{51}$$

3) Formulas for $W^{(0)}_{kk'}$ and $W^{(2)}_{kk'}$ show that the Born boson scattering is essentially nonisotropic, *i. e.* $W^{(0)}_{kk'} = W^{(0)}(\boldsymbol{k}-\boldsymbol{k}', \boldsymbol{k})$, whereas the non-Born one is basically isotropic, *i. e.* $W^{(2)}_{kk'} = W^{(2)}(\boldsymbol{k}-\boldsymbol{k}')$ both for phonon [1, 2] and magnon scattering. Such a structure of small-polaron boson scattering is determined by the strong *EP* coupling. Formulas for $\underset{\circ}{W}_{kk'(\text{pol})}$ describing the scattering by polar phonons were given in [1, 2], in particular ($C_1 \simeq 1$),

$$\underset{\circ}{\Omega}_{\text{pol}} = \frac{1}{N} \sum_{kk'} \underset{\circ}{W}_{kk'(\text{pol})} \approx \frac{z\Delta_e^2}{\delta\omega} [(\Phi_0^{\text{pol}})^2 + z\Delta_e^2 \omega_{\text{pol}}^2 \mathscr{E}^{-4}] e^{-\beta\omega_{\text{pol}}}. \tag{52}$$

Formulas for $W_{kk'(ac)}$ describing the scattering by acoustic phonons are derived from formulas (83) in [1a] as follows:[12]

$$\begin{aligned} \Omega^{(0)}_{ac} &= \frac{1}{N} \sum W^{(0)}_{kk'(ac)} = 4\pi \sum_{\delta} |\Delta_e(\boldsymbol{\delta})|^2 e^{-\Phi} \frac{1}{N^2} \sum_{f_1 f_2} \delta(\omega_1 - \omega_2) \times \\ &\times \prod_{\nu=1,2} \frac{|X(\omega_\nu)|(1-\cos \boldsymbol{f}_\nu \cdot \boldsymbol{\delta})}{sh\, \frac{1}{2} \beta\omega_0} \approx \frac{\pi^3 z \Delta_e^2}{9\omega_D} e^{-2\Phi} (\Phi_0^{ac})^2 \left(\frac{T}{T_0}\right)^7 J_{7/2}; \end{aligned} \tag{53}$$

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[12] The estimates of $\Omega^{(0)}_{ac}$ and $\Omega^{(2)}_{ac}$ in [11] were made in the longwave approximation (6), at $f \sim f_T \ll f_D$, but with $|f|a \gg 1$ (so that $\zeta_\nu(\boldsymbol{s};\, \delta, \delta') \approx 1$ and $\sin^2 \frac{1}{2} \boldsymbol{f} \cdot \boldsymbol{\delta} \approx \frac{1}{2}$ as here a denotes the large average distance between the impurity centres. Here $|f|a \ll 1$ ($\zeta_\nu(\boldsymbol{0};\, \delta, \delta') \sim f^2 a^2$ and $\sin^2 \frac{1}{2} \boldsymbol{f} \cdot \boldsymbol{\delta} \approx (\frac{1}{2} f^2 \delta^2)^2$), this giving the difference of the T-dependences.

$$W^{(2)}_{kk'(ac)} = \sum_{k-k'} e^{i(k-k')\cdot s} W^{(2)}_{ac}(s) \text{ and } \Omega^{(2)}_{ac} \equiv \frac{1}{N} \sum_{kk'} W^{(2)}_{kk'ac} = W^{(2)}_{ac}(s=0)$$

$$\text{with } W^{(2)}_{ac}(s) = \pi \sum_{\delta,\delta'} \frac{|\Delta_e(\delta)|^2 |\Delta_e(\delta')|^2}{(4\mathscr{E})^6} \frac{1}{N^2} \sum_{f_1 f_2} \delta(\omega_1 - \omega_2) \prod_{\nu=1,2} \frac{\omega_\nu^2 \zeta(s;\delta,\delta')}{\operatorname{sh}\frac{1}{2}\beta\omega_\nu}$$

$$\approx \frac{\pi^7 z^2 \Delta_e^4 \omega_D^3 (\Phi_0^{ac})^2}{20(4\mathscr{E})^6} \left(\frac{T}{T_0}\right)^{11} J_{9/2}, \tag{54}$$

where $T_0 \equiv T_0^{(ac)} = \omega_D/2k$ (in [1], $T_0 \equiv T_0^{\rm pol} = \omega_{\rm pol}/2k$. By taking into account that at $T \ll \omega_D/k$, the essential scattering is associated with long-wave acoustic phonons at $f \simeq f_T \simeq \simeq kT/u_0 \ll f_D$ (see (6).) In (54), $\Phi_0^{ac} = \frac{2}{N}\sum_f |X_f^{ac}|^2 (1 - \cos f\cdot\delta) \simeq \frac{1}{2}|X_{ac}|^2$ is the contribution of the acoustic phonons in the long-wave approximation (somewhat overestimated); $\omega_\nu \equiv \omega_{f_\nu}$; $J \equiv \int_0^\infty \frac{dx x^{4\alpha}}{\operatorname{sh}^2 x} = 4(4\alpha)!\,\zeta(4\alpha)$.

The estimate

$$\sum_{\delta}\sum_{\delta'} \int \frac{d\Omega}{4\pi|f|^2} [(f\cdot\delta)(f\cdot\delta') + 2(f\cdot s)(f\cdot(\delta'-\delta))]^2 \approx z^2 q_1(s)$$

with $q_1^2(s) \simeq 1$ is used here. In a similar way, by using (24)–(28), (7) and (15), one can calculate the probability $W_{kk'(M)}$ of magnon scattering (neglecting corrections $\sim (2S)^{-1}$, see footnote 5). The basic two-magnon Born scattering (without spin flip) can be described by the formula

$$\Omega_M^{(0)} \equiv \frac{1}{N}\sum_{kk'} W^{(0)}_{kk'(M)} \approx \frac{\pi z \Delta_e^2}{4} e^{-2\Phi} \frac{1}{N^2} \sum_{f_1 f_2} \delta(\Omega_1 - \Omega_2) \frac{|\Phi_{f_1}|^2 |\Phi_{f_2}|^2}{\operatorname{sh}^2 \frac{\beta\Omega_1}{2}} \times$$

$$\times [3 - 2\cos f_1\cdot\delta - 2\cos f_2\cdot\delta + \cos f_1\cdot\delta \cos f_2\cdot\delta] \approx \frac{z\Delta_e^2 e^{-2\Phi}}{192\pi^3 I S^3}\left(\frac{kT}{2IS}\right)^3 J_{1/2}. \tag{55}$$

Analogous estimates lead to the formulas for $W^{(2)}_{kk'(M)}$, $W^{(2)}_{(M)}(s)$ and $\Omega_M^{(2)} = W_M^{(2)}(0)$. Omitting similar, very lengthy, calculations and using the analogy of the structure of $\Omega_M^{(2)}$ with that of $\Omega_{ph}^{(2)}$ (as a consequence of the strong EP coupling) and formulas (83) of [1a] and (25)–(26), one can obtain that

$$\Omega_M^{(2)} = W_M^{(2)}(s=0) \approx \frac{4z^2\Delta_e^4 I^3 S}{\pi^3 (4\mathscr{E})^6}\left(\frac{kT}{2IS}\right)^7 J_{3/2}. \tag{56}$$

It follows from (53)–(56) that magnon and acoustic phonon scattering, the Born and non--Born processes, can complete with one another as they have essentially different power T-dependences T^r (with $r > 1$), and all these scattering processes can compete with polarization phonon scattering, so that one of the processes dominates in the appropriate intervals of

T. In fact, expressions (53–(54) and (55)–(56) are the basic terms of the expressions of $\mathop{\Omega}\limits_{\circ}{}_{ac}$ and $\mathop{\Omega}\limits_{\circ}{}_M$ in powers of the parameters

$$\Phi_0^{\mathrm{pol}} e^{-\frac{\beta\omega_{\mathrm{pol}}}{2}} \ll 1, \quad \Phi_0^{ac}\left(\frac{T}{T_0}\right)^4 \ll 1 \text{ and } \Phi_{EM}\left(\frac{T}{T_C}\right)^3 \approx \frac{1}{2S}\left(\frac{T}{T_C}\right)^{5/2} \ll 1 \qquad (57)$$

which are the additional factors for each additional boson, polar or acoustic phonon and magnon, respectively.

When the isotropic non-Born scattering dominates, the transport equation (43)–(44) is solved by the Fourier method [1] so that in [46] the essential transport times

$$\tau_r \equiv \tau(\delta) = [W^{(2)}(\mathbf{0}) - W^{(2)}(\delta)]^{-1} \approx [W^{(2)}(\mathbf{0})]^{-1} = [\Omega^{(2)}]^{-1} \qquad (58)$$

are practically equal, and $|v_{x,r}|^2 = |v_x(\delta)|^2 = \delta_x^2 \Delta_e^2(\delta)$.

When the non-isotropic Born scattering dominates, the solution of (43)–(44) can be approximated by assuming that all the essential τ_r in (46) are of the same order of magnitude, *i. e.*

$$\tau_r^{-1} \approx \Omega^{(0)} \qquad (59)$$

Then, at $\omega \ll kT$ (and $\beta D_p \ll 1 \ll \beta G$),

$$\mu_{xx}(\omega) \approx z|e|a^2\beta\Delta_p^2 \frac{\tau_{tr}}{1+\omega^2\tau_{tr}^2} \qquad (60)$$

with the usual approximation

$$\tau_{tr}^{-1} \approx \mathop{\Omega}\limits_{\circ}{}_{ac} + \mathop{\Omega}\limits_{\circ}{}_M + \mathop{\Omega}\limits_{\circ}{}_{\mathrm{pol}} \qquad (61)$$

for $\tau_{tr}^{-1} \ll \Delta_p$ ($\ll kT \ll \{kT_c;\, \omega_D;\, \omega_{\mathrm{pol}}\}$), and $\Delta_p = \Delta_e\, e^{-\Phi}$. For such EP and EM coupling parameters and T when $\tau_{tr}^{-1} \approx \mathop{\Omega}\limits_{\circ}{}_{ac} + \mathop{\Omega}\limits_{\circ}{}_M \gg \mathop{\Omega}\limits_{\circ}{}_{\mathrm{pol}}$, the situation is as follows. Comparing the contributions of the phonon Born and non-Born scattering (a), the magnon Born and non-Born scattering (b), the Born scattering due to magnons and to phonons (d) and the non-Born scattering due to magnons and to phonons (e), one can define the characteristic temperatures:

$$T_a \approx 0.2 \frac{\mathscr{E}^{3/2} e^{-\frac{1}{2}\Phi}}{k\sqrt{\Delta_e}} \approx T_b (\ll \{T_c;\, T_0;\, T_1^{ac}\})$$

with

$$\frac{\Omega_{ph}^{(0)}(T_a)}{\Omega_{ph}^{(2)}(T_a)} \approx \frac{\Omega_M^{(0)}(T_b)}{\Omega_M^{(2)}(T_b)} \approx 1,$$

$$T_d \approx 0.25\, T_0 \left(\frac{T_0}{T_C}\right) \Big/ \sqrt{\Phi_0^{ac}} \text{ with } \frac{\Omega_M^{(0)}(T_d)}{\Omega_{ph}^{(0)}(T_d)} \approx 1,$$

and

$$T_e \approx 1.25\, T_0 \left(\frac{T_0}{T_C}\right) \Big/ \sqrt{\Phi_0^{ac}} \quad \text{with} \quad \frac{\Omega_M^{(2)}(T_e)}{\Omega_{ph}^{(2)}(T_e)} \approx 1. \tag{62}$$

The actual meaning of the estimates is that $T_a \approx T_b$ and $T_d < T_e$ (the number coefficients in T_a, T_d and T_e have a tentative sense). Thus, the Born scattering dominates at $T < T_a$. When $T_d > T_a$ (the case seems to occur at $\Phi \gg 1$) the magnon scattering prevails with $\tau_{tr}^{-1} \sim T^r$ and $r = 3$. When $T_d < T_a$ two regions can be found: at $T_d < T < T_a$ the phonon ($r = 7$) scattering and at $T < T_d$ the magnon ($r = 3$) scattering dominate. The non-Born scattering dominates at $T > T_a$. When $T_e > T_a$ two regions can be found: at $T_a < T < T_e$ the magnon ($r = 7$) scattering and at $T_e < T < T_1^{ac}$ the phonon ($r = 11$) (at $T_e < T_a$, if any, only phonon ($r = 11$) scattering dominates in a region $T > T_a$). If $T_e > T_1^{ac}$ is possible the magnon non-Born mechanism is the basic one at $T_a < T < T_1^{ac}$, with $r = 7$.

The conditions of dominating polar phonon scattering, with $\tau_{tr}^{-1} \approx \overset{\circ}{\Omega}_{\text{pol}} (\sim e^{-\beta\omega_{pol}})$, can be found similarly. Thus, at $\omega\tau_{tr} \ll 1$ the mobility

$$\mu_{xx}(\omega) \approx \mu_{xx} \equiv \mu_{xx}(0) \approx |e| z\beta a^2 \Delta_p^2 \tau_{tr} = \frac{|e|}{m^*} z\tau_{tr}\beta\Delta_p^3$$

$$\sim \left\{ T^{-r-1}, \text{ or } \frac{1}{T}\, e^{\beta\omega_{\text{pol}}} \right\}, \quad \text{with } m^* = (\Delta_p^3 a^2)^{-1}, \tag{63}$$

decreases with T in a power ($r > 1$) and exponential way, depending of the inter-relation of the *EP* and *EM* coupling parameters, whereas at $1 \ll \omega\tau_{tr} (\ll \beta\omega)$

$$\mu_{xx}(\omega) \approx \frac{z|e|\beta a^2 \Delta_p^2}{\omega^2 \tau_{tr}} \sim \left\{ T^{r-1}, \text{ or } \frac{1}{T}\, e^{-\beta\omega_{\text{pol}}} \right\} \tag{64}$$

increases with T in the way noted here. It follows from (62) in particular that the Born magnon scattering dominates at sufficiently low T ($< \min\{T_d; T_a\}$) with $r = 3$, while the non-Born magnon scattering with $r = 7$ can dominate at higher T, see above.

Further more, approximate formulas can be given for describing normal (non-spontaneous) galvanomagnetic effects in an external magnetic field $\boldsymbol{H}$, when using formulas (98)–(101) in [1b] or (101)–(106) in [1a] (at $\boldsymbol{H} = 0$, the operator $\hat{D}L_1\hat{D}\hat{Q}_x$ in (38) describes the effect of the Lorentz force in transport equations (43)–(44), see (92) in [1*a*]). The case of not too high $\boldsymbol{H}$ is implied,

$$H \ll H_0 \equiv \hbar c(|e|a^2)^{-1}, \; i.\,e. \quad \omega_L^* = \frac{|e|H}{m^* c} \ll \Delta_p^3 = \Delta_e e^{-\Phi}, \tag{65}$$

when the H-dependence of scattering probabilities $\overset{\circ}{W}_{kk'}$ can be neglected. Then, the Hall mobility ($\boldsymbol{H} \| 0z$)

$$\mu_H \equiv \frac{c}{H} \frac{\mu_{xy}}{\mu_{xx}} \equiv \mu_H^{(1)} \approx \frac{|e|}{m^*} \frac{\tau_{tr}}{1 + \omega_L^2 \tau_{tr}^2} \quad \text{or}$$

$$\mu_H \equiv \mu_H^{(1)} \approx \frac{|e|}{m^*} \frac{\tau_{tr}\beta\Delta_p}{1 + \omega_L^2 \tau_{tr}^2} \left(\ll \frac{|e|\,\tau_{tr}}{m^*(1 + \omega_L^2 \tau_{tr}^2)} \right) \tag{66}$$

and

$$\Delta\mu_{xx}(\boldsymbol{H}) \equiv \mu_{xx}(\boldsymbol{H})-\mu_{xx} < 0 \text{ and } \frac{|\Delta\mu_{xx}(\boldsymbol{H})|}{\mu_{xx}} \approx \frac{(\omega_L\tau_{tr})^2}{1+(\omega_L\tau_{tr})^2}\,. \tag{67}$$

The case $\mu_H = \mu_H^{(1)}$ is realized for a (non cubic) crystal lattice in which crystal planes orthogonal to magnetic field $\boldsymbol{H}$ contain triades of mutually nearest neighbour appropriate sites and for the region of T where the approximation of one essential transport time, $\tau_r^{-1} \approx \overset{\circ}{\Omega}\delta_{r1}$, is applicable, whereas the case $\mu_H = \mu_H^{(2)}$ is for all the other situations [1c]. Thus,

$$\mu_H^{(1)} \sim T^{-r},\ \mu_H^{(2)} \sim T^{-r-1} \text{ and } \frac{|\Delta\mu_{xx}(\boldsymbol{H})|}{\mu_{xx}} \sim T^{-2r}H^2, \tag{68}$$

$$\text{at } \omega_L\tau_{tr} \ll 1, \text{ with } \frac{\mu_H^{(1)}}{\mu_{xx}} \approx (\beta\Delta_p)^{-1} \gg 1 \text{ and } \frac{\mu_H^{(2)}}{\mu_{xx}} \approx 1.$$

The change of the carrier concentration $N_c(T, \boldsymbol{H})$ with external magnetic field $\boldsymbol{H}$ also contributes to the magnetoconductivity, because actually the gap $G \approx |A_{sd}|\left(2S\dfrac{M(\boldsymbol{H},T)}{M(0,0)}+1\right)$ and the concentration activation energy

$$E_d \approx E_d(H=0)+A_{sd}\left[S\frac{M(H,T)}{M(0,0)}(\gamma_1-1)+(\gamma_2-1)\,\delta_{\sigma,\uparrow}\right]$$

$\boldsymbol{M}(\boldsymbol{H}, T)$ denotes the magnetization, while constants γ_1 and γ_2 describe the split and *EM* shift of the impurity energy level (impurity semiconductors are here implied), see [16]. Then, with $N_c \equiv N_c\ (H=0)$ and $\Delta N_c(\boldsymbol{H}) \equiv \boldsymbol{N}_c(\boldsymbol{H})-N_c$,

$$\frac{\Delta\sigma_{xx}(\boldsymbol{H})}{\sigma_{xx}} \equiv \frac{\sigma_{xx}(\boldsymbol{H})-\sigma_{xx}}{\sigma_{xx}} \approx |e|N_c\frac{\Delta\mu_{xx}(\boldsymbol{H})}{\mu_{xx}} + |e|\mu_{xx}\frac{\Delta N_c(\boldsymbol{H})}{N_c}\,, \tag{69}$$

so that $\dfrac{\Delta N_c(\boldsymbol{H})}{N_c} < 0$, and when this effect dominates (this is usually the case) one obtains positive magnetoconductivity, $\Delta\sigma_{xx}(\boldsymbol{H}) > 0$, even though $\Delta\mu_{xx} < 0$.

Finally, the Seebeck coefficient practically does not depend on the scattering mechanism in the narrow small polaron band ($\beta\Delta_p \ll 1$) and basically is determined by the chemical potential of the small polarons (see (112) in [1c]).

3. Characteristics of high-frequency and high-electric field conduction of small polarons

The split of the small polaron band into two subbands ($\sigma = \pm\frac{1}{2}$) in the ferromagnetic semiconductor causes the characteristic effect of a resonance-type infrared absorption at $\omega \simeq G(\gg kT)$ due to the "inter-subband" transition of the small polarons, with spin flip. As the gap $G(\gg kT)$ actually corresponds to infrared ω ($\omega > \omega_{xx}$), the small polaron longitudinal conductivity $\sigma_{xx}(\omega) = |e|\,N_c\mu_{xx}(\omega)$ and the related absorption coefficient $\eta_p(\omega)$

($\sim \bar{\sigma}_{xx}(\omega)$) are determined by hopping (and not by band-type transport [1, 2]), so that (see (38)–(40) and (35))

$$\bar{\sigma}_{xx}(\omega) \approx \bar{\sigma}^h_{xx}(\omega) = \frac{e^2 N_c}{E_\beta(\omega)} \int\limits_{-\infty}^{+\infty} dt \cos \omega t \; e^{-\eta|t|} \langle v_x(t)_0 \hat{Y} v_x \rangle_0$$

where $\int\limits_{+0}^{\infty} dt\, A(t)_0 = \int\limits_0^\infty dt\, e^{i\mathscr{H}_0 t} A\, e^{-i\mathscr{H}_0 t - \eta t} = \int\limits_0^\infty dt\, e^{iL_0 t - \eta t} A = (\eta - iL_0)^{-1} A$ and $\langle \hat{A} \rangle_0 \equiv \operatorname{tr} \hat{\varrho}_0 \hat{A}$.

The explicit general formula for $\underset{\circ}{\sigma}^h_{xx}(\omega)$ is obtained by calculating trace $\langle A \rangle_0$ in $|\boldsymbol{s} n \sigma\rangle$ basis (equivalently, in $|\boldsymbol{k} n \sigma\rangle$ basis, with $D_p \ll kT$) and considering (19)–(28), (42). This formula contains the contributions of all the ν-phonon and ν-magnon ($\nu = 1, 2, \ldots$) hopping processes and is given, at the low T under consideration, as a series in small parameters of the type (57) (the ν^{th} term describes the ν-boson hopping processes:

$$\bar{\sigma}_{xx}(\omega) \approx \underset{\circ}{\bar{\sigma}}^h_{xx}(\omega) = \frac{e^2 N_c}{E_\beta(\omega)} \sum_{\boldsymbol{\delta}} \delta_x^2 \Gamma_h(\boldsymbol{\delta}; \omega) \tag{70}$$

with [1, 2]

$$\Gamma_h(\boldsymbol{\delta}; \omega) = |\Delta_e(\boldsymbol{\delta})|^2 \operatorname{Re} \int\limits_{+0}^{\infty} dt \cos \omega t \; e^{-i\eta t} \sum_{\sigma = \pm \frac{1}{2}} \varrho_0(\sigma) \langle (T^{\mathbf{0}}(t)_0)^+ T^{\boldsymbol{\delta}}(t)_0 \; \hat{Y} \, (T^{\boldsymbol{\delta}})^+ T^{\mathbf{0}} \rangle_b$$

$$= \tfrac{1}{2} |\Delta_e(\boldsymbol{\delta})|^2 \sum_{\pm} \sum_{\sigma} \varrho_0(\sigma) \sum_n \varrho_0(n) |\langle \mathbf{0} n^{(0)} | \boldsymbol{\delta} n^{(\boldsymbol{\delta})} \rangle|^2 \times$$

$$\times \delta(\varepsilon_\sigma + \varepsilon_n - \varepsilon_{\sigma'} - \varepsilon_{n'} \pm \omega) \, (1 - \delta_{nn'} \delta_{\sigma\sigma'}) = \sum_{\nu=1}^{\infty} \Gamma_h^{(\nu)}(\boldsymbol{\delta}; \omega)$$

and (neglecting corrections $\sim \exp(-\Phi) \ll 1$)

$$\Gamma_h(\omega) \approx \sum_{\sigma\sigma'} \varrho_0(\sigma) \, \Omega^{(0)}_{\sigma\sigma'}(\omega) \tag{71}$$

where $\Omega^{(0)}_{\sigma\sigma'}(\omega) = \dfrac{1}{N} \sum\limits_{kk'} W^{(0)}_{\alpha\alpha'}(\omega)$ and $W^{(0)}_{\alpha\alpha'}(\omega) = \operatorname{Re} \int\limits_0^\infty dt \cos \omega t \; e^{-\eta t} \langle (\mathscr{H}_1)_{\alpha\alpha'} (\mathscr{H}_1(t)_0)_{\alpha\alpha'} \rangle_b$ (see (45) and (53)–(55)). Then, at $\omega \simeq G$, the basic term $\Gamma^{(1)}_{h(M)}(\delta; \omega)$ in $\Gamma_h(\delta; \omega)$ (and in $\underset{\circ}{\bar{\sigma}}^h_{xx}(\omega)$) can be determined to be one-magnon hops with spin flip from the lowest energy level ε_{σ_0} for the $\boldsymbol{O}$-th site to the upper level for the $\boldsymbol{\delta}$-th site, with magnon emission (+) or absorption (−), $\omega - G \pm \Omega_f = 0$.

This term alone determines the absorption effect under consideration, and it can be obtained from (70), (71) and (26) as follows:

$$\Gamma^{(1)}_{h(M)}(\delta; \omega) = |\Delta_e(\delta)|^2 e^{-2\Phi} \sum_\sigma \varrho_0(\sigma) \operatorname{Re} \int\limits_0^\infty dt \cos \omega t e^{-\eta t} \frac{2}{N} \sum_{\boldsymbol{f}} |\Phi_{\boldsymbol{f}}|^2 (1 - \cos \boldsymbol{f} \cdot \boldsymbol{\delta}) \times$$

$$\times \{ \delta_{\sigma\downarrow} \bar{N}_{\boldsymbol{f}} e^{it(\varepsilon_\uparrow - \varepsilon_\downarrow + \Omega_{\boldsymbol{f}})} + \delta_{\sigma\uparrow} (1 + \bar{N}_{\boldsymbol{f}}) e^{-it(\varepsilon_\uparrow - \varepsilon_\downarrow + \Omega_{\boldsymbol{f}})} \} \tag{72}$$

and the contribution of magnon hopping processes

$$\Gamma_{h(M)}(\delta;\omega) \approx \Gamma^{(1)}_{h(M)}(\delta;\omega) \text{ at } \omega \approx G \tag{73}$$

neglecting corrections $\sim (2S)^{-1}\left(\frac{T}{T_C}\right)^{5/2}$ (see (57)).

Applying (7) and the standard transformation $t+i\frac{\beta}{2}\to t$ in (72), and neglecting corrections $\simeq \exp(-\beta G)\exp(\pm\frac{1}{2}\beta(\omega-G))$ at $\beta G \gg 1$ and $\omega\approx G$, one can transform (72) to the form:

$$\Gamma_{h(M)}(\mathbf{\delta};\omega) \approx \Gamma^{(1)}_{h(M)}(\mathbf{\delta};\omega) \approx \frac{\sqrt{2}\,|\Delta_e(\mathbf{\delta})|^2 e^{-2\Phi}}{6\pi S^2 I}\left(\frac{kT}{2IS}\right)^{3/2}\varphi(z') \tag{74}$$

where

$$\varphi(z) = 3\pi^2 S\left(\frac{\beta}{2}\right)^{3/2}(2IS)^{5/2}e^{-z}\frac{1}{N}\sum_{f}\frac{|\Phi_f|^2}{\operatorname{sh}\frac{\beta\Omega_f}{2}}(1-\cos \boldsymbol{f}\cdot\delta)\,\delta\left(z'\pm\frac{\beta\Omega_f}{2}\right)$$

$$\approx \tfrac{3}{2}\pi^2\left(\frac{\beta}{2}\right)^{3/2}(2IS)^{5/2}\int\frac{g_M(\Omega)d\Omega}{\operatorname{sh}\frac{\beta\Omega}{2}}\left(1-\frac{\sin\sqrt{2\mu\Omega a^2}}{\sqrt{2\mu\Omega a^2}}\right)\delta\left(z'\pm\frac{\beta\Omega}{2}\right) = \frac{|z'|}{\operatorname{sh}|z'|}e^{-z'},$$

$g_M(\Omega) = \frac{\mu^{3/2}\sqrt{\Omega}}{\pi^2\sqrt{2}}$ is the density of magnon frequencies Ω. The function $\varphi(z')$ has minimum at $z'=0$ ($\omega=G$), a maximum at $z'\approx 0.5$, *i. e.* at $\omega = G+kT=\omega_m$, and an inflectional point at $z'\approx -0.5$, *i. e.* at $\omega - G\approx -kT$, so that

$$(\varphi(z'))_{\min} \equiv \varphi(z'=0)=0,\ (\varphi(z'))_{\max}=\varphi(z'\simeq 0.5)\simeq 0.42 \tag{75}$$

and $\varphi(z'\approx -0.5)\approx 1.1$.

The schematic curve for $\varphi(z')$ in the neighbourhood of these extremum points is given in Fig. 1. In fact, $\Gamma_{h(M)}(\delta;\omega)$ at $\omega=G$ is very small (but non-zero),

$$\frac{\Gamma_{h(M)}(\mathbf{\delta};\omega=G)}{\Gamma_{h(M)}(\mathbf{\delta};\omega\simeq G+kT)} \lesssim \frac{1}{2S}\left(\frac{T}{T_C}\right)^{5/2} \ll 1.$$

Actually, $\Gamma^{(1)}_{h(M)}(\delta;\omega=G)=0$, so that the absorption peak lies at $\omega\neq G$ because the density $g_M(\Omega)$ of magnon frequencies Ω_f is zero at $\Omega=0$: the absorption peak is shifted to higher $\omega = G+kT$ (at lower ω ($\approx G-kT$), the symmetric maximum of function $|z'|^{3/2}/\operatorname{sh}|z'|$ is transformed into an inflectional point due to the effect ot the "background" $\sim e^{-\beta z'}=e^{\beta z'}$ which grows with $|z'|$ at $z'<0$). Thus, the small polaron hopping conductivity $\bar\sigma_{xx}(\omega)\approx \bar{\sigma}^h_{\overset{\circ}{xx}}(\omega)$ and the related infrared absorption coefficient $\eta_p(\omega)$ at $\omega\approx G$ has the structure shown in Fig. 1. When the conductivity $\bar\sigma_{xx}(\omega)$ is determined in (70) by the one-magnon hops, $\Gamma_h(\omega)\approx\Gamma^{(1)}_{h(M)}(\omega)$, *i. e.*, at $\omega\approx G\gg kT$,

$$\bar\sigma_{xx}(\omega)\approx\frac{2e^2N_c}{\omega}\sum_{\delta}\delta_x^2\Gamma_{h(M)}(\mathbf{\delta};\omega)\approx\frac{2e^2N_c}{\omega}\sum_{\delta}\delta_x^2\Gamma^{(1)}_{h(M)}(\mathbf{\delta};\omega). \tag{76}$$

The absorption peak is nonsymmetric — its left half-width ($\omega < \omega_m \equiv G+kT$) is smaller than the right ($\omega > \omega_m$) one and increases with T as well as the frequency ω_m of the peak, and the height of the peak increases with T as $T^{3/2}$. Now, it is necessary to study the conditions when (76) actually holds at $\omega \approx G$, *i. e.* to make comparative estimates of $\Gamma_{h(M)}(\boldsymbol{\delta}; \omega)$ with the contribution $\Gamma_{h(ph)}(\boldsymbol{\delta}; \omega)$ of phonon hops in $\Gamma_h(\boldsymbol{\delta}; \omega)$. If $\omega_{\text{pol}} \gg G \; (\gg kT)$, the

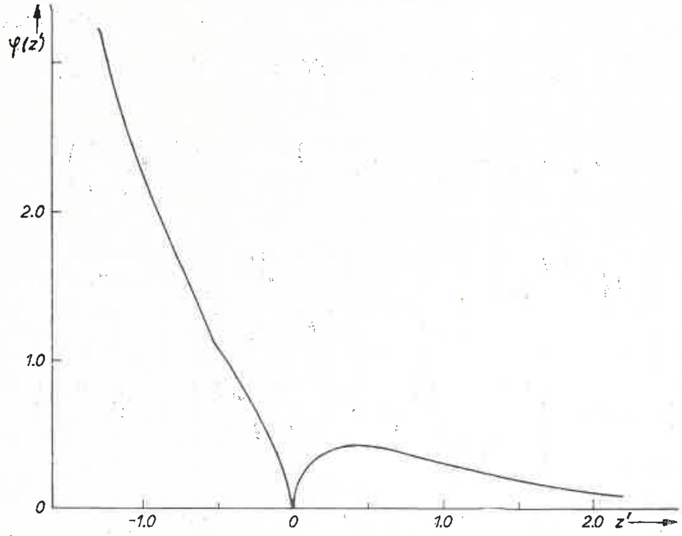

Fig. 1. The curve for $\varphi(z')$ determining $\bar{\sigma}_{xx}(\omega)$ and $\eta_p(\omega)$ near $\omega = G$; $z' = (\omega - G)/2kT$ for the high-frequency conductivity $\bar{\sigma}_{xx}(\omega)$ and $z' = (\omega_E - G)/2kT$ for the high-electric field conductivity $\bar{\sigma}_{x\lambda}(E)$, $\omega_E \equiv |e|Ea$

contribution $\Gamma_{h(\text{pol})}(\boldsymbol{\delta}; \omega)$ of polar phonons can be finite, describing only ν-phonon processes with $\nu > 1$ and, therefore, they are relatively small ($\sim \Phi_0^{\text{pol}} e^{-\beta\omega_{\text{pol}}}$, see (57)), but it can be negligible if $\omega_{\text{pol}} \gg G > \delta\omega$. The contribution $\Gamma_{h(ac)}(\boldsymbol{\delta}; \omega)$ of the acoustic phonons can be significant at $\omega \approx G$ if $G < \omega_D$, so that one-phonon hops dominate and, with approximation (6) (somewhat overestimating this contribution)

$$\Gamma_{h(ac)}(\boldsymbol{\delta}; \omega) \approx \Gamma^{(1)}_{h(ac)}(\boldsymbol{\delta}; \omega) \approx 4\Delta_e^2 e^{-2\Phi} \operatorname{ch} \frac{\beta\omega}{2} \times$$

$$\times \frac{1}{N} \sum_{\boldsymbol{f}} \frac{|X_{\boldsymbol{f}}^{ac}|^2 \sin^2 \dfrac{\boldsymbol{f} \cdot \boldsymbol{\delta}}{2}}{\operatorname{sh} \dfrac{\beta u_0 |\boldsymbol{f}|}{2}} \delta(\omega - u_0 |\boldsymbol{f}|) \approx 2\pi \Delta_e^2 e^{-2\Phi} \Phi_0^{ac} \frac{G}{\omega_D^2} \left(1 - \frac{\left(\sin \dfrac{\pi G}{\omega_0}\right)}{(\pi G/\omega_D)}\right) \tag{77}$$

at $\omega \approx G < \omega_D$. In this way, at $\omega - G \approx kT$ and $G < \omega_D$

$$\frac{\Gamma_{h(M)}(\omega)}{\Gamma_{h(ac)}(\omega)} > 1 \text{ only, when}$$

$$\frac{3\omega_D}{\sqrt{2}\,\pi S^2 I \Phi_0^{ac}} \left(\frac{kT}{2IS}\right)^{3/2} \left(\frac{\omega_D}{\pi G}\right)^3 \varphi(z') > 1. \tag{78}$$

If $\omega_{pol} \gg G > \{\omega_D; \delta\omega\}$, the contribution $\Gamma_{h(ac)}(\delta; \omega)$ is essentially smaller (as far as $\Phi_0^{ac} \left(\frac{T}{T_0}\right)^4 < 1$ in (57)), so that

$$\Gamma_{h(M)}(\omega)/\Gamma_{h(ac)}(\omega) > 1 \text{ if } \frac{3\omega_D \varphi(z')}{\sqrt{2}\,\pi S^2 I (\Phi_0^{ac})^{\nu+1}} \left(\frac{kT}{2IS}\right)^{3/2} \left(\frac{\omega_D}{\pi G}\right)^3 \left(\frac{T_0}{T}\right)^{4\nu} > 1 \qquad (79)$$

where $\nu = [G/\omega_D]$ (this condition is less stringent than (78)). Thus, the infrared small polaron absorption is actually determined by one-magnon hops with splin flip, and (74), (76) take place (Fig. 1), when conditions (78) at $\omega_{pol} > G > \delta\omega$ and $\omega_D > G$ or (79) are fulfilled[13].

Now, one can apply the analogy between the behaviour of the high-frequency small polaron conductivity $\bar{\sigma}_{\underset{\circ}{xx}}(\omega) \approx \bar{\sigma}^h_{\underset{\circ}{xx}}(\omega)$ as a function of ω and that of the small polaron conductivity $\bar{\sigma}_{xx}(\boldsymbol{E}) = \bar{\sigma}^h_{xx}(\boldsymbol{E})$ in high stationary electric fields $\omega_E = |e|\ E\, a > kT$, as stated in [10] (see also [13])[14]. $\sigma_{\underset{\circ}{xx}}(\boldsymbol{E})$ is obtained from $\bar{\sigma}_{\underset{\circ}{xx}}(\omega)$ by replacing in the last formula

$$\omega \to |e\,(\boldsymbol{E}\cdot\boldsymbol{\delta})| \equiv \omega_E(\boldsymbol{\delta}) \qquad (80)$$

Note that both $\bar{\sigma}_{xx}(\omega)$ at $\omega > kT$ and $\sigma_{xx}(\boldsymbol{E})$ at $\omega_E \equiv |e|\ Ea > kT$ are basically due to hopping transport, *i. e.* $\bar{\sigma}_{xx}(\omega) \approx \bar{\sigma}^h_{\underset{\circ}{xx}}(\omega)$ and $\sigma_{xx}(\boldsymbol{E}) \approx \sigma^h_{\underset{\circ}{xx}}(\boldsymbol{E})$. By applying (80) to the formulas (74), (76), one obtains the following formula for $\sigma_{xx}(\boldsymbol{E})$ at $\omega_E \equiv |e|\ Ea \approx G$:

$$\sigma_{xx}(\boldsymbol{E})|_{\omega_E \backsimeq G} \simeq \sigma^h_{\underset{\circ}{xx}}(\boldsymbol{E})|_{\omega_E \backsimeq G} \approx 2e^2 N_c \sum_{\boldsymbol{\delta}} \delta_x^2 \Gamma^{(1)}_{h(M)}(\boldsymbol{\delta}; \omega_E(\boldsymbol{\delta}))$$

$$\approx \frac{2e^2 N_c z \Delta_e^2 a^2}{3\pi I \sqrt{2}\, S^2} \left(\frac{kT}{2IS}\right)^{3/2} e^{-2\Phi} \varphi(z') \qquad (81)$$

with $z' \equiv (\omega_E - G)/2kT$; $\omega_E \equiv |e|\ Ea$, if conditions (78) or (79) are fulfilled. The shematic curve for $\sigma_{\underset{\circ}{xx}}(\boldsymbol{E})|_{\omega_E \backsimeq G}$ is the same as in Fig. 1. Note that the $\sigma_{\underset{\circ}{xx}}(\boldsymbol{E})$ curve has in the region $\omega_E \approx G$ intervals of $\boldsymbol{E}$ in which the differential conductivity $\sum(\boldsymbol{E}) = \frac{dJ_x(\boldsymbol{E})}{dE_x}$ is negative (< 0), at $\omega_E < G$ and $\omega_E < \omega_m$.

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[13] This characteristic infrared absorption peak can be separated from the lattice ones, for either the small polaron concentration $N_c(T)$ can significantly increase with T or the gap G essentially differs from the characteristic frequencies of the lattice-absorption peaks. When $\omega_m \gg \omega_{pol}$, the peak at $\omega_m \approx G + kT$ under consideration can be hardly observed against the background of the basic small polaron Gaussian absorption band which also exists at low T (see [1c]).

[14] At this opportunty let us correct some misprints in [10*a*]: the first formula on p. 84 and the third formula on p. 87 should read

$$g_q(\varepsilon_1 \dots \varepsilon_q) \to \prod_{i,j} g_2(\varepsilon_i, \varepsilon_j) \simeq \prod_{i,j} g_1(\varepsilon_i) g_1(\varepsilon_j) g'(\omega_{ji})$$

and

$$g_2(\varepsilon_1, \varepsilon_2) \simeq g_1(\varepsilon_1) g_1(\varepsilon_2) g'(\omega_{21}),$$

respectively, with $g'(\omega_{21})$ the appropriate energy-difference correlation function (see (8) and (11) in [10]).

An experimental investigation of these characteristic effects predicted for high-frequency conduction $\bar{\sigma}_{xx}(\omega \approx G)$ and for conduction $\bar{\sigma}_{xx}(\boldsymbol{E})$ in a strong electric field, at $\omega_E \approx G$, seems to be interesting both as an observation of the action of the ferromagnetic spin system on the high-frequency and high-field small polaron properties, and as a fairly precise way of determining the parameter $|A_{sd}|$ of the $s-d$ exchange.

Concluding remarks

1. EM coupling in the ferromagnetic semiconductor (in spin-wave region $T \ll T_C$) is not strong at $2S > 1$, even though $|A_{sd}|/I \gg 1$. This is associated with the essential role of the "static" part of the $s-d$ exchange field splitting the electron conduction band in to two subbands ($\sigma = \pm\frac{1}{2}$) with a gap G ($\gg kT$). It can be expected that the qualitative results hold also in the case $2S = 1$.

2. The "static" $s-d$ exchange field and EM coupling cause a characteristic nonmonotonous variation of the high-frequency small polaron conductivity, (the associated absorption coefficient) with ω at $\omega \approx G$ and the high electric field small polaron conductivity with electric field $\boldsymbol{E}$ at $\omega_E \approx G$ (Fig. 1). This specific effect is due to small polaron hops with spin flip and one-magnon processes conserving the z-component of the total spin and it allows an estimate of the $s-d$ exchange parameter $|A_{sd}|$ to be made. A similar characteristic effect should be expected in antiferromagnets (this should be separately discussed elsewhere).

3. For the band regime ($T < T_{xx} \sim T_1$), EM coupling determines the additional magnon scattering of smal polarons of the Born and non-Born type, the first dominating at sufficiently low T (the last can also dominate at higher T under some conditions, see above). This results, together with phonon scattering, in diverse power-type (or exponential) T-dependences of the mobility. In the band regime and spin-wave region under consideration, the characteristic transport time τ_{tr} describing boson scattering of the band small polarons and the mobility determined by (60), (61) is sufficiently large if the well-known criterion of Boltzmann-type transport in a narrow band ($\Delta_p = \Delta_e e^{-\Phi} \ll kT$) are fulfilled

$$\tau_{tr}^{-1} < \Delta_p, \; i.\,e.\; l_{tr} \approx v_p \tau_{tr} > \frac{v_p}{\Delta_p} \approx (m^* \Delta_p)^{-1/2} = \lambda_p \approx a, \tag{82}$$

Here, $v_p \approx a\Delta_p$, Eq. (42), $m^* \approx (a^2 \Delta_p)^{-1}$, Eq. (63), l_{tr} and λ_p are typical free-path (transport) length and be Broglie wave length, respectively, and $\lambda_p \approx a$ for a narrow band ($\Delta_p < kT$). In future work discussion of spontaneous small polaron galvanomagnetic effects will be given.

4. Let us estimate, for comparison, the contribution $\tau_{tr}^{(d)}$ to the transport time τ_{tr} caused by the small polaron scattering on point defects of the lattice (impurities *etc.*) which can dominate at sufficiently low T and high defect concentration $N_d (\ll N)$ in a non-ideal lattice. As the band width $D_p \approx 2z\Delta_p$ can actually be smaller than the typical magnitude of the defect potential the defect scattering, generally speaking, is of a non-Born type. Then, at not very high N_d, when

$$l_{tr}^{(d)} \approx v_p \tau_{tr}^{(d)} \gg \lambda_p \approx a, \tag{83}$$

this scattering can be described by the exact "one-defect" scattering amplitude. In particular,

$$\tau_{tr}^{(d)} \approx (v_p N_d t_d^2)^{-1} \ll \Delta_p^{-1}, \; i.e. \; l_{tr}^{(d)} \approx (N_d t_d^2)^{-1} \gg a, \tag{84}$$

at

$$N_d \ll (a t_d^2)^{-1}$$

for the case of a short-range defect potential with the radius $r_0 (\gtrsim a)$, where t_d is the characteristic amplitude of "forward" scattering, *i.e.* $t_d(\boldsymbol{k}-\boldsymbol{k}') \approx t_d(0) \equiv t_d(\neq 0)$ at small transfer of small-polaron crystal momentum $|\boldsymbol{k}-\boldsymbol{k}'| \ll \lambda_p^{-1} \approx a^{-1}$. In particular, $t_d \approx r_0^2 \gtrsim a^2$ so that $N_d \ll (a t_d^2)^{-1} < a^{-3} \approx N$. Then, comparing $\tau_{tr}^{(d)}$ with (84), one finds that the boson scattering of small polarons prevails and the discussion given in Sec. 2 holds true, with $N_d < \overset{\circ}{N}_d \equiv \overset{\circ}{N}_d(T)$ and $\tau_{tr}^{(d)}(\overset{\circ}{N}_d) \approx \tau_{tr}^{(\mathrm{bos})}(T)$. In particular when conditions (84) are satisfied we have

$$N_d < \overset{\circ}{N}_d \approx \underset{\circ}{\Omega}(v_p t_d^2)^{-1} \quad \text{and} \quad \underset{\circ}{\Omega} = \underset{\circ}{\Omega}_{ac} = \underset{\circ}{\Omega}_M + \underset{\circ}{\Omega}_{\mathrm{pol}}. \tag{85}$$

In the opposite case $N_d > \overset{\circ}{N}_d$ see (Eqs (61)–(62)),

$$\mu_{xx}(\omega) \sim T^{-1}; \; \mu_H^{(1)} \sim T^0; \; \mu_H^{(2)} \sim T^{-1} \text{ and } \frac{|\Delta\sigma_{xx}(\boldsymbol{H})|}{\sigma_{xx}} \sim T^0 \tag{86}$$

with $\tau_{tr} \approx \tau_{tr}^{(d)} \approx (v_p N_d t_d^2)^{-1} \approx T^0$ (defect scattering is the basic scattering for band small polarons).

5. Generally speaking, the effect of the spin system on the small polaron transport in a ferromagnetic crystalline semiconductor outside spin-wave region is more complicated and strong — we hope to discuss this in detail in a future work. (Here $D_p \ll kT$ and $D_p \ll |A_{sd}|$ whereas for magnetofluctuons — magnetic polarons of strong coupling and large radius, $D_e \approx 2z\Delta_e \gg |A_{sd}| \gg kT$ is assumed [14].) The comparatively simple influence of the $s-d$ exchange on the hopping mobility at temperatures near T_C is discussed in [2], [7] actually for the case when $T_C \gg T_{xx} (\simeq T_1)$. The situation is much more difficult for the case when $T_C \ll T_{xx} (\approx T_1)$ and the band-regime region contains a part of the paramagnetic region. Let us make a remark concerning this actual case. Basing on the preliminary analysis of the Kubo formula for this case performed by one of the authors (M.I.K.) it can be concluded that in the region where the localized spins S are essentially disordered (not too far from T_C), in the paramagnetic region at least, at $G = |A_{sd}|(2S+1) \gg kT$ ($|A_{sd}| \gg I$), the small polaron mobility,

$$\mu_{xx} \approx \frac{|e|}{m^* \Delta_p} \beta \Delta_p \approx \frac{|e| a^2 \Delta_p}{kT} = \mu_0 \beta \Delta_p \ll \mu_0 \tag{87}$$

with $\mu_0 = |e| a^2 \approx 1$ cm²/V·s, can be interpreted as the quantum-mechanical non-activation type Brownian diffusion (Markovian process) due to noninterferring individual tunnel transitions ($\sim \Delta_p$) of the small polaron between nearest neighbour equivalent sites and to its strong scattering on short-range ($\approx a$) fluctuations of the disordered spin configurations.[15]

[15] In this respect, the mobility under consideration has similar features with the Brownian diffusion of an electron in a system with disordered atomic configurations as discussed in [15], though the dependences on T and other parameters differ from each other. Thus, it can be expected that (87) holds true for small polarons in systems with disordered atomic configurations as well.

Thus, accepting the transport process discussed, one can conclude that the temperature interval, near T_C, exists for small polarons in a ferromagnetic semiconductor (between the band region, at $T < T_{xx}$, and the hopping one [1, 2]) in which criterion (82) is not fulfilled for the strong spin scattering and the formula (87) is similar to (63) for the band mobility, but with the smallest limit values of effective transport length $l_{tr} \approx \lambda_p \approx a$ and time $\tau_{tr} \approx (\Delta_p)^{-1}$ (The details of the calculations will be published in future)[16].

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[16] Actually, the estimate of μ_{xx} in (87) of the form

$$\mu_{xx} \approx |e|/m^*kT$$

should hold true both for narrow bands, $\Delta_p \ll kT$ and for broad bands, $\Delta_p \gg kT$ (considering m^* as a fixed parameter), so that in (87) $\tau_{tr} \simeq [\min\{\Delta_p; kT\}]^{-1}$. Supposing that this is the case, one can conclude also that (87) should be true both for $\Phi \gg 1$ and for $\Phi \ll 1$.