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DEPENDENCE OF MAGNON RELAXATION ON DISLOCATION DENSITY

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The relaxation time for uniform magnons scattered on a system of parallel dislocation lines with concordant Burgers vectors is calculated. A discussion for high and low concentration of the dislocations shows the reciprocal relaxation time to be proportional, respectively, to the first power and square of the dislocation density in the crystal.

1. *Introduction*

First, we have to define low and, respectively, high dislocation densities in our case. By low density we understand cases when $1/K \ll d$; here, K is the "mean" wave vector of a magnon arising by scattering of a uniform mode on the dislocations, distant by d on the average; the resulting magnon wavelength is much smaller than the mean distance between dislocations. On the other hand, cases when $1/K \gg d$ will be understood to imply high dislocation densities.

For low densities, results are in agreement with Ref. [1], *i. e.* $1/\tau \sim n$ (n — dislocation density, τ — relaxation time). For high densities, however, results differ from those of [1]. This is due to the circumstance that Baryakhtar had assumed the dimensions of dislocation loops as proportional to the distance between the dislocations. With growing density, the loops went over into point defects. In the present paper, however, calculations will be performed for dislocation lines assumed to have dimensions independent of the dislocation density in the crystal and large as compared to the wavelength of non-uniform magnons (thus, $L \gg 1/k, r \gg 1/k$, with L — the length of a dislocation line, r — the dislocation radius, and k — the magnon wave vector). The individual dislocation loops at high dislocation density, in Baryakhtar's paper, were "seen" by the magnons as point defects; this led to the result $1/\tau \sim n^{-1/2}$. The result proposed here for high density, namely $1/\tau \sim n^2$, is derived on the assumption of parallel, large-sized dislocation lines, with uniformly directed Burgers vectors.

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For dislocation dipoles (parallel dislocation lines with anti-parallel Burgers vectors), the contributions from the different dislocations cancel out (*cf.* Ref. [3]). This complicates the $1/\tau$ *vs.* n dependence for real crystals.

2. *Relaxation time of scattering of the uniform mode into a spectrum of non-uniform magnons*

We restrict our present considerations to crystals of cubic symmetry; the crystallographical axes are assumed to coincide with the ellipsoidal axes of the specimen. Magnon-dislocation coupling is described by a magneto-elastic energy. The relaxation time τ of uniform magnons ($k = 0$) scattered by the dislocations into magnons with $k \neq 0$ (taking two-magnon processes into account only) is given by the expression [2]:

$$1/\tau = (2\pi/\hbar) \sum_{k\neq 0} |W_k|^2 \delta(\varepsilon_0 - \varepsilon_k), \tag{1}$$

where the matrix element W_k is equal:

$$W_k = (2\mu_B B_1/MV) \int d\boldsymbol{r} \{\bar{e}_{xx}(\boldsymbol{r}) + \bar{e}_{yy}(\boldsymbol{r}) - 2\bar{e}_{zz}(\boldsymbol{r})\} \exp(i\boldsymbol{k}\cdot\boldsymbol{r}) \tag{2}$$

ε_k is the energy of a non-uniform magnon ($k \neq 0$):

$$\varepsilon_k = 2\mu_B\{(H - 4\pi N_z M + \alpha k^2)(H - 4\pi N_z M + \alpha k^2 + 4\pi M \sin^2 \Theta_k)\}, \tag{3}$$

whereas the energy ε_0 of the uniform magnon ($k = 0$) is:

$$\varepsilon_0 = 2\mu_B\{[H + 4\pi(N_x - N_z)M]\,[H + 4\pi(N_y - N_z)M]\}^{1/2}; \tag{4}$$

other notations are: M — saturation magnetization, V — volume of the specimen, B_1 — magneto-elastic coupling constant, α — exchange parameter, Θ_k — azimuth angle of the wave vector $\boldsymbol{k}$, N_x, N_y, N_z — demagnetisation factors, H — external magnetic field applied along the z-axis.

The strain tensor $\bar{e}_{ij}(\boldsymbol{r})$ is the sum of the strain tensors of all dislocations in the crystal:

$$\bar{e}_{ij}(\boldsymbol{r}) = \sum_R e_{ij}(\boldsymbol{R}, \boldsymbol{r}). \tag{5}$$

The strain field of the $\boldsymbol{R}$-th dislocation, whose axis makes an angle ψ with its Burgers vector is given by:

$$e_{ij}(\boldsymbol{R}, \boldsymbol{r}) = e_{ij}^{\text{screw}} \cos\psi + e_{ij}^{\text{edge}} \sin\psi, \tag{6}$$

where e_{ij}^{screw} and e_{ij}^{edge} are the strain fields of pure-screw and pure-edge dislocations with the same axis. The vector $\boldsymbol{R}$ labelling the dislocation connects its centre and the origin of the co-ordinate axes x, y, z.

3. *Matrix element of scattering of uniform magnons by the dislocation system*

Calculations will be performed for a system of parallel dislocation lines with uniformly directed Burgers vectors. The lines subtend the angle ϑ with the z-axis, the Burgers vector of the pure-edge dislocation being parallel to the x-axis. It is of advantage to introduce co-ordinate systems (x_1^R, x_2^R, x_3^R) attached to the dislocations, with the origin at the centre of the

$\boldsymbol{R}$-th dislocation, the x_3^R-axis coinciding with the dislocation line, and the x_1^R-axis parallel to x. The strain tensor field of the $\boldsymbol{R}$-th dislocation is non-zero for $|x_3^R| \leqslant L/2$, $(x_1^R)^2 + (x_2^R)^2 \leqslant r^2$.

By Eqs (2) and (5), and with regard to the preceding considerations, the matrix element W_k for scattering of uniform magnons by a system of parallel dislocations with identically directed Burgers vectors, takes the form:

$$W_k = (2\mu_B B_1/MV) \sum_R \exp(i\boldsymbol{k}\cdot\boldsymbol{R}) \int_{V_R} d\boldsymbol{r}_R \exp(i\boldsymbol{k}\cdot\boldsymbol{r}_R) \times$$
$$\times \{e_{11}(\boldsymbol{R}, \boldsymbol{r}_R) + (1-3\sin^2\vartheta)\, e_{22}(\boldsymbol{R}, \boldsymbol{r}_R) + (1-3\cos^2\vartheta)\, e_{33}(\boldsymbol{R}, \boldsymbol{r}_R) -$$
$$-3\sin 2\vartheta\, e_{23}(\boldsymbol{R}, \boldsymbol{r}_R)\}, \tag{7}$$

where integration extends over the volume of the cylinder inside which the strain tensor field of $\boldsymbol{R}$-th dislocation is non-vanishing and $\boldsymbol{r}_R = \boldsymbol{r} - \boldsymbol{R}$.

For the $\boldsymbol{R}$-th pure screw and, respectively, edge dislocation, the tensor components are [4]:

$$e_{13}^{\text{screw}} = -(b/4\pi)(1/\varrho)\sin\varphi,$$
$$e_{23}^{\text{screw}} = (b/4\pi)(1/\varrho)\cos\varphi, \tag{8}$$
$$e_{11}^{\text{edge}} = -[b/4\pi(1-\nu)](1/\varrho)\sin\varphi(\cos^2\varphi - \sin^2\varphi) - (b/2\pi)(1/\varrho)\sin\varphi,$$
$$e_{22}^{\text{edge}} = [b/4\pi(1-\nu)](1/\varrho)\sin\varphi(3\cos^2\varphi + \sin^2\varphi) - (b/2\pi)(1/\varrho)\sin\varphi,$$
$$e_{12}^{\text{edge}} = [b/4\pi(1-\nu)](1/\varrho)\cos\varphi(\cos^2\varphi - \sin^2\varphi), \tag{9}$$

where ϱ, φ are polar coordinates ($\varrho\cos\varphi = x_1^R$, $\varrho\sin\varphi = x_2^R$), ν is Poisson's constant, and b — the length of the Burgers vector. All the other tensor components vanish.

On integration, we obtain W_k in the form:

$$W_k = (W_k^{\text{screw}}\cos\psi + W_k^{\text{edge}}\sin\psi) \sum_R \exp(i\boldsymbol{k}\cdot\boldsymbol{R}). \tag{10}$$

W_k^{screw} and W_k^{edge} are matrix elements for scatterings of the uniform mode by a single, pure screw or edge dislocation (*cf. Ref.* [2]):

$$W_k^{\text{screw}} = i(6\mu_B b B_1/MV)\sin 2\vartheta\cos\Phi\sin(k_3L/2)\{1 - J_0(rk_0)\}(1/k_0k_3), \tag{11}$$
$$W_k^{\text{edge}} = -i[4\mu_B b B_1/MV(1-\nu)]\sin(k_3L/2)(1/k_0k_3)\sin\Phi \times$$
$$\times\{(1-2\nu+3\nu\sin^2\vartheta - 3\sin^2\vartheta\cos^2\Phi)[1-J_0(rk_0)] +$$
$$+(2/3)\sin^2\vartheta(4\cos^2\Phi - 1)J_2(k_0)\}; \tag{12}$$

k_0, k_3, Φ are cylindrical co-ordinates of the wave vector $\boldsymbol{k}$ ($k_0\cos\Phi = k_1$, $k_0\sin\Phi = k_2$; the k_1, k_2, k_3 — components of the wave vector lie in the directions of the axes x_1^R, x_2^R, x_3^R).

4. *Relaxation time of the uniform mode scattered on a system of parallel dislocations*

By Eqs (1) and (10), the relaxation time is:

$$1/\tau = (2\pi/\hbar)\sum_k |W_k^{\text{screw}}\cos\psi + W_k^{\text{edge}}\sin\psi|^2 \sum_R\sum_{R'}\cos\{\boldsymbol{k}\cdot(\boldsymbol{R}-\boldsymbol{R}')\}\,\delta(\varepsilon_k - \varepsilon_0). \tag{13}$$

We replace the sum $\sum_k (\ldots)$ by the integral $(V/8\pi^3)\int d\boldsymbol{k}(\ldots)$, and approximate the factor $\{\sin (k_3L/2)/k_3\}^2$ in $|W_k^{\text{screw}} \cos \psi + W_k^{\text{edge}} \sin \psi|^2$ by $(\pi/2)\, L\delta(k_3)$ (*cf.* Ref. [2]). On integration, the expression for the relaxation time becomes:

$$1/\tau = \frac{\mu_B b^2 B_1^2 L}{16\pi^2\hbar(1-\nu)^2 M^3 V} \int_0^{2\pi} d\Phi \frac{\Theta(u^2)}{u^2 \sqrt{1+\chi^2(1-\sin^2\vartheta \sin^2\Phi)^2}} \times$$

$$\times\{3(1-\nu) \sin 2\vartheta \cos \Phi \cos \psi\, [1-J_0(\lambda u)] - 3 \sin^2\vartheta \sin 3\Phi \sin \psi\, J_2(\lambda u) -$$

$$-2(1-2\nu+3\nu \sin^2\vartheta - 3 \sin^2\vartheta \cos^2\Phi) \sin \Phi \sin \psi [1-J_0(\lambda u)]\}^2 \times$$

$$\times \sum_R \sum_{R'} \cos \{\sqrt{2\pi M/\alpha}\, u|\boldsymbol{R}_0' - \boldsymbol{R}_0| \cos (\Phi - \zeta)\}, \tag{14}$$

where

$$u = \{\sin^2\vartheta \sin^2\Phi + [1/\chi^2 + (1-\sin^2\vartheta \sin^2\Phi)^2]^{1/2} - N_z - [(N_x - N_y)^2 - 1/\chi^2]^{1/2}\}^{1/2}, \tag{15}$$

$$\chi = 2\pi M/(\varepsilon_0/2\mu_B), \tag{16}$$

$$\lambda = r(2\pi M/\alpha)^{1/2}. \tag{17}$$

$\boldsymbol{R}_0$ is the projection if $\boldsymbol{R}$ on the plane perpendicular to the dislocation lines, ζ is the angle between the x-axis and the vector $(\boldsymbol{R}_0' - \boldsymbol{R}_0)$, and $\Theta(x)$ is the step function equal to unity for $x \geqslant 0$ and to zero for $x < 0$.

For our further discussion, we transform equation (14) by resorting to the mean value theorem of integral calculus

$$1/\tau = (1/\tau_0) \sum_R \sum_{R'} \cos \{K|\boldsymbol{R}_0' - \boldsymbol{R}_0| \cos (\Phi_0 - \zeta)], \tag{18}$$

where τ_0 is the relaxation time for scattering of the uniform mode, by a single dislocation line, into a spectrum of non-uniform magnons (see [2]):

$$1/\tau_0 = \frac{\mu_B b^2 B_1^2 L}{16\pi^2\hbar\, (1-\nu)^2 M^3 V} \int_0^{2\pi} d\Phi \frac{\Theta(u^2)}{u^2 \sqrt{1+\chi^2(1-\sin^2\vartheta \sin^2\Phi)^2}} \times$$

$$\times\{3(1-\nu) \sin^2\vartheta \cos \Phi \cos \psi\, [1-J_0(\lambda u)] - 3 \sin^2\vartheta \sin 3\Phi \sin \psi\, J_2(\lambda u) -$$

$$-2(1-2\nu+3\nu \sin^2\vartheta - 3 \sin^2\vartheta \cos^2\Phi) \sin \Phi \sin \psi\, [1-J_0(\lambda u)]\}^2, \tag{19}$$

where we understand $K = (2\pi M/\alpha)^{1/2} u(\Phi = \Phi_0)$ as the mean wave vector of a magnon arising by scattering of the uniform mode on the dislocations; Φ_0 is a parameter fulfilling the condition $0 \leqslant \Phi_0 \leqslant \pi$.

We now proceed to consider two particular cases:

(*i*) *Low dislocation density*, $d \gg 1/K$. The dislocations are assumed to be uniformly distributed throughout the crystal. With regard to the identity

$$\exp \{iKd \cos (\Phi_0 - \zeta)\} = \sum_{n=-\infty}^{\infty} i^n \exp \{in\, (\Phi_0 - \zeta)\}\, J_n(Kd) \tag{20}$$

and the asymptotic behaviour of Bessel's functions of large argument ($Kd \gg 1$), we obtain in a first approximation for low concentrations of dislocations in the crystal:

$$\sum_R \sum_{R'} \cos\{K|\boldsymbol{R}_0 - \boldsymbol{R}'_0|\cos(\Phi_0 - \zeta)\} \simeq N \tag{21}$$

N denotes the number of dislocation lines in the crystal. From the preceding and the shape of $1/\tau_0$, we draw the conclusion: $1/\tau = (1/\tau_0)\, N \sim n$. In this instance each dislocation scatters magnons independently (*cf.* [2]).

(*ii*) *High dislocation density in the specimen*, $d \ll 1/K$. We replace the sum $\Sigma_R(\ldots)$ by the integral $(1/S)\int\int dxdy\,(\ldots)$ (this is not permitted for low densities); S is the area *per* dislocation line of the plane perpendicular to the lines;

$$\sum_R \sum_{R'} \cos\{K|\boldsymbol{R}_0 - \boldsymbol{R}'_0|\cos(\Phi_0 - \zeta)\} \simeq$$

$$\simeq n^2 \int\int dxdy \int\int dx'dy' \cos\{K(x-x')\cos\Phi_0 + K(y-y')\sin\Phi_0\}. \tag{22}$$

For the present configuration of the dislocations, S is expressed in terms of the dislocation density as follows: $1/S = n$. We perform integration for the case of circular cross-section of the lines, leading to the simplest result:

$$\sum_R \sum_{R'} \cos\{K|\boldsymbol{R}_0 - \boldsymbol{R}'_0|\cos(\Phi_0 - \zeta)\} = 4\pi^2 n^2 L_0^2 J_1^2(KL_0)/K^2. \tag{23}$$

L_0 is a parameter equal to the radius of the specimen. From Eqs (18) and (23), we obtain the result $1/\tau \sim n^2$ — a conclusion meaningful, provided that the mean wave vector K of magnons arising by the scattering of the uniform mode on dislocations does not vary strongly with the dislocation density through the parameter Φ_0. Such a simple result may be obtained only for the dislocation configuration which we have discussed. For the dislocation configuration existing in real crystals we should take into account: dislocation dipoles and nodes, and the magnon scattering cross-section dependence on dislocation dimensions.

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