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MICROSCOPIC THEORY OF DILUTE He^3—He II SOLUTIONS. II. LINEARIZED HYDRODYNAMIC EQUATIONS AND GREEN FUNCTIONS

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After solving the linearized hydrodynamic equations the expressions for the retarded thermodynamic Green functions for He^3 — He II solutions are obtained in the long-wavelength low-frequency domain. Thed ensity-density Green functions have singularities only for the energies of the first sound quanta.

Introduction

In paper [1], referred to here as Part I, the hydrodynamic equations for dilute He^3—He II solutions were obtained on the basis of microscopic theory. The theory was proposed by Bogolyubov [2] for He II. From the full hydrodynamic equations it is easy to obtain linearized equations in the so-called acoustic approximation. These equations can be solved exactly, what enables us to consider the propagation of sound in a superfluid.

1. Linearization of hydrodynamic equations

Consider, by means of Eqs (55)—(59), (I), an infinitesimal deviation from thermodynamic equilibrium. The deviation is caused by infinitesimal scalar potential $\delta U(t, r)$ and infinitesimal "sources of particles" $\delta\eta(t, r)$ and $\delta\eta^*(t, r)$ introduced adiabatically into the basic Hamiltonian.

We put into the hydrodynamic equations ((55)—(59), I) the following:

$$\varrho = \varrho^0 + \delta\varrho(t, r), \quad \theta = \theta^0 + \delta\theta(t, r), \quad S = S^0 + \delta S(t, r), \tag{1}$$
$$\boldsymbol{v}_s = \delta\boldsymbol{v}_s(t, r), \quad \boldsymbol{v}_n = \delta\boldsymbol{v}_n(t, r), \quad \boldsymbol{v}_s^0 = \boldsymbol{v}_n^0 = 0,$$
$$\zeta \approx \delta\eta(t, r), \quad U(t, r) = \delta U(t, r).$$

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The terms with products of two variations are neglected, what gives the linearized hydrodynamic equations

$$\frac{\partial \delta \varrho^m}{\partial t} + \varrho_s^m \nabla \delta \boldsymbol{v}_s + \varrho_n^m \nabla \delta \boldsymbol{v}_n = i m_B \sqrt{\varrho_0}\,(\delta \eta^* - \delta \eta), \tag{2}$$

$$\varrho^m \frac{\partial \delta c}{\partial t} + c \frac{\partial \varrho^m}{\partial t} + \varrho^m c \nabla \delta \boldsymbol{v}_n = 0, \tag{3}$$

$$\varrho_s^m \frac{\partial \delta v_s^{(\alpha)}}{dt} + \varrho_n^m \frac{\partial \delta v_n^{(\alpha)}}{\partial t} + \frac{\partial \delta \mathscr{P}}{\partial r_\alpha} = -\left(\varrho_B^m \frac{1}{m_B} + \varrho_F^m \frac{1}{m_F}\right) \frac{\partial \delta U}{\partial r_\alpha}. \tag{4}$$

$$\frac{\partial \delta v_s^{(\alpha)}}{\partial t} + \frac{\partial \delta \mu_B}{\partial r_\alpha} = -\frac{1}{m_B} \frac{\partial \delta U}{\partial r_\alpha}, \tag{5a}$$

$$\frac{\partial \delta v_s^{(\alpha)}}{\partial t} + \frac{\partial}{\partial r_\alpha}(\mu - zc) = -\frac{1}{m_B} \frac{\partial \delta U}{\partial r_\alpha}, \tag{5b}$$

$$\varrho^m \frac{\partial \delta S}{\partial t} + S \frac{\partial \delta \varrho^m}{\partial t} + \varrho^m S \nabla \delta \boldsymbol{v}_n = 0 \tag{6}$$

(the superscript "0", referring to thermodynamic equilibrium, is omitted; see also (40), I).

Thus far it makes no difference what three independent parameters, (ϱ^m, θ, c) or $(\mathscr{P}, \theta, c)$, are chosen. For the calculation of the Green functions it is more convenient to take (ϱ^m, θ, c), but we shall express our formulae also in terms of the $(\mathscr{P}, \theta, c)$ parameters.

We have the following equations:

$$\mu = \frac{\partial(\varrho^m F)}{\partial \varrho^m} = F + \varrho^m \frac{\partial F}{\partial \varrho^m} = F + \frac{1}{\varrho^m} \mathscr{P}, \tag{7}$$

$$\mathscr{P} = (\varrho^m)^2 \frac{\partial F}{\partial \varrho^m}, \qquad S = -\frac{\partial F}{\partial \theta}, \; z = \frac{\partial F}{\partial c}.$$

From (7) it follows that

$$\delta \mu = \frac{1}{\varrho^m} \frac{\partial \mathscr{P}}{\partial \varrho^m} \delta \varrho^m + \left(-S + \frac{1}{\varrho^m} \frac{\partial \mathscr{P}}{\partial \theta}\right) \delta \theta + \\ + \left(z + \frac{1}{\varrho^m} \frac{\partial \mathscr{P}}{\partial c}\right) \delta c = \frac{1}{\varrho^m} \delta \mathscr{P} - S \delta \theta \tag{8}$$

and

$$\frac{\partial z}{\partial \varrho^m} = \frac{1}{(\varrho^m)^2} \frac{\partial \mathscr{P}}{\partial c}, \; \frac{\partial z}{\partial \theta} = -\frac{\partial S}{\partial c}, \; \frac{\partial S}{\partial \varrho} = -\frac{1}{(\varrho^m)^2} \frac{\partial \mathscr{P}}{\partial \theta}; \tag{9}$$

hence,

$$\delta z = \frac{\partial z}{\partial \varrho} \delta \varrho + \frac{\partial z}{\partial \theta} \delta \theta + \frac{\partial z}{\partial c} \delta c = \frac{1}{(\varrho^m)^2} \frac{\partial \mathscr{P}}{\partial c} \delta \varrho - \frac{\partial S}{\partial c} \delta \theta + \frac{\partial z}{\partial c} \delta c. \tag{10}$$

The second term in (5b) has the form

$$\delta(\mu - zc) = \delta\mu_B = \left(\frac{1}{\varrho^m}\frac{\partial\mathscr{P}}{\partial\varrho^m} - \frac{c}{(\varrho^m)^2}\frac{\partial\mathscr{P}}{\partial c}\right)\delta\varrho^m - \bar{S}\delta\theta +$$

$$+\left(\frac{1}{\varrho^m}\frac{\partial\mathscr{P}}{\partial c} - c\frac{\partial z}{\partial c}\right)\delta c, \quad \bar{S} = S - c\frac{\partial S}{\partial c}. \tag{11}$$

The variations $\delta v_s^{(\alpha)}$ and $\delta v_n^{(\alpha)}$ can be eliminated from (2)—(6), and we get

$$\frac{1}{S}\frac{\varrho_n^m}{\varrho_s^m}\delta\ddot{S} - S\nabla^2\delta\theta - c\nabla^2\delta z = -\frac{im_B}{\varrho^m}\frac{\varrho_n^m}{\varrho_s^m}\Delta\dot{\eta} +$$

$$+\frac{m_B - m_F}{m_F m_B}c\nabla^2\delta U, \quad \Delta\eta = \sqrt{\varrho_0}(\delta\eta^* - \delta\eta), \tag{12}$$

$$\delta\ddot{\varrho}^m - \nabla^2\delta\mathscr{P} = \varrho\nabla^2\delta U + im_B\Delta\dot{\eta}, \tag{13}$$

$$S\delta\dot{c} - c\delta\dot{S} = 0. \tag{14}$$

We express now all variations in (12)—(14) in terms of independent variations $\delta\varrho^m$, $\delta\theta$ and δc, *e. g.*,

$$\delta\ddot{S} = \frac{\partial S}{\partial\varrho}\delta\ddot{\varrho} + \frac{\partial S}{\partial\theta}\delta\ddot{\theta} + \frac{\partial S}{\partial c}\delta\ddot{c} = \ddot{S},$$

$$\nabla^2\delta\mathscr{P} = \frac{\partial\mathscr{P}}{\partial\varrho^m}\nabla^2\delta\varrho^m + \frac{\partial\mathscr{P}}{\partial\theta}\nabla^2\delta\theta + \frac{\partial\mathscr{P}}{\partial c}\nabla^2\delta c = \nabla^2\mathscr{P}. \tag{15}$$

Eqs (12)—(14) are linear, therefore, all variations can be written in the form

$$\delta f(t, r) = e^{-i\omega t + \varepsilon t + ikr}\delta t(k) + e^{i\omega t + \varepsilon t - ikr}\delta f(-k), \tag{16}$$

$$\omega + i\varepsilon = E, \ \varepsilon > 0, \ \varepsilon \to 0.$$

They vanish when $t \to -\infty$, *i. e.* at equilibrium.

The Fourier components $\delta\theta(k)$, $\delta\varrho^m(k)$ and $\delta c(k)$ can be found from the following algebraic equations

$$\left[-E^2\frac{\varrho_n^m}{\varrho_s^m}\frac{1}{S}\left(\frac{\partial S}{\partial\theta}\right)_{\varrho,c} + k^2\bar{S}\right]\delta\theta(k) + k^2\frac{1}{(\varrho^m)^2}c\left(\frac{\partial\mathscr{P}}{\partial c}\right)_{\varrho,\theta}\delta\varrho^m(k) +$$

$$+\left[-E^2\frac{\varrho_n^m}{\varrho_s^m}\frac{1}{S}\left(\frac{\partial S}{\partial c}\right)_{\varrho,\theta} + k^2 c\left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta}\right]\delta c(k) = \frac{m_B - m_F}{m_B m_F}ck^2\varrho\delta U(k) + E\frac{\varrho_n^m}{\varrho_s^m}\frac{m_B}{\varrho^m}\Delta\eta(k), \tag{17}$$

$$0.\ \delta\theta(k) + \left[-E^2 + k^2\left(\frac{\partial\mathscr{P}}{\partial\varrho^m}\right)_{\varrho,c}\right]\delta\varrho^m(k) + k^2\left(\frac{\partial\mathscr{P}}{\partial c}\right)_{\varrho,\theta}\delta c(k) = k^2\varrho\delta U(k) - Em_B\Delta\eta(k), \tag{18}$$

$$c\left(\frac{\partial S}{\partial\theta}\right)_{\varrho,c}\delta\theta(k) + 0.\ \delta\varrho^m(k) - \bar{S}\delta c(k), \quad \bar{S} = S - c\frac{\partial S}{\partial c}. \tag{19}$$

2. *The velocity of the first and second sound*

The determinant of the system of Eqs (17)—(19) is

$$\mathrm{Det} = \frac{\varrho_n^m}{\varrho_s^m}\frac{\partial S}{\partial \theta}\,\mathrm{Det}\,(E^2),$$

$$\mathscr{D}(E^2) = E^4 - E^2k^2\left[\frac{\partial \mathscr{P}}{\partial \varrho^m} + \frac{\varrho_s^m}{\varrho_n^m}\frac{\bar{S}^2}{\frac{\partial S}{\partial \theta}} + \frac{\varrho_s^m}{\varrho_n^m}c^2\left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta}\right] + \tag{20}$$

$$+k^4\frac{\partial \mathscr{P}}{\partial \varrho^m}\frac{\varrho_s^m}{\varrho_n^m}\left[\frac{\bar{S}^2}{\frac{\partial S}{\partial \theta}} + c^2\left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta} - \left(\frac{c}{\varrho^m}\frac{\partial \mathscr{P}}{\partial c}\right)^2_{\varrho,\theta}\Bigg/\frac{\partial \mathscr{P}}{\partial \varrho^m}\right].$$

In order to express the thermodynamic derivatives in $(\mathscr{P}, \theta, c)$-parameters we have the following formulae

$$\left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta} = \left(\frac{\partial z}{\partial c}\right)_{\mathscr{P},\theta} + \left(\frac{\partial \mathscr{P}}{\partial \varrho^m}\right)_{\theta,c}\left(\frac{1}{\varrho^m}\frac{\partial \varrho^m}{\partial c}\right)^2_{\mathscr{P},\theta} = \left(\frac{\partial z}{\partial c}\right)_{\mathscr{P},\theta} - \left(\frac{\partial \mathscr{P}}{\partial \varrho^m}\right)^{-1}_{\theta,c}\left(\frac{1}{\varrho^m}\frac{\partial \mathscr{P}}{\partial c}\right)^2_{\varrho,\theta},$$

$$\left(\frac{\partial z}{\partial c}\right)_{\mathscr{P},\theta} = \left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta} + \left(\frac{\partial \mathscr{P}}{\partial \varrho^m}\right)^{-1}_{\theta,c}\left(\frac{1}{\varrho^m}\frac{\partial \mathscr{P}}{\partial c}\right)^2_{\varrho,\theta} = \left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta} - \left(\frac{\partial \mathscr{P}}{\partial \varrho^m}\right)_{\theta,c}\left(\frac{1}{\varrho^m}\frac{\partial \varrho^m}{\partial c}\right)^2_{\mathscr{P},\theta},$$

$$\left(\frac{\partial \mathscr{P}}{\partial c}\right)_{\varrho,\theta} = -\left(\frac{\partial \mathscr{P}}{\partial \varrho^m}\right)_{\theta,c}\left(\frac{\partial \varrho^m}{\partial c}\right)_{\mathscr{P},\theta}, \tag{21}$$

$$\left(\frac{\partial S}{\partial \theta}\right)_{\varrho,c} = \left(\frac{\partial S}{\partial \theta}\right)_{\mathscr{P},c} + \left(\frac{\partial S}{\partial \varrho}\right)_{\theta,c}\left(\frac{\partial \mathscr{P}}{\partial \theta}\right)_{\varrho,c} \approx \left(\frac{\partial S}{\partial \theta}\right)_{\mathscr{P},c}.$$

Thus, the determinant (20) can be written in the form

$$\mathscr{D}(E^2) = E^4 - E^2k^2\left\{\left[1 + \frac{\varrho_s^m}{\varrho_n^m}\left(\frac{c}{\varrho^m}\frac{\partial \varrho^m}{\partial c}\right)^2_{\mathscr{P},\theta}\right]\frac{\partial \mathscr{P}}{\partial \varrho^m} + \right.$$

$$\left. + \frac{\varrho_s^m}{\varrho_n^m}\left[\frac{\bar{S}^2}{\frac{\partial S}{\partial \theta}} + c^2\left(\frac{\partial z}{\partial c}\right)_{\mathscr{P},\theta}\right]\right\} + k^4\left(\frac{\partial \mathscr{P}}{\partial \varrho^m}\right)_{\theta,c}\frac{\varrho_s^m}{\varrho_n^m}\left[\frac{\bar{S}^2}{\frac{\partial S}{\partial \theta}} + c^2\left(\frac{\partial z}{\partial c}\right)_{\mathscr{P},\theta}\right], \tag{22}$$

in agreement with Khalatnikov's results [3].

We are interested now in the roots of the equation

$$\mathscr{D}(E^2) = 0. \tag{23}$$

For pure He II they are of the form

$$E_1^2 = c_1^2k^2, \quad E_2^2 = c_2^2k^2, \tag{24}$$

$$c_1^2 = \frac{\partial \mathscr{P}}{\partial \varrho^m}, \quad c_2^2 = S^2\varrho_s^m\Bigg/\frac{\partial S}{\partial \theta}\varrho_n^m, \quad c_2^2 \ll c_1^2.$$

We start from the values

$$\mathring{c}_1^2 = \frac{\partial\mathcal{P}}{\partial\varrho^m}, \qquad \mathring{c}_2^2 = \frac{\bar{S}^2\overset{m}{\varrho}_s}{\dfrac{\partial S}{\partial\theta}\overset{m}{\varrho}_n}, \qquad \mathring{c}_2^2 \ll \mathring{c}_1^2 \tag{25}$$

and calculate the corrections to E_1^2 and E_2^2 (arising from the additional dependence on concentration c) as follows

$$\mathcal{D}(E^2+\delta E^2) = 0 = \mathcal{D}(E^2) + \left(\frac{d\mathcal{D}}{dE^2}\right)_E \delta E^2 + +,$$

$$\delta E_i^2 = -\mathcal{D}(E_i^2) \Big/ \left(\frac{d\mathcal{D}}{dE^2}\right)_{E_i}. \tag{26}$$

From (22) we have

$$\mathcal{D}(E_1^2) = -k^2 \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n}\left(\frac{c}{\varrho^m}\frac{\partial\mathcal{P}}{\partial c}\right)^2_{\varrho,\theta} = -k^2 \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n}(\mathring{c}_1 k)^2 \left(\frac{c}{\varrho^m}\frac{\partial\varrho^m}{\partial c}\right)^2_{\mathcal{P},\theta},$$

$$\left(\frac{d\mathcal{D}(E^2)}{dE^2}\right)_{E_1} = (\mathring{c}_1^2-\mathring{c}_2^2)k^2 - k^2 \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n} c^2 \left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta} \approx \mathring{c}_1^2 k^2,$$

$$\mathcal{D}(E_2^2) = -(\mathring{c}_2 k)^2 k^2 \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n} c^2 \left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta} + (\mathring{c}_1 k)^2 k^2 \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n} c^2 \left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta} - \tag{27}$$

$$-k^4 \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n}\left(\frac{c}{\varrho^m}\frac{\partial\mathcal{P}}{\partial c}\right)^2_{\varrho,\theta} \approx (\mathring{c}_1 k)^2 k^2 \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n}\left[c^2\left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta} - \frac{1}{\left(\dfrac{\partial\mathcal{P}}{\partial\varrho^m}\right)_{\theta,c}}\left(\frac{c}{\varrho^m}\frac{\partial\mathcal{P}}{\partial c}\right)^2_{\varrho,\theta}\right]$$

$$= (\mathring{c}_1 k^2) \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n} c^2 \left(\frac{\partial z}{\partial c}\right)_{\mathcal{P},\theta} k^2,$$

$$\left(\frac{d\mathcal{D}(E^2)}{dE^2}\right)_{E_2} = -(\mathring{c}_1 k)^2 + (\mathring{c}_2 k)^2 - k^2 \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n} c^2 \left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta} \approx -\mathring{c}_1^2 k^2.$$

Hence

$$\delta E_1^2 = \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n}\left(\frac{c}{\varrho^m}\frac{\partial\mathcal{P}}{\partial c}\right)^2_{\varrho,\theta} k^2 \mathring{c}_1^{-2} = \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n}\left(\frac{c}{\varrho^m}\frac{\partial\varrho^m}{\partial c}\right)_{\mathcal{P},\theta}(\mathring{c}_1 k)^2,$$

$$\delta E_2^2 = \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n}\left[c^2\left(\frac{\partial z}{\partial c}\right)_{\varrho,\theta} - \frac{1}{\left(\dfrac{\partial\mathcal{P}}{\partial\varrho^m}\right)_{\theta,c}}\left(\frac{c}{\varrho^m}\frac{\partial\mathcal{P}}{\partial c}\right)^2_{\varrho,\theta}\right]k^2 = \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n} c^2 \left(\frac{\partial z}{\partial c}\right)_{\mathcal{P},\theta} k^2. \tag{28}$$

So for the velocity of the first and second sound we have

$$c_1^2 = \frac{\partial\mathcal{P}}{\partial\varrho^m}\left(1 + \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n}\left(\frac{c}{\varrho^m}\frac{\partial\varrho^m}{\partial c}\right)^2_{\mathcal{P},\theta}\right), \tag{29}$$

$$c_2^2 = \frac{\overset{m}{\varrho}_n\bar{S}^2}{\overset{m}{\varrho}_n\dfrac{\partial S}{\partial\theta}} + \frac{\overset{m}{\varrho}_s}{\overset{m}{\varrho}_n} c^2 \left(\frac{\partial z}{\partial c}\right)_{\mathcal{P},\theta}.$$

From these considerations we see that the additional term in Khalatnikov's formula for c_2 can be omitted.

In order to calculate S and $c^2 \frac{\partial z}{\partial c}$ it is necessary to know the (θ, c)—dependence of μ_B and μ_F. Fortunately, dilute solutions can be treated as ideal ones and we can put

$$\mu_B = \mu_{BO} + \frac{k_B \theta}{m_B} \ln(1-C), \tag{30}$$

$$\mu_F = \mu_{FO} + \frac{k_B \theta}{m_F} \ln C,$$

where μ_{BO} is the chemical potential of pure He II, μ_{FO} of pure He^3 and C is the molar concentration of He^3. We have the general formula connecting C with c,

$$\frac{1}{C} - 1 = \frac{m_F}{m_B}\left(\frac{1}{c} - 1\right). \tag{31}$$

For dilute solutions, *i. e.* for small C and c, we have

$$C = \frac{m_B}{m_F} c. \tag{32}$$

Wilks [4] remarks that Khalatnikov writes in (30) the mass concentration c instead of the molar concentration C. But all final Khalatnikov's results are correct. With the help of (30) and (32) we find

$$S = cS_{FO} + (1-c)S_{BO} - \frac{k_B}{m_F} c \ln\left(c\frac{m_B}{m_F}\right) - \frac{k_B}{m_B}(1-c)\ln\left(1 - c\frac{m_B}{m_F}\right),$$

$$\bar{S} = S - c\frac{\partial S}{\partial c} = S_{BO} - \frac{k_B}{m_B}\ln\left(1 - c\frac{m_B}{m_F}\right) + \frac{k_B}{m_F} c -$$

$$- c(1-c)\frac{k_B}{m_B}\frac{m_B/m_F}{1 - \frac{m_B}{m_F} c} \approx S_{BO} + \frac{k_B}{m_F} c, \tag{33}$$

$$c^2 \frac{\partial z}{\partial c} = c^2 \frac{\partial}{\partial c}(\mu_F - \mu_B) = \frac{k_B \theta c}{m_F} = \frac{R\theta C}{M_B}$$

where M_B is the molar mass of He^4. Hence,

$$c_2^2 = \frac{\varrho_s^m}{\varrho_n^m}\left[\frac{(S_{BO} + k_B c/m_F)^2}{\frac{\partial S}{\partial \theta}} + \frac{k_B \theta c}{m_F}\right] = \frac{\varrho_s^m}{\varrho_n^m}\left[\frac{(S_{BO} + RC/M_B)^2}{\frac{\partial S}{\partial \theta}} + \frac{R\theta C}{M_B}\right]. \tag{34}$$

3. Solution of the linearized hydrodynamic equations and calculation of the Green functions

The solution of Eqs (17)—(19) has the form

$$\delta\varrho^m(k) = \frac{1}{\mathscr{D}(E^2)}\left[(E^2-(c_2k)^2)+k^2c^2\frac{\partial\mathscr{P}}{\partial c}\frac{\varrho_s^m}{\varrho_n^m}\frac{m_B-m_F}{m_Bm_F\varrho}\right]\varrho k^2\delta U(k)-$$

$$-\frac{1}{\mathscr{D}(E^2)}\left[(E^2-(c_2k)^2)+k^2c\frac{\partial\mathscr{P}}{\partial c}\frac{1}{\varrho^m}\right]Em_B\Delta\eta(k), \tag{35}$$

$$\delta\theta(k) = \frac{1}{\mathscr{D}(E^2)}\frac{\varrho_s^m}{\varrho_n^m}\frac{\overline{S}}{\dfrac{\partial S}{\partial\theta}}c\left[(E^2-(\mathring{c}_1k)^2)\frac{m_B-m_F}{m_Bm_F}+k^2\frac{1}{(\varrho^m)^2}\frac{\partial\mathscr{P}}{\partial c}\right]\varrho k^2\delta U(k)+$$

$$+\frac{1}{\mathscr{D}(E^2)}\frac{\overline{S}}{\varrho^m\dfrac{\partial S}{\partial\theta}}\left[(E^2-(\mathring{c}_1k)^2)-k^2\frac{\varrho_s^m}{(\varrho^m)^2}c\frac{\partial\mathscr{P}}{\partial c}\right]Em_B\Delta\eta(k), \tag{36}$$

$$\mathscr{D}(E^2)\approx[E^2-(c_1k)^2]\,[E^2-(c_2k)^2] \tag{37}$$

where $c_{1,2}$ are given by (29), (34).

We see that the numerator of $\delta\varrho^m(k)$ is for $E=c_2k$ vanishingly small. Hence, the density amplitude in the second sound wave is very small in comparison to that in the first sound wave.

On the other hand, the numerator of $\delta\theta(k)$ is vanishingly small for $E=c_1k$. Therefore, the temperature amplitude is significant in the second sound wave.

In [5] it was demonstrated that

$$\delta c(k) = \frac{c\dfrac{\partial S}{\partial\theta}}{\overline{S}}\,\delta\theta(k). \tag{38}$$

From (38) we see that the excitation of the standing temperature waves leads automatically to the appearance of standing concentraction waves in the He^3—He II solutions. A source with periodically varying temperature can excite not only temperature but also concentration waves. The same conclusion follows from the formula in [6] and remark in [7]. We hope that this effect, however small, may be observed during ultrasound attenuation or optical experiments.

If the variation of the Hamiltonian $\delta\hat{H}_t$ (see [2]) is adiabatically introduced we have for $\delta\varrho^m(k)$, [8]

$$\delta\varrho^m(k) = 2\pi\{\langle\langle\hat{\varrho}_k^m;a_{-k}\rangle\rangle_{E=\omega+i\varepsilon}\delta\eta^*(-k)+\langle\langle\hat{\varrho}_{-k}^m;a_k^+\rangle\rangle_{E=\omega+i\varepsilon}\delta\eta(k)+ \tag{39}$$

$$+\langle\langle\hat{\varrho}_k^m;\hat{\varrho}_{-k}\rangle\rangle_{E=\omega+i\varepsilon}\delta U(k)$$

where a_k and a_k^+ are the Bose annihilation and creation operators, and $\hat{\varrho}_k^m$ the Fourier components of the mass density operator (see [8]), $\langle\langle\,;\,\rangle\rangle$ are the Fourier components of the retarded temperature Green functions

$$\langle\langle \hat{A}(t); \hat{B}(\tau)\rangle\rangle^r = G_r(t-\tau) = -i\,\Theta(t-\tau)\,\langle[\hat{A}(t), \hat{B}(\tau)]\rangle_{eq} \tag{40}$$

$$\Theta(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

where $\langle\ldots\rangle_{eq}$ denotes averaging for thermodynamic equilibrium.

From (35) and (39) we have

$$\frac{\delta\varrho^m(k)}{\delta U(k)} = 2\pi\,\langle\langle\hat{\varrho}_k^m; \hat{\varrho}_{-k}\rangle\rangle_{E=\omega+i\varepsilon}$$

$$= \frac{1}{\mathscr{D}(E^2)}\left[(E^2-(c_2k)^2)+k^2c^2\,\frac{\partial\mathscr{P}}{\partial c}\,\frac{\varrho_s^m}{\varrho_n^m}\,\frac{m_B-m_F}{m_Fm_B}\right]k^2, \tag{41}$$

$$\frac{\delta\varrho^m(k)}{\delta\eta^*(-k)} = 2\pi\langle\langle\hat{\varrho}_k^m; a_{-k}\rangle\rangle_{E=\omega+i\varepsilon} = \frac{Em_B\sqrt{\varrho_0}}{\mathscr{D}(E^2)}\left[(E^2-(c_2k)^2)+k^2c\,\frac{\partial\mathscr{P}}{\partial c}\,\frac{1}{\varrho^m}\right]. \tag{42}$$

Analysis of (41) and (42) shows that the Green functions $\langle\langle\hat{\varrho}_k^m; \hat{\varrho}_{-k}\rangle\rangle_E$ and $\langle\langle\hat{\varrho}_{-k}^m; a_{-k}\rangle\rangle_E$ have poles only for the energies of the first sound quanta.

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