# A SOLUTION OF THE PROBLEM OF UNSTEADY VISCOUS COMPRESSIBLE FLOW THROUGH A STRAIGHT CHANNEL WITH POROUS WALLS

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(Received July 3, 1970)

Under the time varying pressure, the problem of unsteady, viscous, compressible fluid through a straight channel with flat porous walls was studied and a solution for the problem obtained.

## *Introduction*

Rosenhead [3] obtained an exact solution for the steady viscous flow through a flat plate at zero incidence with uniform solution. Satya Prakash [2] obtained a solution under a time varying pressure gradient of unsteady incompressible viscous flow in a straight channel with two parallel porous walls, when one wall is injecting the fluid the other is sinking the same amount of fluid. In the present paper the authors discuss the solution of an unsteady compressible viscous flow through a channel with porous walls.

## *Formulation of the problem*

Physics of compressible flow is much more complicated than in the case of incompressible flow. To deal with the compressible flow we require imposing certain conditions in order for the equation of motion to be written in a simpler form. The walls of the channel are porous, one injects the fluid and the other absorbs it. The velocity of the fluid injected is smaller than the main stream. Assume that the Reynolds number based on the free stream is large; the viscous effects are then confined to a narrow region near the walls. Therefore we can deal with the present problem under boundary layer theory. The fundamental equations of the problem are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\varrho}\frac{\partial p}{\partial x} + \frac{1}{\varrho}\frac{\partial}{\partial x}\left\{\mu\left[2\frac{\partial u}{\partial x} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right]\right\} + \\ + \frac{1}{\varrho}\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] \tag{1}$$

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$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\varrho}\frac{\partial p}{\partial x}+\frac{1}{\varrho}\frac{\partial}{\partial y}\left\{\mu\left[2\frac{\partial v}{\partial y}-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]\right\}+$$
$$+\frac{1}{\varrho}\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right] \tag{2}$$

$$\frac{\partial \varrho}{\partial t}+\frac{\partial \varrho u}{\partial x}+\frac{\partial \varrho v}{\partial y}=0 \tag{3}$$

where $u$, $v$ are the components of the velocity in the $x$ and $y$ directions respectively. $p$, $t$, $\varrho$ and $\mu$ denote pressure, time, density and viscosity of the fluid, respectively. From the equation of continuity for the steady case the magnitude of $u$ and $v$ is given by [1]

$$\frac{v}{u}=-\frac{\delta}{l}\,[0(1)] \tag{4}$$

where $\delta$ is the measure of the thickness of the boundary layer and $l$ is the characteristic length of the wall. Neglecting the term of order of $\delta$ and smaller in the equation obtained from (1) and (2) after making them non-dimensional, we get [1]

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\varrho}\frac{\partial p}{\partial x}+\frac{1}{\varrho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{5}$$

$$\frac{\partial p}{\partial y}=0. \tag{6}$$

Now suppose that the boundary and initial conditions for the problem under consideration are:

$$t \leqslant 0: u=0 \text{ and } v=0 \text{ for } 0 \leqslant y \leqslant d \tag{7}$$

$$t>0: u=0 \text{ and } v=v_0=\text{const.}>0 \text{ for } y=0, d \tag{8}$$

where $d$ is the distance between the two walls.

From the above conditions we see that

$$\frac{\partial u}{\partial x}=0 \quad \text{and} \quad \frac{\partial v}{\partial x}=0 \tag{9}$$

and inserting (9) into (5), we have

$$\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial y}=-\frac{1}{\varrho}\frac{\partial p}{\partial x}+\frac{1}{\varrho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right). \tag{10}$$

The equations of the problem obtained above are still non-linear. Let us write them again in non-dimensional form by taking the characteristic velocity $v_0$, the characteristic length $d$, the characteristic pressure $\varrho v_0^2$ and the characteristic time $d/v_0$. Inserting:

$$u^*=\frac{u}{v_0},\quad x^*=\frac{x}{d},\quad y^*=\frac{y}{d},\quad p^*=\frac{p}{\varrho v_0^2},\quad t^*=\frac{t}{(d/v_0)} \tag{11}$$

under the above assumptions, the first of equations (9), equation (10) and equation (6) after simplification become:

$$\frac{\partial u^*}{\partial x^*} = 0 \tag{12}$$

$$\frac{\partial u^*}{\partial t^*} + \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\text{Re}}\frac{\partial 2u^*}{\partial y^{*2}} \tag{13}$$

$$\frac{\partial p^*}{\partial y^*} = 0 \tag{14}$$

where $R_0 = \frac{v_0 d}{\nu}$ is the Reynolds number. The initial and boundary conditions (7) and (8) become

$$t^* \leqslant 0 : u^* = 0 \text{ for } 0 \leqslant y^* < 1 \tag{15}$$

$$t^* > 1 : u^* = 0 \text{ for } y^* = 0, 1. \tag{16}$$

Equation (14) shows that $p^*$ is independent of $y^*$ and hence from equation (13) we see that $\frac{\partial p^*}{\partial x^*}$ is only a function of $t^*$. Therefore we assume that

$$\frac{\partial p^*}{\partial x^*} = -f(t^*).$$

Then equation (13) reduces to

$$\frac{\partial u^*}{\partial t^*} + \frac{\partial u^*}{\partial y^*} = f(t^*) + \frac{1}{\text{Re}}\frac{\partial^2 u^*}{\partial y^{*2}}. \tag{17}$$

This equation is the same as that obtained by Satya Prakash for incompressible flow. The solution for equation (17) is given by

$$u^* = \frac{1}{2\pi i}\int_{\nu - i\infty}^{\nu + i\infty} \bar{u}^* e^{\lambda t^*} d\lambda$$

where $\nu$ is greater than the real part of all the singularities of $\bar{u}^*$ and bar (—) denotes the Laplace Transform [2].

## *Conclusion*

We conclude that for a large Reynolds numbers the compressible viscous flow has the same solution as that of the incompressible viscous flow in the case of porous walls through one of which the fluid is injected the other sinking the same amount.

*Remark*

Suppose that there are two parallel walls of a channel and both are injecting fluid with high velocities. Thus we have an impact of injecting fluids in the channel making the flow stationary at those points. Thus a stationary layer is formed in the flow of channel. The physics of this layer must be developed under the boundary layer theory. Also, due to impact, heat is produced, thus the flow will be diabatic rather than adiabatic. The only explicit difference in the fundamental equations of diabatic flow lies in the energy equation. Hick found that diabatic flow can be easily studied in terms of the Crocco vector [2].

This work is financially assisted by the University Grants Commission of India.

REFERENCES

[1] S. I. Pai, *Introduction of theory of compressible flow*, D. Van Nostrand Company, Inc. New York.
[2] Satya Prakash, *Proc. Natn. Inst. Sci. India*, **35**, 123 (1969).
[3] L. Rosenhead, *Laminar Boundary Layers*, Clarendon Press, Oxford, p. 141 (1963).