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APPROXIMATE GROUND STATE OF A UNIAXIAL NÉEL ANTIFERRIMAGNET IN A LONGITUDINAL MAGNETIC FIELD

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The field-induced transitions between the antiferrimagnetic, canted-spin and paramagnetic phases of a two-sublattice Néel antiferrimagnet at zero temperature are studied. The approximate ground state is obtained for all three phases, and the magnetization and susceptibility in these phases as a function of the magnetic field is determined and discussed.

1. *Introduction*

In this paper we examine the zero-temperature magnetic properties of a uniaxial two-sublattice Néel antiferrimagnet in an external magnetic field parallel to the easy axis. In particular, we determine the critical field strengths for the phase transitions and study such thermodynamic quantities as the magnetization and susceptibility. We confine ourselves to the case $T = 0$ and defer the case $T \neq 0$ to a subsequent paper.

We consider only such crystal structures which can be split into two sublattices, such that all nearest neighbours of an atom belong to the other sublattice. The atoms of the two sublattices, denoted by 1 and 2 respectively, have different maximum spin eigenvalues S_1 and S_2 and are coupled by nearest-neighbour exchange interaction of the Heisenberg type. The exchange interaction is assumed to be anisotropic, favouring spin alignment along a single crystallographic direction which we choose as the z-axis of our co-ordinate system.

As the true ground state of an antiferrimagnet is unknown, there is usually the problem of choosing a suitable reference state (spin wave vacuum) when applying spin wave theory [1–3]. We determine the approximate ground state of the spin Hamiltonian by minimizing its expectation value in a class of trial states corresponding to complete sublattice spin alignment with arbitrary direction (sublattice saturation state). Strict solutions for the field-dependence of the direction of the sublattice magnetization are obtained.

It is shown that for small magnetic fields the spins of the two sublattices are antiparallel to each other and lie in the field direction (antiferrimagnetic phase). As the field increases, a phase transition occurs to the canted-spin phase in which the spins deviate from the field

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direction. Upon further increasing the field, all the spins become aligned in the direction of the external field (paramagnetic phase; see *e. g.* [4–7]). In some ways the situation is reminiscent of that in the spin-flop phase of an antiferromagnet [8–11], except that in the present case the antiferrimagnetic phase exists for fields below a certain critical field, even in the absence of magnetic anisotropy. Furthermore, in an antiferrimagnet, both transitions are of second order.

When specified to the isotropic case, our results are shown to agree with those obtained in [4–7, 12]. It is also shown that the results obtained in [13] correspond to the weak-anisotropy limit.

2. *Trial ground state*

The Hamiltonian of the system is assumed to be of the form:

$$\mathscr{H} = \sum_{\langle f,g \rangle} A_{ab} \tilde{S}_f^a \tilde{S}_g^b - \mu_1 \sum_f H_a \tilde{S}_f^a - \mu_2 \sum_g H_a \tilde{S}_g^a, \tag{1}$$

where $\langle f, g\rangle$ denotes summation over nearest-neighbours only, and f and g are lattice sites in the first and second sublattice, respectively (each of them having N sites); μ_1 and μ_2 are the effective magnetic moments per lattice atom in the first and second sublattice; $\tilde{\mathbf{S}}_f$ and $\tilde{\mathbf{S}}_g$ are spin operators asigned to the lattice sites f and g; $\boldsymbol{H} = (0, 0, H)$ is the uniform external magnetic field which is parallel to the co-ordinate axis z. The interaction tensor A_{ab} represents isotropic and anisotropic exchange interactions between neighbouring atoms (belonging to different sublattices); it may be written as

$$A_{ab} = J \begin{pmatrix} X, & 0, & 0 \\ 0, & 1, & 0 \\ 0, & 0, & Z \end{pmatrix}, \tag{2}$$

where $X = 1 + K_x J^{-1}$, $Z = 1 + K_z J^{-1}$. We assume that $Z > X$. Here, $J > 0$ is the nearest--neighbours exchange integral; the constants $K_x \geqslant 0$, $K_z \geqslant 0$ represent anisotropic exchange interaction in the x and z direction, respectively. In Eq. (1), Einstein's summation convention is applied to the tensor indices a, $b (= x, y, z)$.

Similarly as in [2, 3, 11], we perform the following rotations of the spins around the axis y (see Fig. 1), with the help of the transformation

$$\begin{aligned} \tilde{S}_f^x &= S_f^x \cos\Theta_1 + S_f^z \sin\Theta_1 & \tilde{S}_g^x &= S_g^x \cos\Theta_2 - S_g^z \sin\Theta_2 \\ \tilde{S}_f^y &= S_f^y & \tilde{S}_g^y &= S_g^y \\ \tilde{S}_f^z &= -S_f^x \sin\Theta_1 + S_f^z \cos\Theta_1, & \tilde{S}_g^z &= S_g^x \sin\Theta_2 + S_g^z \cos\Theta_2. \end{aligned} \tag{3}$$

This transformation introduces two different co-ordinate systems (x', y, z') and (x'', y, z'') in the sublattices 1 and 2, respectively. For simplicity, we omit the prime and double-prime superfixes over vector indices x, y, z of the transformed spin components in (3).

We define the sublattice saturation states $|0\rangle_1$ and $|0\rangle_2$ (homogeneous spin- deviation reference states; cp. [2]), in which the spins in the two sublattices are aligned along the new z' and z'' axes, as follows:

$$S_f^z|0\rangle = S_1|0\rangle, \quad S_f^+|0\rangle = 0. \tag{4}$$
$$S_g^z|0\rangle = -S_2|0\rangle, \quad S_g^-|0\rangle = 0,$$

where $|0\rangle = |0\rangle_1\, |0\rangle_2$, $\langle 0|0\rangle_1 = \langle 0|0\rangle_2 = 1$, $S_h^{\pm} = S_h^x \pm i S_h^y$ $(h = f, g)$.

The approximate ground state is determined by minimizing the mean value of the Hamiltonian (1) in the state $|0\rangle$ with respect to the parameters Θ_1 and Θ_2, *i. e.*,

$$\min \langle 0\, |\mathcal{H}|\, 0\rangle \equiv \min E_0(\Theta_1, \Theta_2) \equiv E_0. \tag{5}$$

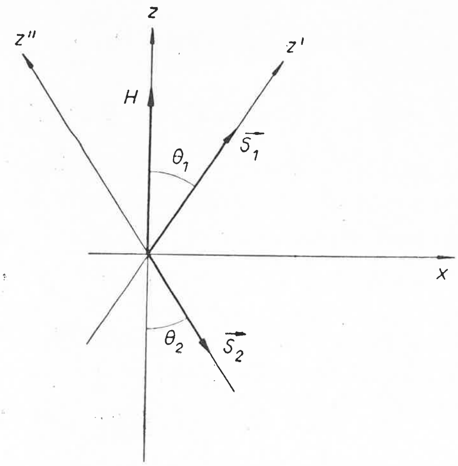

Fig. 1. Co-ordinate systems as introduced by Eq. (3)

By taking into account Eqs (1)–(5) we obtain for E_0 (Θ_1, Θ_2) the following expression:

$$E_0(\Theta_1, \Theta_2) = d(X \sin \Theta_1 \sin \Theta_2 - Z \cos \Theta_1 \cos \Theta_2 -$$
$$- h \cos \Theta_1 + \varkappa h \cos \Theta_2), \tag{6}$$

where $h = \mu_1 H / J S_2 \gamma_0$, $d = N S_1 S_2 J \gamma_0$, $\varkappa = \mu_2 S_2 / \mu_1 S_1$, and γ_0 is the number of nearest--neighbours. Without loss of generality, we assume $\varkappa < 1$.

3. *Stable magnetic phases and critical fields*

The approximate ground-state energy is obtained by solving the necessary and examining the sufficient conditions for the existence of a minimum of $E_0(\Theta_1, \Theta_2)$, in dependence of the parameters $(X, Z, h, \varkappa)$:

$$\frac{\partial E_0}{\partial \Theta_1} = \frac{\partial E_0}{\partial \Theta_2} = 0, \tag{7}$$

$$\Delta \equiv \left(\frac{\partial^2 E_0}{\partial \Theta_1^2}\right)\left(\frac{\partial^2 E_0}{\partial \Theta_2^2}\right) - \left(\frac{\partial^2 E_0}{\partial \Theta_1 \partial \Theta_2}\right)^2 > 0, \quad \frac{\partial^2 E_0}{\partial \Theta_2^2} > 0. \tag{8}$$

The equilibrium conditions (7) can be written in the form

$$\sin \Theta_1 = \eta R \sin \Theta_2 \tag{9}$$

and

$$\sin \Theta_1 (w \cos \Theta_2 + \eta \varkappa h X R^{-1} + hZ) = 0, \tag{10a}$$

or

$$\sin \Theta_2 (w \cos \Theta_1 - \eta h X R - \varkappa h Z) = 0, \tag{10b}$$

where

$$R^2 = (\varkappa^2 h^2 - w)/(h^2 - w), \quad w = Z^2 - X^2, \quad \eta = \pm 1. \tag{11}$$

From (9) and (10a) or (10b) we obtain the following solutions:

$$\sin \Theta_1 = \sin \Theta_2 = 0 \tag{12}$$

or

$$\cos \Theta_1 = h w^{-1}(\varkappa Z + \eta X R), \cos \Theta_2 = -h w^{-1}(Z + \eta \varkappa X R^{-1}). \tag{13}$$

Without loss of generality, we assume $h \geqslant 0$. By substituting Eqs (12) and (13) into (8) one easily verifies that there are real and stable solutions in the whole interval $0 \leqslant h < \infty$ for the external magnetic field. Namely, for

$$0 \leqslant h < h_c = (2\varkappa)^{-1}\{Z(\varkappa - 1) + [Z^2(1-\varkappa)^2 + 4\varkappa w]^{1/2}\} \tag{14a}$$

there are only two stable solutions that follow from (12), $\Theta_1 = \Theta_2 = 0$ and $\Theta_1 = \Theta_2 = \pi$, which we denote by (0,0) and (π, π), respectively. In the interval

$$h_c < h < h_l = (2\varkappa)^{-1}\{Z(1-\varkappa) + [Z^2(1-\varkappa)^2 + 4\varkappa w]^{1/2}\} \tag{14b}$$

the only stable solution is (0,0). For

$$h > h_u = (2\varkappa)^{-1}\{Z(1+\varkappa) + [Z^2(1+\varkappa)^2 - 4\varkappa w]^{1/2}\} > h_l \tag{14c}$$

there is also one stable solution which follows from (12), namely, $\Theta_1 = 0$ and $\Theta_2 = \pi$, *i.e.*, $(0, \pi)$.

As regards the interval $h_l < h < h_u$, the stable solution which we shall denote by (Θ_1, Θ_2) is given by Eq. (13) for which we have, upon inserting in (8),

$$\partial^2 E_0 / \partial \Theta_2^2 = -\eta d X R, \tag{15}$$

$$\Delta = d^2(\varkappa^2 h^2 - w)(1 - \cos^2 \theta_2). \tag{16}$$

It can easily be seen that in this case there is a minimum only if $\eta = -1$. At the same time, the obvious conditions $|\cos \Theta_1| \leqslant 1$ and $|\cos \Theta_2| \leqslant 1$ along with the minimum condition $\Delta > 0$, restrict the external field precisely to the interval $h_l < h < h_u$, as required. One easily proves that in this interval the reality condition $R^2 \geqslant 0$ following from (11) is automatically satisfied.

The ground-state energies E_0 corresponding to the stable solutions (0,0), (π, π), (Θ_1, Θ_2) and $(0, \pi)$ are easily obtained from (6):

$$E_0^{0,0} = -d[Z + h(1-\varkappa)] \equiv E_0^{A_1},$$

$$E_0^{\pi,\pi} = -d[Z+h(\varkappa-1)] \equiv E_0^{A_2},$$
$$E_0^{\Theta_1,\Theta_2} = -d[XR+h\cos\Theta_1] \equiv E_0^C,$$
$$E_0^{0,\pi} = -d[-Z+h(1+\varkappa)] \equiv E_0^P. \tag{17}$$

From Fig. 1 it is seen that the solutions (0, 0) and (π, π) describe the so-called antiferrimagnetic phase (A) at zero temperature corresponding to an antiparallel configuration along the external field (that is, along the direction of easiest magnetization) of the sublattice saturation magnetizations. The solution Θ_1, Θ_2 represents the so-called canted-spin phase (C) in which the sublattice magnetizations form the angles Θ_1 and Θ_2 with the field direction, while the solution $(0, \pi)$ describes the so-called paramagnetic phase (P) corresponding to complete spin-alignment along the external field.

As regards the antiferrimagnetic phase, in consistency with our assumptions $\varkappa = \mu_2 S_2/\mu_1 S_1 < 1$, $h \geqslant 0$ and the definitions (4) the solution (0,0) describes the case when the external magnetic field is directed along the larger magnetic moments (sublattice 1), while the state (π, π) corresponds to the opposite case. From (17) it is seen that $E_0^{\pi,\pi} > E_0^{0,0}$ except for $h = 0$ when those energies coincide (two-fold degeneracy of the ground-state energy). Thus, in the interval $h < h_c$ in which both the solutions exist, one has to distinguish between the antiferrimagnetic configuration A_1 with lower energy corresponding to the solution (0, 0), and the opposite and energetically less favourable antiferrimagnetic configuration A_2 described by the solution (π, π).

Our analysis shows that, depending on the direction in which the longitudinal external magnetic field is switched on, the magnetization process can take place either according to the scheme $A_1 \to C \to P$ or $A_2 \to A_1 \to C \to P$, the critical fields for the phase transitions $A_2 \to A_1$, $A_1 \leftrightarrow C$ and $C \leftrightarrow P$ being respectively h_c, h_l and h_u. In the isotropic case $K_x = K_z = 0$ (*i.e.*, $X = Z = 1$) only the first magnetization mechanism is possible, as $h_c = 0$ according to (14a), *i.e.*, the phase A_2 does not exist.

4. *Magnetization and susceptibility*

In order to determine what kind of phase transitions occur at the critical points h_c, h_l and h_u, we examine the energy E_0, the magnetization M and the susceptibility χ of the system in the approximate ground state (4) as functions of the external field h and study their behaviour at those points.

The components of the sublattice magnetization vectors M_i^a $(i = 1, 2)$ in the state (4) are defined as follows:

$$M_1^a = \mu_1 \sum_f \langle 0|\tilde{S}_f^a|0\rangle, \qquad M_2^a = \mu_2 \sum_g \langle 0|\tilde{S}_g^a|0\rangle, \ (a = x, y, z) \tag{18}$$

and those of the total magnetization $\boldsymbol{M}$ accordingly,

$$M_{||} = M_1^z + M_2^z \equiv M_1^{||} + M_2^{||}, \quad M_\perp = M_1^x + M_2^x \equiv M_1^\perp + M_2^\perp,$$
$$M = (M_{||}^2 + M_\perp^2)^{1/2}, \tag{19}$$

where $M_\perp$, $M_i^\perp$ and $M_{||}$, $M_i^{||}$ denote the components of the magnetization vectors in the direction perpendicular and parallel to the external magnetic field (transversal and longitudinal magnetizations), respectively. For the respective phases one easily obtains, *e.g.*, the following expressions for the components of the total magnetization:

$$M^{A_1} = M_{||}^{A_1} = \mu_1 S_1 N(1-\varkappa), \; M_{\perp}^{A_1} = 0, \tag{20}$$

$$M^{A_2} = M_{||}^{A_2} = \mu_1 S_1 N(\varkappa-1), \; M_{\perp}^{A^2} = 0, \tag{21}$$

$$M^{C} = \mu_1 S_1 N[1+\varkappa^2-2\varkappa\cos(\Theta_1+\Theta_2)]^{1/2}, \tag{22}$$

$$M_{||}^{C} = \mu_1 S_1 N(\cos\Theta_1-\varkappa\cos\Theta_2)$$

$$= \mu_1 S_1 N h w^{-1}[2\varkappa Z - XR(1+\varkappa^2 R^{-2})], \tag{23}$$

$$M_{\perp}^{C} = \mu_1 S_1 N(\sin\Theta_1+\varkappa\sin\Theta_2) = \mu_1 S_1 N(\varkappa-R)\sin\Theta_2, \tag{24}$$

$$M^{P} = M_{||}^{P} = \mu_1 S_1 N(1+\varkappa), \; M_{\perp}^{P} = 0. \tag{25}$$

We define the differential sublattice susceptibility as

$$\chi_{ia} \equiv \frac{\partial M_i^a}{\partial H} \quad (i = 1, 2; a = ||, \perp), \tag{26}$$

and the total susceptibility accordingly:

$$\chi_{||} = \chi_{1||}+\chi_{2||}, \quad \chi_{\perp} = \chi_{1\perp}+\chi_{2\perp}, \quad \chi = \frac{M_{||}}{M}\chi_{||}+\frac{M_{\perp}}{M}\chi_{\perp}. \tag{27}$$

After simple calculations we obtain for the respective phases

$$\chi^{A_1} = \chi_{||}^{A_1} = \chi_{\perp}^{A_1} = \chi^{A_2} = \chi_{||}^{A_2} = \chi_{\perp}^{A_2} = \chi^{P} = \chi_{||}^{P} = \chi_{\perp}^{P} = 0, \tag{28}$$

$$\chi_{1||}^{C} = \mu_1^2 S_1 N(S_2 J\gamma_0)^{-1}[h^{-1}\cos\Theta_1 - h^2 XR^{-1}(1-\varkappa^2)(h^2-w)^{-2}], \tag{29}$$

$$\chi_{2||}^{C} = -\mu_1^2 S_1 N\varkappa(S_2 J\gamma_0)^{-1}[h^{-1}\cos\Theta_2 +$$

$$+h^2 XR\varkappa(\varkappa^2-1)(\varkappa^2 h^2-w)^{-2}], \tag{30}$$

$$\chi_{1\perp}^{C} = -\chi_{1||}^{C}\operatorname{ctg}\Theta_1, \; \chi_{2\perp}^{C} = \chi_{2||}^{C}\operatorname{ctg}\Theta_2, \tag{31}$$

and $\chi_{||}^{C}$, $\chi_{\perp}^{C}$, χ^{C} follow from (27) upon substituting the values (29)–(31) and (22)–(24).

From (14b), (17), (20), (22) and (27)–(30) we obtain for the $A_1 \leftrightarrow C$ transition point h_l

$$E_0^{A_1}(h_l) = E_0^{C}(h_l) = -d(2\varkappa)^{-1}\{Z(1+\varkappa^2)+$$

$$+(1-\varkappa)[Z^2(1-\varkappa)^2+4\varkappa w]^{1/2}\}, \tag{32}$$

$$M^{A_1}(h_l) = M^{C}(h_l) = \mu_1 S_1 N(1-\varkappa), \tag{33}$$

$$\chi_{||}^{A_1}(h_l) = 0, \; \chi_{||}^{C}(h_l) = \mu_1^2 S_1 N(1-\varkappa)(S_2 J\gamma_0 h_l)^{-1}\times$$

$$\times\left\{1+\left[\left(\frac{Z}{X}\right)^2-1\right]\left(\frac{1+\varkappa}{1-\varkappa}\right)^2\right\} \neq 0. \tag{34}$$

From (32)–(34), it results that for $h = h_l$ the ground-state energy of the system and the zero-temperature total magnetization are continuous, however, in the longitudinal susceptibility a jump occurs. Therefore, the phase transition is of second order. The system behaves analogously at the $C \leftrightarrow P$ transition point h_u, namely,

$$E_0^C(h_u) = E_0^P(h_u) = -d(2\varkappa)^{-1}\{Z(1+\varkappa^2)+ +(1+\varkappa)[Z^2(1+\varkappa)^2-4\varkappa w]^{1/2}\}, \tag{35}$$

$$M^C(h_u) = M^P(h_u) = \mu_1 S_1 N(1+\varkappa), \tag{36}$$

$$\chi_{||}^C(h_u) = \mu_1^2 S_1 N(1+\varkappa)(S_2 J\gamma_0 h_u)^{-1}\times \times\left\{1+\left[\left(\frac{Z}{X}\right)^2-1\right]\left(\frac{\varkappa-1}{\varkappa+1}\right)^2\right\}\neq 0, \quad \chi_{||}^P(h_u) = 0, \tag{37}$$

which follows from (14c), (17), (22), (25) and (27)–(30). For the $A_2 \leftrightarrow A_1$ transition point h_c we obtain from (14a) and (17) that in the ground-state energy of the system a jump occurs:

$$\Delta E = E_0^{A_2}(h_c) - E_0^{A_1}(h_c) = 2dh_c(1-\varkappa), \tag{38}$$

and from (20), (21) it follows that in the magnetization a jump also occurs:

$$\Delta M = M^{A_1}(h_c) - M^{A_2}(h_c) = 2\mu_1 S_1 N(1-\varkappa). \tag{39}$$

This means that the transition is of first order.

Schematic curves of the system's ground-state energy, transversal and longitudinal total and sublattice magnetizations and susceptibilities as functions of the external field

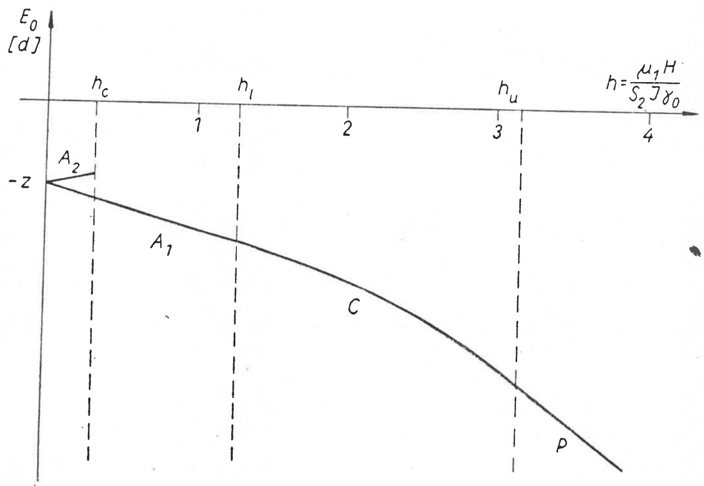

Fig. 2. Ground-state energy E_0 as function of the external magnetic field H, according to Eq. (17)

are given in Figs 2–6, respectively. Numerical curves for the absolute value M of the total magnetization and the corresponding susceptibility χ are plotted in Figs 7, 8 for $\varkappa = 0.5$, $K_x = 0$ and $K_z = 0$, $K_z = 0.01J$ (weak anisotropy) and $K_z = 0.1J$ (strong anisotropy).

5. *Discussion of results*

As is seen from Fig. 2, the magnetization process $A_2 \rightarrow A_1 \rightarrow C \rightarrow P$ is irreversible, as the demagnetization process always follows the scheme $P \rightarrow C \rightarrow A_1 \rightarrow 0$, omitting the phase A_2. This implies that a supercooling effect should take place at the phase transition

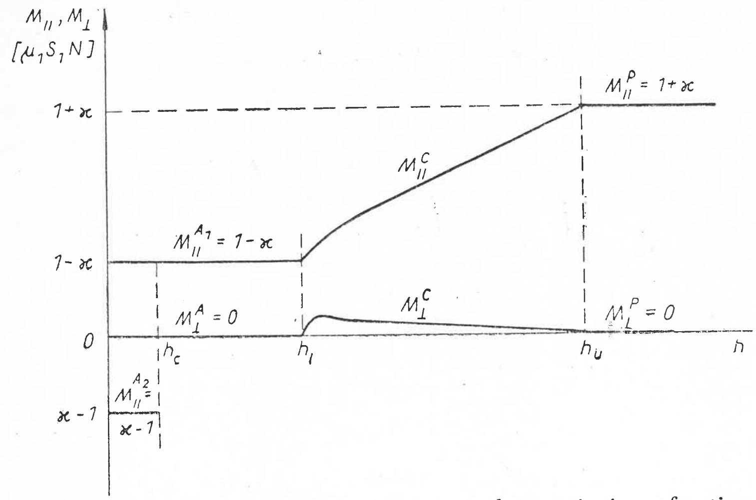

Fig. 3. Longitudinal $M_{||}$ and transversal $M_{\perp}$ components of the total magnetization as functions of the external magnetic field, according to Eqs (20)–(25)

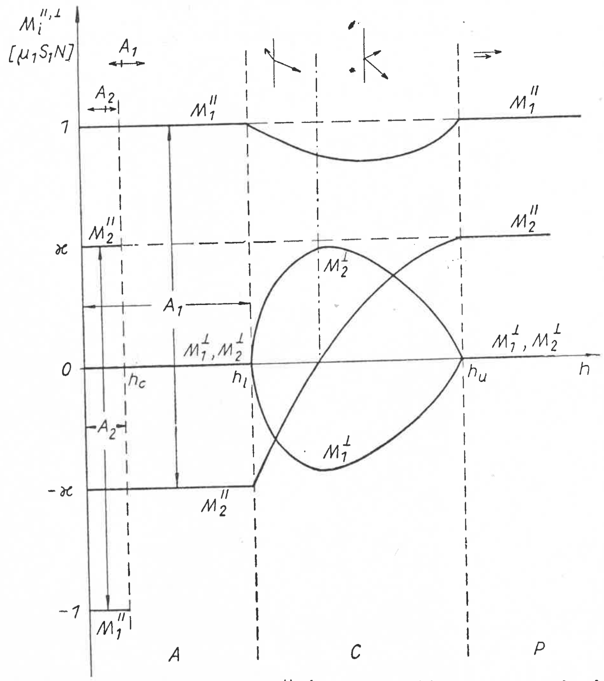

Fig. 4. Longitudinal and transversal components $M_i^{||,\perp}$ of the sublattice magnetizations as functions of the external magnetic field

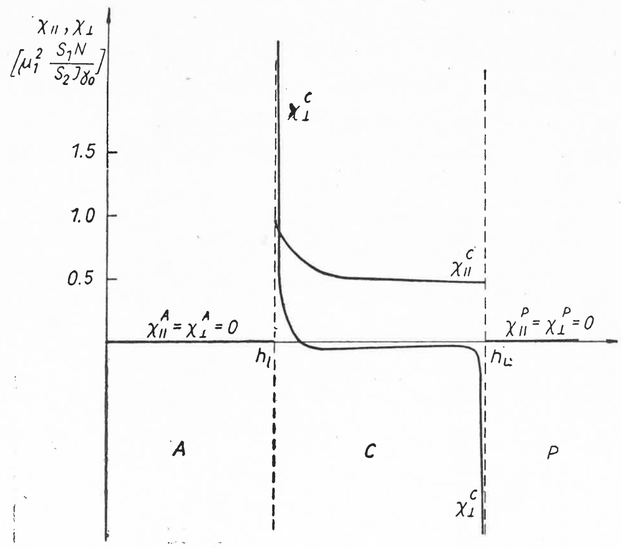

Fig. 5. Longitudinal $\chi_{||}$ and transversal $\chi_{\perp}$ components of the total magnetic susceptibility as functions of the external magnetic field, according to Eqs (27)–(31)

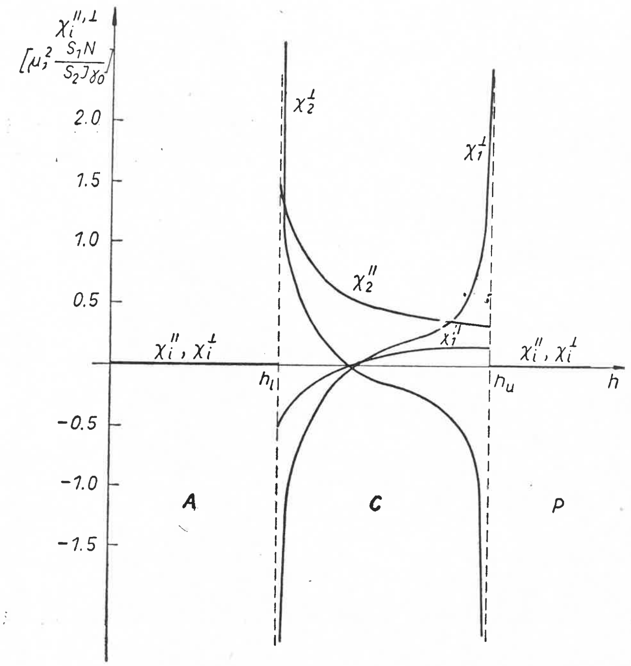

Fig. 6. Longitudinal and transversal components $\chi_i^{||,\perp}$ of the sublattice magnetic susceptibilities as functions of the external magnetic field

$A_2 \to A_1$, as the system's ground-state energy drops by ΔE according to (38). In other words, when starting the magnetization process with the field parallel to the smaller spins (*i.e.*, antiparallel to the larger ones), upon exceeding the first critical field h_c, the spins of the two sublattices change their signs and the system assumes the energetically favourable antiferrimagnetic configuration A_1. The supercooling can therefore be based on the cycle $A_2 \to A_1 \to 0 \to A_2 \to$ *etc.*

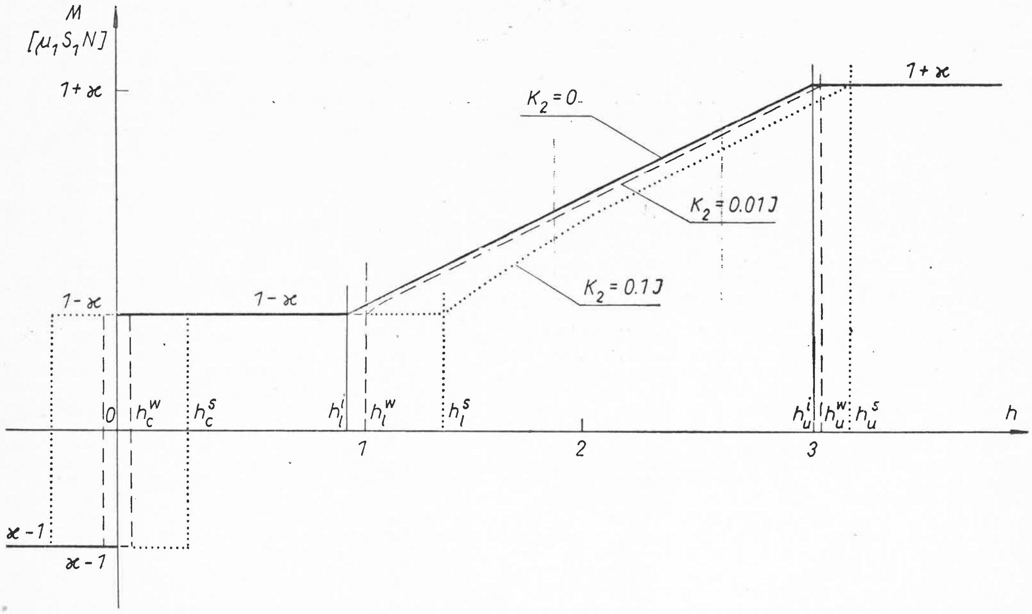

Fig. 7. Numerical curves of the total magnetization M as function of the external magnetic field, for the values $K_z = 0$, $K_z = 0.01\ J$ and $K_z = 0.1\ J$ of the anisotropy constant

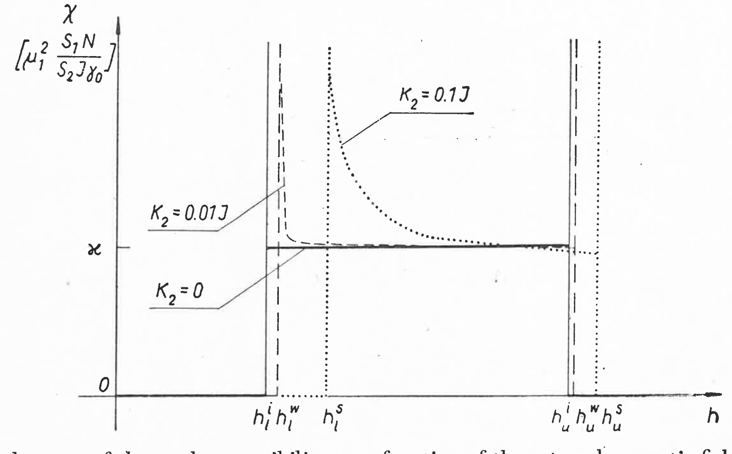

Fig. 8. Numerical curves of the total susceptibility χ as function of the external magnetic field, for the values $K_z = 0$, $K_z = 0.01\ J$ and $K_z = 0.1\ J$ of the anisotropy constant

As regards the longitudinal and transversal components of the total (Fig.3) and sublattice (Fig. 4) magnetizations, it is seen from Figs 3 and 4 that they are continuous functions of the external field h except for h_c, if the phase transition $A_2 \to A_1$ occurs. Furthermore, numerical calculations based on the formula (24) show that for weak anisotropy ($K_z = 0.01J$) the transversal component $M_\perp$ of the total magnetization is, in the C phase, negligible, as it does not exceed 1% of the absolute value M of the total magnetization. This means that the total magnetization is in this phase practically parallel to the external field, the deviation being less than 40′. When the anisotropy is strong ($K_z = 0.1J$) then $M_\perp^C$ increases up to 9.2% of the total magnetization which deviates from the field direction up to 5°20′. Therefore, the customary assumption $\boldsymbol{M} \parallel \boldsymbol{H}$ for the C phase in the molecular field approach (cp. [7]) is justified only in the weak-anisotropy approximation.

The curves of the total (Fig. 5) and sublattice (Fig. 6) longitudinal and transversal susceptibilities show clearly the existence of second-order phase transitions at the critical fields h_l and h_u for the $A_1 \leftrightarrow C$ and $C \leftrightarrow P$ transitions, respectively.

The influence of the anisotropy on the magnetization process of a uniaxial two-sublattice antiferrimagnet in a longitudinal external field is for $K_x = 0$ illustrated by the total magnetization and susceptibility curves in Figs 7 and 8, respectively, for the isotropic ($K_z = 0$;

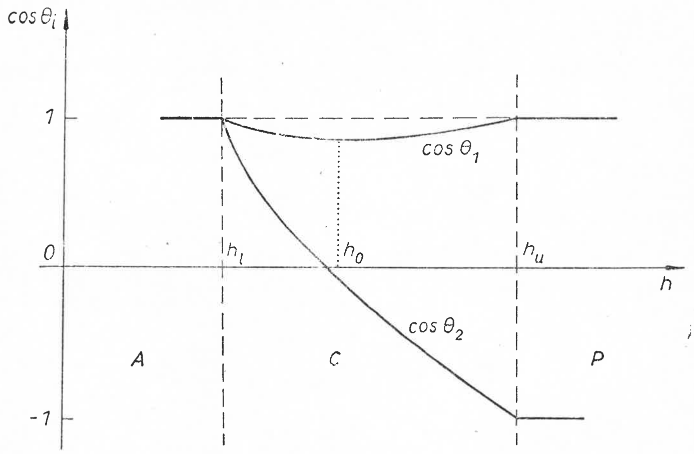

Fig. 9. The stable solutions θ_i as functions of the external magnetic field, according to Eqs (12), (13)

solid curves), weak anisotropy ($K_z = 0.01J$; dashed curves) and strong anisotropy ($K_z = 0.1J$; dotted curves) cases. The corresponding critical fields h_c, h_l, h_u are respectively marked with the superscripts i, w, s. It is seen that with increasing anisotropy the critical fields also increase. However, at the same time the interval $h_u - h_l$ in which the C phase exists decreases, and approaches zero for infinite anisotropy. The "hysteresis loop" indicated in Fig. 7 exists only in the anisotropy case (as $h_c^i = 0$) and grows wider with increasing anisotropy. Note also that the magnetization in the C phase is an approximately linear function of the external field for weak anisotropy only, the deviation from linearity becoming stronger with increasing anisotropy. This result, again, restricts the considerations in [13] to the weak-anisotropy approximation.

As regards the motion of the sublattice spins in the C phase, our strict solutions (13) plotted schematically in Fig. 9 show evidently that smaller spin S_2 (described by the function Θ_2) rotates with increasing field monotonously from the antiparallel to the parallel position with respect to the external field, while the larger spin S_1 (described by the function Θ_1) rotates at first out off the (positive) field direction and only upon exceeding a certain field strength h_0 falls gradually back onto the direction of the field. This characteristic swingback motion of the larger spins confirms the conclusions drawn in [14] from a numerical analysis of the same process described by phenomenological methods. One easily verifies that no phase transition take place at h_0.

For comparison with previous results we specify briefly the isotropic case for which $X = Z = 1$ and $w = 0$. According to (11) Eq. (9) simplifies

$$\sin \Theta_1 = -\varkappa \sin \Theta_2. \tag{40}$$

Of course, Eqs (13) are no longer valid and must be replaced by

$$\cos \Theta_1 = (2h\varkappa)^{-1}(1-\varkappa^2+\varkappa^2h^2),$$

$$\cos \Theta_2 = (2h\varkappa^2)^{-1}(1-\varkappa^2-\varkappa^2h^2). \tag{41}$$

These solutions are the same as those obtained in [7, 12]. Furthermore, from (14 a, b, c) one easily obtains

$$h_c = 0, \quad h_l = (1-\varkappa)/\varkappa, \quad h_u = (1+\varkappa)/\varkappa. \tag{42}$$

From (40) and (24) it is immediately seen that $M_{\perp}^{C} = 0$, which means that in the canted-spin phase the total magnetization lies along the external field. This is the standard assumption in the molecular field approach [7] (see also [4–6, 12]).

From (23), (19) and (41) we obtain that

$$M^C = M_{\parallel}^{C} = \mu_1 S_1 N \varkappa h, \tag{43}$$

i.e., the total magnetization in the C phase is a linear function of h. The same result was obtained in [4–7, 12, 13].

The linear relationship between $\boldsymbol{M}$ and $\boldsymbol{H}$ in the canted-spin phase was found experimentally in several magnetic materials as, *e.g.*, in rare earth orthoferrites [15], in single-crystalline $Ba\,Fe_{12}\,O_{19}$ [16], in $Mn_5\,Ge_3$ [17], and in pyrhottite $Fe_7\,S_8$ [14] and YbIG [7]. The existence of two critical fields and a similar shape of the magnetization curve as obtained by us were experimentally observed in $Mn\,Cr_2\,S_4$ [18].

Finally, we may point out that for $\varkappa = 1$ and $S_1 = S_2$ our results correspond to those for two-sublattice uniaxial antiferromagnets (see *e.g.* [11]).

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