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FUNCTIONAL DESCRIPTION OF QUANTUM ELECTRODYNAMICS IN COULOMB AND FEYNMAN GAUGES

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The off-mass shell generalization of scattering amplitudes in quantum electrodynamics (QED) is considered. It is shown that different ways of extrapolating scattering amplitudes produce different gauges. For the extrapolations corresponding to Feynman and Coulomb gauges respective unitarity and causality conditions are formulated. The formal solution of these conditions coincide with the conventional S-matrix in both gauges. The equivalence of general classes of gauges specified by the parameters a_μ, Λ and M (see B. Zumino, *J. Math. Phys.*, **1**, 1 (1960)) to the Coulomb gauge, is given. In the proof of equivalence the functional integration over nontransversal degrees of freedom is used. The formula connecting any Coulomb Green function with the transversal parts of an infinite set of Feynman Green functions is derived.

Introduction

The problem of the proper choice of the subsidiary condition in the Fermi formulation of QED is not clear.[1] Usually this condition is written as follows $\partial_\mu A_\mu(x)|\varphi\rangle = 0$ and is used for the elimination of the nonphysical states involving any number of time photons. The necessity of imposing the subsidiary condition in QED follows from the construction of the theory based on field operators and vector states as the original notions. In this paper we shall describe QED in terms of the scattering amplitudes as the fundamental notions. We shall show that one can construct QED in Feynman (Lorentz) gauge without any appearence of the awkward subsidiary condition and indefinite Hilbert space. The method which we shall use is the functional Rzewuski's method presented in [1], [2], [3] for the scalar and spinor field, and further generalized by the author to the massive vector field [4].

We formulate QED in the two most common gauges: in the radiation (Coulomb) gauge and in the Feynman gauge. The equivalence of operator formulation of QED in these two

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[1] See the paper of Haller and Landovitz [5] and the critical remarks against their alternative subsidiary condition given by Polubarinov [6].

gauges was shown in many ways (see for example [7], [8], [9], [10]). The most exhaustive proof of equivalence was given by Tatur [11] taking into account the renormalization procedure. Here we give another equivalence proof, which is simple but formal, by the use of the functional integration technique. The same technique allows us to obtain a compact dependence between the Green functions in Coulomb and in Feynman gauges. The general dependence between the Green functions in various gauges was discussed in detail by Zumino [12] and Białynicki [13]. Our method and also our point of view are slightly different.

The notion of generating functional in QED, firstly on the mass shell and further off the mass shell, is introduced in Sec. 1. The unitarity condition is formulated and the cases of Feynman and Coulomb gauges are distinguished. In Sec. 2 we impose on the off mass shell generating functional in the Feynman gauge the causality condition. We show that the formal solution of the causality and unitarity conditions, with the usually chosen interaction functional, gives correct expressions for scattering amplitudes. Similar considerations for Coulomb gauge are carried out in Sec. 3. It is shown there that the generating functional with nonlocal Coulomb interaction term also solves (at least in the special Lorentz framework) the unitarity and causality conditions. In Sec. 4 we use the functional integration method for showing the equivalence between both formulations. We start from the Feynman generating functional, integrate it over time and longitudinal degrees of freedom and obtain a functional which gives the same expression for scattering amplitudes on the mass shell as the one defined in the Coulomb gauge. The use of the functional integration method represents the main point of our paper, which is different from other methods of equivalence proofs. The equivalence between QED in general gauges defined by functions Λ, a_μ and M (see [12], [13]), and QED in Coulomb gauge is shown in the Appendix.

1. *Fundamental notions and definitions*

We start with the scattering amplitudes for QED. These amplitudes depend on three-momenta and polarizations of the ingoing and outgoing electrons, positrons and photons. Here, at the beginning, arises the question about the number of photons occuring in the process. Because photons are massless, they can have any small energy. Such "soft photons" cannot be detected and their number may be optional. This difficulty is related with the know infrared catastrophy in QED. We shall not analyze this difficulty here, but merely try to avoid it by assuming that we take into account only the photons with energies greater than $\Delta\varepsilon$ — the threshold energy of the photon detectors. Our assumption is in agreement with Heitler's discussion on the infrared catastrophy in QED (see [14]).

We use the following notation for the scattering amplitudes

$$S_{nm}^{i_1\ldots i_n, j_1\ldots j_m}(\ ; \vec{p}_1 \ldots \vec{p}_n, \vec{k}_1 \ldots \vec{k}_m) \equiv S_{nm}^{ij}(\ ; \vec{p}, \vec{k}). \tag{1.1}$$

The empty place before the semicolon corresponds to variables refering to electrons and positrons. The functional description of electrons and positrons was given in [2] ,[3], therefore, at present, we omit variables related with these particles. The indices i_k, j_k describe the polarization of photons and take the values 1, 2.

The generating functional for scattering amplitudes $S[;\alpha_i, \beta_i]$ is defined as follows

$$S[;\alpha_i, \beta_i] \equiv \sum_{lr}^{\infty} \sum_{\substack{i_1 \dots i_l \\ j_1 \dots j_r}} \frac{1}{\sqrt{l!r!}} \int d\vec{p}_1 \dots d\vec{p}_l d\vec{k}_1 \dots d\vec{k}_r \times$$

$$\times S_{lr}^{ij}(;\vec{p}, \vec{k}) \alpha_{i_1}(\vec{p}_1) \dots \alpha_{i_l}(\vec{p}_l) \beta_{j_1}(\vec{k}_1) \dots \beta_{j_r}(\vec{k}_r). \tag{1.2}$$

In the above definition the integration over $d\vec{p}$, $d\vec{k}$ runs from $-\infty$ to $+\infty$. We take into consideration only photons with energy greater than $\Delta\varepsilon$. The extension of integration from $-\infty$ to $+\infty$ can be achieved by the assumption that the packets $\alpha_i(\vec{p})$, $\beta_j(\vec{k})$ do not contain momenta smaller than $\Delta\varepsilon$, *i.e.* $\alpha_i(\vec{k}) = 0$ and $\beta_i(\vec{k}) = 0$ for $|\vec{k}| < \Delta\varepsilon$.

The probabilistic interpretation of scattering amplitudes imposes the following unitarity condition on the generating functional (see [1], [3])

$$S^+\left[;\alpha_i, \frac{\delta}{\delta\varrho_i}\right] S[;\varrho_i, \beta_i] = e^{(;\alpha_i\beta_i)}.$$

Here

$$S^+[;\alpha_i, \beta_i] = S^*[;\beta_i, \alpha_i] \tag{1.3}$$

$$\alpha_i\beta_i = \sum_{i=1}^{2} \int d\vec{p}\alpha_i(p)\beta_i(\vec{p}).$$

Now, we introduce a special, physically distinguished, complete, but nonorthogonal base in the momentum space[2]. We indicate the base versors by e_1, e_2, p, n. They obey the following orthogonality and completeness relations

$$(e_i, e_j) = \delta_{ij} \qquad n^2 = -1$$

$$(e_i, n) = (e_i, p) = 0 \tag{1.4}$$

$$\sum_{i=1}^{2} e_{i\mu}e_{i\nu} + \frac{[p_\mu + (np)n_\mu]\,[p_\nu + (np)n_\nu]}{p^2 + (np)^2} - n_\mu n_\nu = \delta_{\mu\nu}. \tag{1.5}$$

We call this base physically distinguished, because one can choose two versors e_1, e_2 as the polarization four-vectors for photons with momentum equal to p.

The extrapolation off the mass shell for the scattering amplitudes is restricted by the requirement that the transversal part of the amplitudes generalized off the mass shell after multiplication and integration with orthonormal solutions of the d'Alambert equation are equal to the scattering amplitudes on the mass shell. Namely

$$S_{lr}^{\prime ij}(;\vec{p}, \vec{k}) = \frac{i^{l+r}}{\sqrt{l!r!}} \sum_{\mu_1 \dots \mu_{l+r}}^{4} \int dx_1 \dots dx_{l+r} \times$$

$$\times \mathscr{S}_{l+r}^{\mu_1 \dots \mu_{l+r}}(;x_1 \dots x_{l+r}) \prod_{n=1}^{l} f^*(\vec{p}_n, x_n)\, e_{i_n\mu_n}(\vec{p}_n) \prod_{m=1}^{r} f(\vec{k}_m, x_{l+m})\, e_{j_m\mu_{l+m}}(\vec{k}_m). \tag{1.6}$$ [3]

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[2] A detailed discussion of such a base is given in [8].

[3] The relation (1.6) is like an "asymptotic condition" expressed in terms of the scattering amplitudes.

Here

$$\mathscr{S}_n^{\mu_1 \dots \mu_n}(; x_1 \dots x_n) \equiv \mathscr{S}_n^{\mu}(; x)$$

are symmetric generalized off-mass shell scattering amplitudes. $S'^{ij}_{lr}(; \vec{p}, \vec{k})$ are obtained from $S^{ij}_{lr}(; \vec{p}, \vec{k})$ by the separation of the freely propagating particles. The generating functional for $S'^{ij}_{lr}(; \vec{p}, \vec{k})$ is defined by $S[; \alpha_i, \beta_i]$

$$S'[; \alpha_i, \beta_i] = e^{(;-\alpha_i \beta_i)} S[; \alpha_i, \beta_i]. \tag{1.7}$$

The generalized amplitudes $\mathscr{S}_n^{\mu}(;x)$ are used for the definition of the generating functional

$$\mathscr{S}[; A_\mu] = \sum_n^{\infty} \sum_{\mu}^{4} \frac{i^n}{n!} \int dx_1 \dots dx_n \mathscr{S}_n^{\mu}(: x) A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n). \tag{1.8}$$

By substitution of (1.6) into (1.7) and (1.2) one can verify the following equality

$$\mathscr{S}[; {}^tA_\mu^0] [\alpha_i, \beta_i] = e^{(;-\alpha_i\beta_i)} S[; \alpha_i, \beta_i] \tag{1.9}$$

where ${}^tA_\mu^0 [x, \alpha_i, \beta_i]$ forms the transversal part of the solution of the d'Alambert equation

$${}^tA_\mu^0[x, \alpha_i, \beta_i] = \sum_i \int d\vec{p}\{e_{i\mu}(\vec{p})\alpha_i(\vec{p})f^*(\vec{p}, x) + e_{i\mu}(\vec{p})\beta_i(\vec{p})f(\vec{p}, x)\}. \tag{1.10}$$

The unitarity condition (1.3) expressed by the functional $\mathscr{S}[; A_\mu]$ takes the form (*cf.e.g.* [1], [4])

$$\mathscr{S}^*[; {}^tA_\varrho^0] \otimes \mathscr{S}[; {}^tA_\varrho^0] = 1. \tag{1.11}$$

The definition of the star product $\otimes$ is the following

$$\otimes \equiv \exp\left\{i \sum_{\nu\mu} \int dx dy \frac{\overleftarrow{\delta}}{\delta\, {}^tA_\mu^0(x)}\, {}^t\Delta_{\mu\nu}^+(x-y) \frac{\vec{\delta}}{\delta\, {}^tA_\nu^0(y)}\right\} \equiv \exp\left\{i \frac{\delta}{\delta {}^tA_\mu^0}\, {}^t\Delta_{\mu\nu}^+ \frac{\delta}{\delta\, {}^tA_\nu^0}\right\} \tag{1.12}$$

where

$${}^t\Delta_{\mu\nu}^+(x-y) = \left(\delta_{\mu\nu} - \frac{[\partial_\mu + n_\mu(n\partial)]\,[\partial_\nu + n_\nu(n\partial)]}{\Box + (n\partial)^2} + n_\mu n_\nu\right) \Delta^+(x-y) \equiv O_{\mu\nu}^n \Delta^+. \tag{1.13}$$

It is easy to see that

$$n_\mu\, {}^t\Delta_{\mu\nu}^+ = 0 \quad \text{and} \quad \partial_\mu\, {}^t\Delta_{\mu\nu}^+ = 0.$$

We are now able to impose on $\mathscr{S}[; A_\mu]$ the condition of generalized unitarity (see [3] p. 113) but we wish firstly to clarify the notion of functional derivative with respect to the transversal functions. At first sight there are two possibilities of definition of such a derivative

i) We can use the usual definition used for nonrestricted functions, putting everywhere after differentiation in the place od A_μ, the function ${}^tA_\mu$.

ii) Another definition takes into consideration the transversal property of these functions and looks similar to the one introduced in ([4] Sec. 5) for functions restricted by Lorentz condition, namely

$$\frac{\delta\, {}^tA_\mu(x)}{\delta\, {}^tA_\nu(y)} = O_{\mu\nu}^n \delta(x-y). \tag{1.14}$$

This second possibility seems to us more natural and more adequate for the description of the physical content of the theory. In this paper we shall apply the definition (1.14). Now we simplify the star product $\otimes$. Since, as it is easy to show

$$O^n_{\mu\nu}O^n_{\nu\varrho} = O^n_{\mu\varrho} \tag{1.15}$$

we can change ${}^t\Delta^+_\mu(x-y)$ by $\Delta^+_{\mu\nu}(x-y) = \delta_{\mu\nu}\Delta^+(x-y)$ without any change of product (1.12). After this change the star product simplifies to

$$\otimes = \exp\left\{i \frac{\delta}{\delta\, {}^tA^0_\mu} \Delta^+_{\mu\nu} \frac{\delta}{\delta\, {}^tA^0_\nu}\right\}.$$

Let us now consider how to generalize the unitarity condition (1.11). At least, two ways may be seen. The first one is based on the rejection of the condition that A_μ must be a solution of d'Alambert equation, but preserves its transversality. Therefore, we impose on $\mathscr{S}[;{}^tA_\mu]$ the following generalized unitarity condition

$$\mathscr{S}^*[;{}^tA_\varrho](\times)\mathscr{S}[;{}^tA_\varrho] = 1 \tag{1.16}$$

where

$$(\times) = \exp\left\{i \frac{\delta}{\delta\, {}^tA_\mu} \Delta^+_{\mu\nu} \frac{\delta}{\delta\, {}^tA_\nu}\right\}. \tag{1.17}$$

This generalization corresponds to the formulation of QED in Coulomb gauge. The second way relies on rejecting both the fullfilment by A_μ of the d'Alambert equation and its transversality. Namely, we impose on $\mathscr{S}[;A_\mu]$ the following condition

$$\mathscr{S}^*[;A_\mu] * \mathscr{S}[;A_\mu] = 1 \tag{1.18}$$

where

$$* = \exp\left\{i \frac{\delta}{\delta A_\mu} \Delta^+_{\mu\nu} \frac{\delta}{\delta A_\nu}\right\}. \tag{1.19}$$

It is easy to understand that the condition (1.18) is more restrictive than (1.16). While (1.16) restricts only the transversal part of generalized amplitudes $\mathscr{S}^\mu_n(;x)$, the condition (1.18) restricts the whole $\mathscr{S}^\mu_n(;x)$. Both (1.16) and (1.18) on the mass shell (*i.e.* for $A_\mu = {}^tA_\mu = {}^tA^0_\mu$) go over into (1.11). This second generalization corresponds to QED in Feynman gauge.

2. *QED in Feynman gauge*

The Feynman gauge is characterized by the complete independence of the four components of the electromagnetic potential. In the operator formulation of QED the transversality of the external photons is restored by imposing on the physically admissible states the Lorentz subsidiary condition. We want to show that in the functional formulation one can obtain the same S-matrix elements, as were obtained in operator formulation, without the apparent use of the subsidiary condition. We give the functional scheme of the theory.

First of all, remaining on the mass shell, we extend the generating functional $\mathscr{S}[;{}^tA^0_\mu]$ to longitudinal and time degrees of freedom. Consider the functional $\mathscr{S}[;{}^tA^0_\mu]$ where

$$A^0_\mu[x, \alpha_\mu, \beta_\mu] = \int d\vec{p}\{\alpha_\mu(\vec{p})f^*(\vec{p}, x) + \beta_\mu(\vec{p})f(\vec{p}, x)\}. \tag{2.1}$$

Here $\alpha_\mu(\vec{p})$, $\beta_\mu(\vec{p})$ are any four vector functions. We can decompose $\alpha_\mu(\vec{p})$ and $\beta_\mu(\vec{p})$ according to the formula (1.5)

$$\alpha_\mu(\vec{p}) = {}^t\alpha_\mu(\vec{p}) + \frac{[(p\alpha)+(np)\,(n\alpha)]\,[p_\mu+(np)n_\mu]}{p^2+(np)^2} - (n\alpha)n_\mu$$

$${}^t\alpha_\mu(p) = \sum_i^2 (e_i\alpha)e_{i\mu}(\vec{p}). \tag{2.2}$$

So

$$A^0_\mu[x, \alpha_\mu, \beta_\mu] = {}^tA^0_\mu[x, \alpha_i, \beta_i] + \int d\vec{p}\left\{f^*(\vec{p}, x) \times\right.$$

$$\left.\times\left[\frac{[(p\alpha)+(np)\,(n\alpha)]\,[p_\mu+(np)n_\mu]}{p^2+(np)^2} - (n\alpha)n_\mu\right] + f(\vec{p}, x)\,[\alpha\to\beta]\right\}. \tag{2.3}$$

Here

$$\alpha_i = (e_i\alpha), \qquad \beta_i = (e_i\beta).$$

In our scheme functional $\mathscr{S}[;\ A^0_\mu]$ plays an analogous role to the role of the functional $\mathscr{S}[q_0]$ in the scalar case (see [1]). However, it is necessary to keep in mind that in order to obtain the generating functional for amplitudes on the mass shell, we must replace A^0_μ by ${}^tA^0_\mu$ after calculation have been performed.

In order to formulate the second fundamental condition — the causality condition — let us introduce the functional matrix

$$S[;\mathrm{A}_\mu, \alpha_\mu, \beta_\mu] \equiv e^{\alpha_\mu\beta_\mu}\mathscr{S}[;A_\mu + A^0_\mu[\alpha_\mu, \beta_\mu]] \tag{2.4}$$

and also the matrix

$$T[;J_\mu, \alpha_\mu, \beta_\mu] \equiv S^+\left[;0, \alpha_\mu, \frac{\delta}{\delta\varrho_\mu}\right] S[;\Delta^F_{\mu\nu}J_\nu, \varrho_\mu, \beta_\mu]\Big|_{\varrho=0}. \tag{2.5}$$

In abbreviated matrix notation

$$\boldsymbol{T}[;J_\mu] = \boldsymbol{S}^+[;0]\,\boldsymbol{S}[;\Delta^F_{\mu\nu}J_\nu]. \tag{2.6}$$

We impose on $\boldsymbol{T}[;J_\mu]$ the causality condition which has the following differential form

$$\frac{\delta}{\delta J_\mu(x)}\left(\boldsymbol{T}[;J_\varrho]\,\frac{\delta \boldsymbol{T}^+[;J_\varrho]}{\delta J_\nu(y)}\right) = 0 \qquad \text{for } x \leqslant y. \tag{2.7}$$

In this paper we shall not try to solve perturbatively the unitarity and causality equations[4].

[4] One can find the formulation of the divergence free method leading to the perturbative solution of causality and unitarity condition for the scalar case in [15].

Here we are interested only in the general construction of theory. With the aid of the same formal consideration as in the scalar case (see [3] Chapter 4) one can see that the general formal solution of the unitarity and causality conditions is given by the following functional[5]

$$\tau[\eta, \bar{\eta}; J_\mu] = \exp\left\{iI\left[-i\frac{\delta}{\delta\eta^*}, i\frac{\delta}{\delta\eta}\gamma_4; -i\frac{\delta}{\delta J_\mu}\right]\right\} e^{i\eta^* D^{-1}}\eta e^{-\frac{i}{2}J_\varrho \Delta^F_{\varrho\nu}J_\nu}. \tag{2.8}$$

Where $\tau[\eta, \bar{\eta}; J_\mu]$ is expressed by the generating functional for the off-mass shell amplitudes as follows

$$\tau[\eta, \bar{\eta}; J_\mu] = \mathscr{S}[-S^F\gamma_4\eta, -\bar{\eta}\gamma_4 S^F; \Delta^F_{\mu\nu}J_\nu]. \tag{2.9}$$

The functional $I[\psi\,\bar{\psi}; A_\mu]$ must be real and local. It plays the role of interaction functional. One can check that for$I[\psi_0, \bar{\psi}_0, A^0_\mu] = 0$ there is no interaction, namely $\mathscr{S}[\psi_0, \bar{\psi}_0, A^0_\mu] = 1$.

We have given here briefly the general framework of the theory. The equivalence of this formulation to that given by Feyman — Dyson can be clearly seen if we pass to the so-called canonical formalism (see [3] page 181). Performing exactly the same calculations as in the scalar ([1] Sec. 9) or in the spinor cases ([2] Sec. 11) we obtain

$$\begin{aligned} S[\alpha, \beta^*, \beta, \alpha^*; \alpha_\mu, \beta_\mu] &= e^{\alpha\alpha^*+\beta^*\beta+\alpha_\mu\beta_\mu} \times \\ &\times \mathscr{S}[\psi_0[\alpha, \beta], \bar{\psi}_0[\alpha^*, \beta^*]; A^0_\mu[\alpha_\mu, \beta_\mu]] \\ &= T\exp\{iI[\boldsymbol{\chi}_0, \bar{\boldsymbol{\chi}}_0, \boldsymbol{A}_{\mu 0}]\}. \end{aligned} \tag{2.10}$$

Here $\boldsymbol{\chi}_0, \bar{\boldsymbol{\chi}}_0, \boldsymbol{A}_{\mu 0}$ are the free field operators for spinor and photon fields respectively. In the functional matrix notation they have the following form

$$\begin{aligned} \chi_0[\alpha, \beta^*, \beta, \alpha^*, x] &= -e^{\alpha\alpha^*+\beta\beta^*}\gamma_4\psi_0[\alpha, \beta, x] \\ A_{\mu 0}[\alpha_\mu, \beta_\mu, x] &= -e^{\alpha_\mu\beta_\mu}A^0_\mu[\alpha_\mu, \beta_\mu; x] \end{aligned} \tag{2.11}$$

and they obey the usual commutation relations, namely

$$[\boldsymbol{\chi}_0(x), \bar{\boldsymbol{\chi}}_0(y)]_+ = iS(x-y) \tag{2.12}$$

$$[\boldsymbol{A}_{\mu 0}(x), \boldsymbol{A}_{\nu 0}(y)] = ig_{\mu\nu}\Delta(x-y). \tag{2.13}$$

Other commutator vanish.

The operators $\boldsymbol{\chi}_0(x), \boldsymbol{A}_{\mu 0}(x)$ are defined by the functional matrix $\boldsymbol{T}[\eta, \bar{\eta}, J_\nu]$ in the following way

$$\boldsymbol{\chi}_0(x) = -i\left.\frac{\delta\boldsymbol{T}[0, \eta, \bar{\eta}, J_\nu]}{\delta\bar{\eta}(x)}\right|_{\substack{\eta=\bar{\eta}=0\\ J_\nu=0}} \tag{2.14}$$

$$\boldsymbol{A}_{\mu 0}(x) = -i\left.\frac{\delta\boldsymbol{T}[0, \eta, \bar{\eta}, J_\nu]}{\delta J_\mu(x)}\right|_{\substack{\eta=\bar{\eta}=0\\ J_\nu=0}} \tag{2.15}$$

when the subscript zero inside $\boldsymbol{T}[\eta\,\bar{\eta}, J_\nu]$ means switching-off of the interaction.

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[5] In this place we explicitly introduce variables refering to electrons and positrons. The notation is taken from [2] and [3] $D^{-1} = -S^F\gamma_4$.

The expression (2.10) together with (2.12), (2.13) give for internal lines of S-matrix elements an agreement with coventional QED provided that one chooses the usual form for the interaction functional

$$I[\psi, \bar{\psi}, A_\mu] = j_\mu A_\mu \tag{2.16}$$

where

$$j_\mu = \frac{ie}{2} [\bar{\psi}, \gamma_\mu \psi].$$

In order to obtain complete agreement with conventional theory also for external lines, one is obliged to change[6] α_μ and β_μ into ${}^t\alpha_\mu$ and ${}^t\beta_\mu$ (see formula (2.2)) after the evaluation of the T-product is performed. This procedure means that we allow for external photons only two degrees of freedom. Conventionally, such an effect is reached by imposing on physically admissible state vectors the subsidiary condition[7]. Thus the equivalence of our formulation to the conventional theory is demonstrated.

3. *QED in Coulomb gauge*

In the formulation of QED in the Coulomb gauge we deal with transverse photons only. This description, though close to physics, is rather more complicated then the Feynman one. First of all the theory looses its manifestly covariant character due to the presence of the distinguished time-like vector n_μ. Nevertheless the theory, as it was shown in several ways (see [8], [9], [12]), is Lorentz covariant and its covariance follows from the freedom of choice of the vector n_μ [8]. This freedom is connected with the arbitrariness in the choice of polarization four-vectors $e_{i\mu}$, namely the two set of vectors related through transformations

$$e'_{\mu i}(\vec{p}) = e_{\mu i}(\vec{p}) + p_\mu \alpha_i(\vec{p})$$

describe the same physical photon states. Another complication comes from the nonlocality of the interaction. Absence of the longitudinal and scalar photons is compensated by the Coulomb interaction term. This term in general framework looks as follows

$$I_c = \frac{1}{2} \frac{(nj)(nj)}{\Box + (n\partial)^2}$$

where j_μ is defined in (2.16). The operator $(\Box + (n\partial)^2)^{-1}$ is nonlocal:

$$I_c = \frac{1}{2} \int dx dy n_\mu j_\mu(x) [\Box + (n\partial)^2]^{-1} (x-y) n_\nu j_\nu(y). \tag{3.1}$$

In a special framework with $n_\mu = (0, 0, 0, 1)$ the above relation simplifies to

$$I_c = \frac{1}{2} \int dx dy \frac{1}{4\pi} \frac{1}{|\vec{x}-\vec{y}|} j_0(x) j_0(y) \delta(x_0 - y_0). \tag{3.2}$$

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[6] Such change corresponds to the change A^0_μ into ${}^tA^0_\mu$.

[7] The role of subsidiary condition is explained in [8], [6].

From (3.2) it is evident that the Coulomb interaction is, at least in this special framework, nonlocal only in space variables and local in time. The formal proof of the validity of unitarity and causality conditions can be carried out if the interaction functional is real and only local in time. Looking at the method of the proof ([3] Chapter [4]) one can see that the nonlocality in space changes the sequence of space variables in closed cycles of retarded functions, but the sequence of time variables, which is responsible for vanishing of such cycles is preserved. The proof was based on the discussion of these cycles and the property that they vanish. So, in the framework with $n_\mu = (0, 0, 0, 1)$ the functional

$$\tau[\eta, \bar{\eta}; {}^tJ_\mu] = \mathscr{S}[-S^F\gamma_4\eta, -\bar{\eta}\gamma_4 S^F; \Delta^F_{\mu\nu}J_\nu]$$

$$= \exp\left\{iI\left[-i\frac{\delta}{\delta\eta^*}, i\frac{\delta}{\delta\eta}\gamma_4; -i\frac{\delta}{\delta^tJ_\mu}\right]\right\} e^{i\eta^*D^{-1}\eta - i/_2\, {}^tJ_\varrho\Delta_{\varrho\nu}{}^tJ_\nu} \tag{3.3}$$

where

$$I[\psi, \bar{\psi}; {}^tA_\mu] = j_\mu{}^tA_\mu + \frac{1}{2}\frac{(nj)(nj)}{\Box + (n\partial)^2} \tag{3.4}$$

formally solves the unitarity (1.16) and causality conditions, which in the simplest form look as follows

$$\frac{\delta}{\delta^tJ_\mu(x)}\left(\boldsymbol{T}[\eta, \bar{\eta}; {}^tJ_\varrho]\frac{\delta\boldsymbol{T}^+[\eta, \bar{\eta}; {}^tJ_\varrho]}{\delta^tJ_\nu(y)}\right) = 0 \quad \text{for} \quad x \lesssim y \tag{3.5}$$

$$\frac{\delta}{\delta\eta_i(x)}\left(\boldsymbol{T}[\eta, \bar{\eta}; {}^tJ_\varrho]\frac{\delta\boldsymbol{T}^+[\eta, \bar{\eta}, {}^tJ_\varrho]}{\delta(\eta_j(y))}\right) = 0 \tag{3.6}$$

$$\eta_1 = \eta, \ \eta_2 = \bar{\eta}^* \qquad i, j = 1, 2.$$

Here

$$\boldsymbol{T}[\eta, \bar{\eta}; {}^tJ_\varrho] \equiv \mathbf{S}^+[0, 0; 0]\mathbf{S}[-S^F\gamma_4\eta, -\bar{\eta}\gamma_4 S^F; \Delta^{F\,t}_{\varrho\nu}J_\nu] \tag{3.7}$$

and the functional matrix

$$S[\psi, \bar{\psi}; {}^tA_\mu; \alpha, \beta^*, \beta, \alpha^*; \alpha_i, \beta_i] = e^{\alpha\alpha^* + \beta^*\beta + \alpha_i\beta_i} \times$$

$$\times \mathscr{S}[\psi + \psi_0[\alpha, \beta], \bar{\psi} + \bar{\psi}_0[\alpha^*, \beta^*]; {}^tA_\mu + {}^tA^0_\mu[\alpha_i, \beta_i]]. \tag{3.8}$$

We are not able to verify directly the solvability of the unitarity and causality conditions by (3.3) in any framework, but keeping in mind the Lorentz covariance of the theory we suppose that such solvability occurs.

Following the same[8] rules as in the scalar or spinor cases ([1] Sec. 9, [2] Sec. 11) one can obtain the canonical form of the theory outlined in the formulae (3.3)–(3.8). We have:

$$S[\alpha, \beta^*, \beta, \alpha^*; \alpha_i, \beta_i] = e^{\alpha\alpha^* + \beta^*\beta + \alpha_i\beta_i} \times$$

$$\times \mathscr{S}[\psi_0[\alpha, \beta], \bar{\psi}[\alpha^*, \beta^*]; {}^tA^0_\mu[\alpha_i, \beta_i]] = T\exp\{iI[\boldsymbol{\chi}_0, \boldsymbol{\chi}_0, {}^t\boldsymbol{A}_{\mu 0}]. \tag{3.10}$$

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[8] Here we use the functional derivative over transversal functions given in (1.14). This complicates slightly the calculations. However, due to the validity of Volterra's formula (see [4] Sec. 5)

$$F[{}^tA_\mu + {}^tA'_\mu] = \exp\left\{{}^tA'_\mu \frac{\delta}{\delta^tA_\mu}\right\} F[{}^tA_\mu] \tag{3.9}$$

all calculation can be carried through.

Here the definition of $\boldsymbol{A}_{\mu 0}(x)$ is the following

$$ {}^t\boldsymbol{A}_{\mu 0}(x) = -i\,\frac{\delta \boldsymbol{T}[0;\ \eta, \bar{\eta};\ {}^tJ_\nu]}{\delta^t J_\mu(x)}\Bigg|_{\substack{\eta=\bar{\eta}=0\\ {}^tJ_\nu=0}}. \tag{3.11} $$

The functional matrix ${}^tA_{\mu 0}[\alpha_i, \beta_i, x] \equiv {}^t\boldsymbol{A}_{\mu 0}(x)$ can be expressed by the transversal solution of the d'Alambert equation (1.10)

$$ {}^tA_{\mu 0}[\alpha_i, \beta_i, x] = -e^{\alpha_i\beta_i}\,{}^tA^0_\mu[\alpha_i, \beta_i, x]. \tag{3.12} $$

Now one can easily obtain the commutation rules

$$ [{}^t\boldsymbol{A}_{\mu 0}, {}^t\boldsymbol{A}_{\nu 0}(y)]_- = iO^n_{\mu\nu}\Delta(x-y). \tag{3.13} $$

4. *The comparison of Feynman and Coulomb gauges*

We would like to check whether the functionals generating off-mass shell amplitudes in Feynman (Eq. (2.8)) and Coulomb (Eq. (3.3)) gauges lead to the same scattering amplitudes. The direct comparison between formulae (2.10) and (3.10) is rather difficult (see [9]) and not too transparent. Hence, we shall start from the off-mass shell functional in the Feynman gauge and, after simple transformation, we shall obtain the coincidence of generating functionals on mass shell in both gauges. We are going to carry on our considerations with the aid of the functional integration method (see [3] Chapter 7). First of all we rewrite (2.8) and (3.3) in alternative, integral forms. Namely[9]

$$ \tau_F[\eta, \bar{\eta}; J_\nu] = N_F^{-1}\int \delta A_\nu,\ \delta\psi\delta\bar{\psi}e^{i/_2 A_\nu \Box A_\nu} \times $$
$$ \times e^{i\psi^* D\psi}e^{ij_\nu A_\nu}e^{i\psi^*\eta + i\eta^*\psi + iJ_\nu A_\nu} \tag{4.1} $$

and

$$ \tau_C[\eta, \bar{\eta}, {}^tJ_\nu] = N_C^{-1}\int \delta^t A_\nu \delta\psi\delta\bar{\psi}e^{i/_2 {}^tA_\nu \Box {}^tA_\nu} \times $$
$$ e^{i\psi^* D\psi}e^{i\left\{j_\nu {}^tA_\nu + \frac{1}{2}\frac{(nj)(nj)}{\Box + (n\partial)^2}\right\}}e^{i\psi^*\eta + i\eta^*\psi + i\,{}^tJ_\nu {}^tA_\nu}. \tag{4.2} $$

N_F and N_C are normalization constants

$$ N_F = \tau_F[0, 0; 0];\ N_C = \tau_C[0, 0; 0]. \tag{4.3} $$

Now we integrate (4.1) over time and longitudinal components. For that purpose we decompose A_μ according to (1.5)

$$ A_\mu = \sum_{i=1}^{2} e_{i\mu}(e_iA) + \frac{[\partial_\mu + n_\mu(n\partial)]\,[\partial A + (nA)\,(n\partial)]}{\Box + (n\partial)^2} + n_\mu(nA). \tag{4.4} $$

Therefore

$$ A_\mu = {}^tA_\mu + n_\mu{}^SA + \partial_\mu{}^lA \tag{4.5a} $$

[9] In this section signs F and C label quantities in Feynman and Coulomb gauges respectively.

where

$$
\begin{aligned}
{}^{t}A_{\mu} &= \sum_{i=1}^{2} e_{i\mu}(e_i A) \\
{}^{S}A &= \frac{(n\partial)\,(\partial A) + \Box(nA)}{\Box + (n\partial)^2} \\
{}^{l}A &= \frac{(\partial A) + (nA)\,(n\partial)}{\Box + (n\partial)^2}\,.
\end{aligned}
\tag{4.5b}
$$

The above expressions describe a linear transformation relating the components A_μ with ${}^{t}A_\mu$, ${}^{S}A$ and ${}^{l}A$. Introduction of such transformation into the integral (4.1) change the measure of integration. Nevertheless, due to the linearity of our transformation, the change of measure results in the multiplication of integral by an infinite constant. The same constant occurs in the denominator by performing the same transformation in N_F and, therefore, the net effect of the transformation will be none. After the use of the transformation (4.5) we obtain

$$
\begin{aligned}
\tau_F[\eta, \bar{\eta}; J_\mu] = N_F^{-1} \int \delta\,{}^{t}A_\mu\, \delta\,{}^{S}A\, \delta\,{}^{l}A \delta\psi \delta\bar{\psi} \times \\
\times e^{i/_2({}^{t}A_\mu + n_\mu {}^{S}A + \partial_\mu {}^{l}A)\Box({}^{t}A_\mu + n_\mu {}^{S}A + \partial_\mu {}^{l}A)} e^{i\psi^* D\psi} e^{ij_\mu({}^{t}A_\mu + n_\mu {}^{S}A + \partial_\mu {}^{l}A)} \times \\
\times e^{i\psi^*\eta + i\eta^*\psi + iJ_\mu({}^{t}A_\mu + n_\mu {}^{S}A + \partial_\mu {}^{l}A)}
\end{aligned}
\tag{4.6}
$$

We integrate (4.6) over $\delta^S A$ and $\delta^l A$ in a straightforward way and obtain (see Appendix I)

$$
\begin{aligned}
\tau_F[\eta, \bar{\eta}; J_\mu] = N_F^{-1} \int \delta^t A_\mu \delta\psi \delta\bar{\psi} e^{1/_2 {}^{t}A_\mu \Box {}^{t}A_\mu} e^{i\psi^* D\psi} e^{i\eta\psi^* + i\eta^*\psi + iJ_\mu {}^{t}A_\mu} \times \\
\times e^{i\left\{j_\mu {}^{t}A_\mu + \frac{1}{2}\frac{(nj)\,(nj)}{\Box + (n\partial)} - \frac{2(nj)\,(n\partial)\,(\partial j) + (\partial j)^2}{2\,\Box(\Box + (n\partial)^2)}\right\}} \times \\
\times e^{i\left\{\frac{1}{2}\frac{(nJ)\,(nJ)}{\Box + (n\partial)^2} - \frac{2(nJ)(n\partial)(\partial J) + (\partial J)^2}{2\Box(\Box + (n\partial)^2)}\right\}} \times \\
\times e^{i\left\{\frac{(nj)(nJ)}{\Box + (n\partial)^2} - \frac{(nJ)(n\partial)(\partial j) + (nj)(n\partial)(\partial J) + (\partial j)(\partial J)}{\Box(\Box + (n\partial)^2)}\right\}}\,.
\end{aligned}
\tag{4.7}
$$

The off mass shell generating functional in the Feynman gauge contains transversal, time--like and longitudinal parts. If we wish to compare that functional with the one in the Coulomb gauge it is sufficient to consider the transversal parts only. In the Coulomb gauge the time and longitudinal parts do not occur. We obtain the transversal part of $\tau_F[\eta, \bar{\eta}; J_\mu]$ if we put instead of J_μ only its transversal part ${}^{t}J_\mu$. Because $(n^t J) = (\partial^t J) = 0$ we can write

$$
\begin{aligned}
\tau_F[\eta, \bar{\eta}; {}^{t}J_\mu] = N_F^{-1} \int \delta^t A_\mu \delta\psi \delta\bar{\psi} e^{i/_2 {}^{t}A_\mu \Box {}^{t}A_\mu} e^{i\psi^* D\psi} e^{i\eta^*\psi + i\psi^*\eta + i\,{}^{t}J_\mu {}^{t}A_\mu} \times \\
\times e^{i\left\{j_\mu {}^{t}A_\mu + \frac{1}{2}\frac{(nj)(nj)}{\Box + (n\partial)^2} \frac{2(nj)(n\partial)(\partial j) + (\partial j)^2}{2\Box(\Box + (n\partial)^2)}\right\}}\,.
\end{aligned}
\tag{4.8}
$$

Comparing (4.8) and (4.2) we see that the difference consists in the existence of extra interaction terms in (4.8). Apart from the Coulomb interaction and interaction with transversal photons, Eq. (4.8) contains two exeptional terms. However, Eq. (4.8) and Eq. (4.2) give the same expressions for the S-functional on the mass shell. This becomes evident if we look for the canonical form of S-functional on the mass shell corresponding to the formula (4.8). The only difference between the canonical form derived from (4.8) and the analogous canonical form in the Coulomb gauge (3.10) is shown by the term

$$I_n = -\frac{2(nj)(n\partial)(\partial j)+(\partial j)^2}{2\Box(\Box+(n\partial)^2)} \tag{4.9}$$

present in the interaction functional. The functional matrices $\chi_0[\alpha, \beta^*, \beta, \alpha^*; x]$ and $\bar{\chi}_0[\alpha, \beta^*, \beta, \alpha^*; x]$ obey the Dirac equations, thus the current $j_\mu(x)$ formed from it must beconserved, *i.e.* $(\partial j) = 0$, and therefore $I_n = 0$.

The formula (4.8) permits to obtain the connection between the transversal part of the off-mass shell functional in the Feynman gauge and the off-mass shell functional in the Coulomb gauge. Namely

$$\tau_F[\eta, \bar{\eta}; {}^tJ_\mu] = \frac{N_C}{N_F} e^{\frac{i\{(n\tilde{j})(n\partial)(\partial\tilde{j})+\frac{1}{2}(\partial\tilde{j})^2)\}}{\Box(\Box+(n\partial)^2)}} \cdot \tau_C[\eta, \bar{\eta}; {}^tJ_\mu] \tag{4.10}$$

where

$$\tilde{j}_\mu(x) = \frac{ie}{2}\left[\frac{\delta}{\delta\eta(x)}, \gamma_\mu \frac{\delta}{\delta\bar{\eta}(x)}\right]. \tag{4.11}$$

The expression (4.10) connects, in a compact form, the transversal part of the Green functions in the Feynman gauge with the Green functions in the Coulomb gauge. The connection is rather involved, because the transversal part of any Green function in Feynman gauge is given by an infinite sum of suitable Green functions in the Coulomb gauge.

Final remarks

We gave the formulation of QED based on the scattering amplitudes and treated operators as a secondary notion. In this way we avoid some difficulties which occur in the conventional approach to QED. However, the difficulties connected with the calculation of scattering amplitudes are of course further present in our formulation, exactly as in the conventional one.

APPENDIX I

Integration (4.6) over ${}^lA$ gives the result

$$\tau_F[\eta, \bar{\eta}; J_\mu] = N_F^{-1} \int \delta^t A_\mu \delta^S A \delta\psi\delta\bar{\psi} \times$$

$$\times e^{-i/2 \frac{[(n\partial)\Box^S A+(\partial j)+(\partial J)]^2}{\Box^2}} \cdot e^{i/2\,({}^tA_\mu+n_\mu{}^SA)\Box({}^tA_\mu+n_\mu{}^SA)} \times$$

$$\times e^{i\psi^* D\psi} e^{ij_\mu({}^tA_\mu+n_\mu{}^SA)} e^{i\psi^*\eta+i\eta^*\psi+iJ_\mu({}^tA_\mu+n_\mu{}^SA)}$$

$$= N_F^{-1}\int \delta^t A_\mu \delta^S A\delta\psi\delta\bar{\psi} e^{i/2\,{}^tA_\mu\Box{}^tA_\mu - i/2\,{}^SA(\Box+(n\partial)^2){}^SA} \times$$

$$\times e^{-i/2\left\{\frac{(\partial j)^2}{\Box^2}+\frac{(\partial J)^2}{\Box^2}+2\frac{(\partial j)(\partial J)}{\Box^2}\right\}} e^{-i\left\{\frac{(n\partial)(\partial j){}^SA}{\Box}+\frac{(n\partial)(nJ){}^SA}{\Box}\right\}} \times$$

$$\times e^{i(j_\mu{}^tA_\mu+(nj){}^SA)} e^{i(J_\mu{}^tA_\mu+(nJ){}^SA)} e^{i\psi^* D\psi} e^{i\psi^*\eta+i\eta^*\psi}.$$

Integrating over ${}^{S}A$ we obtain

$$\tau_F[\eta, \bar{\eta}; J_\mu] = N_F^{-1} \int \delta^t A_\mu \delta\psi \delta\bar{\psi}^{i/_2 {}^tA_\mu \Box {}^tA_\mu} e^{i\psi^* D\psi} e^{i\eta^*\psi + i\psi^*\eta} \times$$

$$\times e^{-i/2\left\{\frac{(\partial j)^2}{\Box^2} + \frac{(\partial j)^2}{\Box^2} + 2\frac{(\partial J)(\partial j)}{\Box^2}\right\}} e^{i/2\left\{\frac{\left[(nj) + (nJ) - \frac{(n\partial)(\partial j)}{\Box} - \frac{(n\partial)(\partial J)}{\Box}\right]^2}{\Box + (n\partial)^2}\right\}} \times$$

$$\times e^{ij_\mu {}^tA_\mu} e^{iJ_\mu {}^tA_\mu} = N_F^{-1} \int \delta^t A_\mu \delta\psi \delta\bar{\psi} e^{i/2 {}^tA_\mu \Box {}^tA_\mu} e^{i\psi^* D\psi} e^{i\eta^*\psi + i\psi^*\eta + iJ_\mu {}^tA_\mu} \times$$

$$\times e^{i\left\{j_\mu {}^tA_\mu + \frac{1}{2}\frac{(nj)(nj)}{\Box + (n\partial)^2} - \frac{(nj)(n\partial)(\partial j)}{\Box(\Box + (n\partial)^2)} - \frac{1}{2}\frac{(\partial j)^2}{\Box(\Box + (n\partial)^2)}\right\}} \times$$

$$\times e^{\left\{\frac{1}{2}\frac{(nJ)(nJ)}{\Box + (n\partial)^2} - \frac{(nJ)(n\partial)(\partial J)}{\Box(\Box + (n\partial)^2)} - \frac{1}{2}\frac{(\partial J)^2}{\Box(\Box + (n\partial)^2)}\right\}} \times$$

$$\times e^{i\left\{\frac{(nj)(nJ)}{\Box + (n\partial)^2} - \frac{(nJ)(n\partial)(\partial j)}{\Box(\Box + (n\partial)^2)} - \frac{(nj)(n\partial)(\partial J) + (\partial j)(\partial J)}{\Box(\Box + (n\partial)^2)}\right\}}.$$

APPENDIX II

We are interested whether QED in general gauges, as the ones introduced by Zumino [12] with specified a_μ, Λ and M functions, is equivalent to the QED in Coulomb gauge. We write the generating off-mass shell functional in the form given by Białynicki [13].

$$\tau[\eta, \bar{\eta}; J_\mu, \Lambda, M] = N^{-1} \int \delta A_\mu \delta\psi \delta\bar{\psi} \delta\lambda e^{-i/2\lambda M\lambda} \times$$

$$\times e^{i\lambda(a_\mu A_\mu + \Lambda)} e^{i/2 A_\mu(\Box\delta_{\mu\nu} - \partial_\mu\partial_\nu)A_\nu} e^{i\psi^* D\psi} e^{ij_\mu A_\mu} e^{i\psi^*\eta + i\eta^*\psi + iJ_\mu A_\mu}. \qquad \text{(II. 1)}$$

Now we decompose A_μ according to (4.5), integrate over ${}^lA$, ${}^SA$ and λ, and look for the transversal part of the τ-functional. For simplicity we omit spinor variables by putting instead of them the semicolon.

$$\tau[; {}^tJ_\mu, \Lambda, M] = N^{-1} \int \delta^t A_\mu \delta^S A \delta^l A \delta\lambda; e^{i/2\lambda M\lambda} \times$$

$$\times e^{i\lambda a_\mu(n_\mu {}^SA + \partial_\mu {}^lA)} e^{i\lambda\Lambda} e^{i/2({}^tA_\mu + n_\mu {}^SA)(\Box\delta_{\mu\nu} - \partial_\mu\partial_\nu)({}^tA_\nu + n_\nu {}^SA)} e^{i{}^tJ_\mu {}^tA_\mu} e^{ij_\mu({}^tA_\mu + n_\mu {}^SA + \partial_\mu {}^lA)}. \qquad \text{(II. 2)}$$

Here we take into account that $a_\mu {}^tA_\mu = 0$ and $\delta_\mu(\Box\delta_{\mu\nu} - \partial_\mu\partial_\nu) = 0$. We integrate over ${}^SA$. We get

$$\tau[; {}^tJ_\mu, \Lambda, M] = N^{-1} \int \delta^t A_\mu \delta^l A \delta\lambda;$$

$$e^{i/2\lambda\left[\frac{(an)(an)}{\Box + (n\partial)^2} - M\right]\lambda} e^{i\lambda\Lambda - \lambda^l A} e^{i\lambda\frac{(an)(nj)}{\Box + (n\partial)^2}} e^{i/2\frac{(nj)(nj)}{\Box + (n\partial)^2}} e^{i{}^tJ_\mu {}^tA_\mu} e^{ij_\mu({}^tA_\mu + \partial_\mu {}^lA)}.$$

Integrating over λ we obtain

$$\tau[; {}^tJ_\mu, \Lambda, M] = N^{-1} \int \delta^t A_\mu \delta^l A; e^{i/2\frac{(nj)(nj)}{\Box + (n\partial)^2}} e^{ij_\mu {}^tA_\mu} e^{i{}^tJ_\mu {}^tA_\mu} \times$$

$$\times e^{-\frac{i}{2}\left\{\frac{\Lambda^2(\Box + (n\partial)^2)}{(an)^2 - M(\Box + (n\partial)^2)} + \frac{(an)^2(nj)^2}{(\Box + (n\partial)^2)[(an)^2 - M(\Box + (n\partial)^2)]} + \frac{2\Lambda(an)(nj)}{(an)^2 - M(\Box + (n\partial)^2)}\right\}} \times$$

$$\times e^{-\frac{i}{2}\frac{{}^lA^2(\Box + (n\partial)^2)}{(an)^2 - M(\Box + (n\partial)^2)}} \cdot e^{i\left\{\frac{{}^lA\Lambda(\Box + (n\partial)^2) + {}^lA(an)(nj)}{(an)^2 - M(\Box + (n\partial)^2)} + (\partial j){}^lA\right\}}.$$

Finally we integrate over ${}^{l}A$

$$\tau[;{}^{t}J_{\mu}, \Lambda, M] = N^{-1} \int \delta\, {}^{t}A_{\mu};\ e^{i/2\,{}^{t}A_{\mu}\Box{}^{t}A_{\mu}} e^{\frac{i}{2}\frac{(nj)(nj)}{\Box+(n\partial)^2} + ij_{\mu}{}^{t}A_{\mu} + i\,{}^{t}J_{\mu}{}^{t}A_{\mu}} \times$$

$$\times e^{-\frac{i}{2}\frac{1}{(an)^2 - M(\Box+(n\partial)^2)}\left\{\Lambda^2(\Box+(n\partial)^2) + 2\Lambda(an)(nj) + \frac{(an)^2(nj)^2}{\Box+(n\partial)^2} - \frac{(an)^2(nj)^2}{\Box+(n\partial)^2} - 2\Lambda(an)(nj) - \Lambda^2(\Box+(n\partial)^2)\right\}} \times$$

$$\times e^{i\left\{\frac{(\partial j)(\Lambda+(an)(nj))}{\Box+(n\partial)^2} + \frac{1}{2}\frac{(\partial j)^2[(an)^2 - M(\Box+(n\partial)^2)]}{\Box+(n\partial)^2}\right\}}. \tag{II.3}$$

We see that, with the exception of the Coulomb interaction term, there remain only terms multiplied by (∂j); the other terms cancel each other out. Thus, using the same argument as in the case of Feynman gauge we conclude that the formulation of QED in all general gauges are equivalent. The comparison of (II.3) with (4.8) show that the Feynman gauge is defined by $\Lambda = 0$, $a_{\mu} = \frac{\partial_{\mu}}{\Box}$ and $M = \frac{1}{\Box^2}$.

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