QUANTUM THEORY OF SPIN-WAVE RESONANCE IN THIN FERROMAGNETIC FILMS. PART II. SPIN-WAVE RESONANCE SPECTRUM

By H. Puszkarski

Section of Theoretical Physics, Physics Institute, A. Mickiewicz University, Poznań*

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Applying the first approximation of perturbation calculus, spin-wave resonance (SWR) spectra are calculated for various values of the surface parameter and compared with experiment. A condition for the critical phenomenon in SWR is derived. Conditions for the occurrence of the uniform mode peak or a surface mode peak in the SWR spectrum are discussed in detail, and the conclusions are confronted with experiment. A method of identifying the surface mode peak and of resorting to this peak for measuring the surface anisotropy energy is proposed.

I. THEORY

1. Perturbation Hamiltonian

In Part I of this paper [1], we calculated the functions and energies of spin-waves for a thin ferromagnetic film satisfying the assumptions of the surface inhomogeneity (SI) model. We shall now use the results of Part I for calculating the spectra of spin-wave resonance (SWR).

The perturbation Hamiltonian (Eq. (I.2.2)[1]) in diagonal representation is of the form:

$$\hat{W} = \hat{W}_0 e^{i\omega t},$$

$$\hat{W}_0 = -ig\mu_B h_0 \sqrt{\frac{S}{2}} \sum_{ij} \sum_{\vec{\varkappa},\tau} [u_{ij}(\vec{\varkappa}, \tau)\hat{\xi}^{+}_{\vec{\varkappa}\tau} - u^{*}_{ij}(\vec{\varkappa}, \tau)\hat{\xi}_{\vec{\varkappa}\tau}]. \tag{1.1}$$

The probability (per unit time) of excitation of a spin-wave (*i.e.* the probability of the transition, under the influence of the perturbation $\hat{W}$, from the state $|0\rangle$ to $|\vec{\varkappa}\tau\rangle$) calculated in the first approximation of perturbation calculus is:

$$P_{|0\rangle \to |\vec{\varkappa}\tau\rangle} = \frac{2\pi}{\hbar} |\langle \vec{\varkappa}\tau | \hat{W}_0 | 0 \rangle|^2 \delta(E(\vec{\varkappa}, \tau) - \hbar\omega) \tag{1.2}$$

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* Address: Instytut Fizyki, Uniwersytet A. Mickiewicza, Pozań, Grunwaldzka 6, Polska.

[1] meaning Eq. (2.2) of Part I.

and, with (1.1):

$$P_{|0\rangle\to|\vec{\varkappa}\tau\rangle} = \frac{\pi}{\hbar} S(g\mu_B h_0)^2 \left| \sum_{lj} u_{lj}(\vec{\varkappa}, \tau) \right|^2 \delta(E(\vec{\varkappa}, \tau) - \hbar\omega). \tag{1.3}$$

Thus, spin-wave excitation by an electromagnetic field takes place only if the following resonance condition is fulfilled:

$$E(\vec{\varkappa}, \tau) = \hbar\omega, \tag{1.4}$$

the power absorbed at resonance by the system (*i.e.* the resonance line intensity) being proportional to $|\sum_{lj} u_{lj}(\vec{\varkappa}, \tau)|^2$. With (I.4.3), we furthermore obtain:

$$P_{|0\rangle\to|\vec{\varkappa}\tau\rangle} \sim \delta(\vec{\varkappa}) |\sum_{l} e^{-i\varphi l} u_l(\tau')|^2, \tag{1.5}$$

showing that only standing spin-waves (modes) can be excited (*i.e.* waves for which $\vec{\varkappa} = 0$, involving $\varphi = 0$). This condition results directly by the assumption of periodicity conditions in the plane of the film. This idealisation, however, does not go too far since periodicity conditions, as we have shown in a separate paper [2], correspond to natural surface defect. Thus, finally, the mode peak intensities are found to be proportional to the expressions

$$P(\tau') = |\sum_{l} u_l(\tau')|^2. \tag{1.6}$$

2. *Intensities and positions of resonance peaks*

$P(\tau')$ is obtained explicitly by addition of Eqs (I.3.14a) and subsequent elimination of the fictitious planes by means of the boundary equations (I.3.14b); putting $\vec{\varkappa} = 0$, we have

$$P(\tau') = \frac{1}{16} (A-1)^2 [u_0(\tau') + u_{L-1}(\tau')]^2 \sin^{-4}\left(\frac{1}{2}\tau'\right) \tag{1.7}$$

or, on resorting to Eqs (I.3.22a) and (I.3.24a),

$$P(\tau') = 2\left(L + \frac{\sin L\tau'}{\sin \tau'}\right)^{-1} \left(\frac{\sin \frac{1}{2} L\tau'}{\sin \frac{1}{2}\tau'}\right)^2. \tag{1.8}$$

These expressions permit the following conclusions:

1) The intensities of resonance lines corresponding to the various spin-wave modes depend solely on the surface parameter A and the amplitudes, at the surface, of these modes.

2) Antisymmetric modes ($u_0(\tau') = -u_{L-1}(\tau')$) cannot be excited in SWR, as involving $P(\tau') = 0$. *Only symmetric modes are excited*; with regard to the latter, the following conclusions hold:

3) For natural defect ($A = 1$), the intensities corresponding to $\tau' \neq 0$ vanish, and only the line intensity $\tau' = 0$ is non-zero. In other words, *this case presents but one resonance, corresponding to the uniform spin-wave mode.*

4) "Multi-peak" SWR occurs only if $A \neq 1$, the resonance line intensities growing

as $A-1$ increases; in other words, these *intensities are the larger the more surface spin pinning differs from natural pinning.*

The shape of the function $P(\tau)'$ defined by Eq. (1.8) depends on the "type" of the quantum numbers τ' (see Eqs (I.3.20)), as shown in Fig. 1. For space modes, this is a "modulated k^{-2} dependence", whereas for surface modes $P(t)$ is monotonous, its values *decreasing* approximately as t^{-1} for acoustic modes but growing for optical modes. Obviously, in order to obtain the "discrete" SWR spectrum, one has to "cut out" of the curves of Fig. 1 the ordinates in the points corresponding to allowed values of τ'_n.

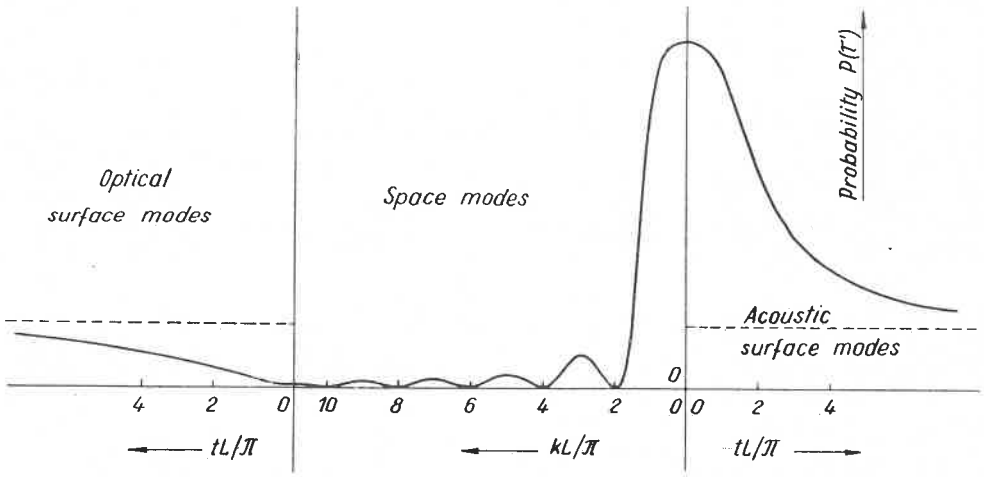

Fig. 1. Excitation probability for a spin-wave mode as a function of the quantum number τ', for space modes ($\tau' = k$, in the middle), surface acoustic modes ($\tau' = ik$, to the right), and surface optical modes ($\tau' = \pi + it$, to the left), at film thickness $L = 11$ (in lattice units)

The positions of resonance lines have to be determined from the resonance condition (1.4). The energy of spin-wave modes is defined, in accordance with Eq. (I.3.17), by the formula:

$$E(\tau') \equiv E(0, \tau') = 4Sz_1J_1(1-\cos\tau') + g\mu_B(\vec{\gamma} \cdot \boldsymbol{H}^{eff}), \tag{1.9}$$

where z_1 is the number of nearest neighbours of the site under consideration lying in the next layer, and J_1 the exchange intergral between two nearest neighbours lying each in a different layer. Thus, the resonance condition takes the form:

$$\frac{\omega}{\gamma} = \frac{4Sz_1J_1}{g\mu_B}(1-\cos\tau') + H\cos(\Phi-\varphi) - 4\pi M\cos^2\varphi \tag{1.10}$$

(here, γ is the spectroscopic splitting ratio). In the case of very thick films, τ' can be considered to be small; in this approximation, one obtains (*cf.* Eq. (I.3.24)) a space mode peak spacing which obeys an $(n-\delta)^2$ law, where

$$\delta = \delta(n, A, L). \tag{1.11}$$

Thus, the theory based on the SI model predicts deviations from the n^2 law. We shall consider the relation (1.11) in detail in subsection 4.

3. *Dependence of the SWR spectrum on the surface parameter*

Since the discrete spectrum of allowed τ'_n values depends on the surface parameter A, the SWR spectrum calculated with Eqs (1.8) and (1.9) will also depend on A. Fig. 2 shows SWR spectra calculated for the following values of the surface parameter (at $L = 11$):

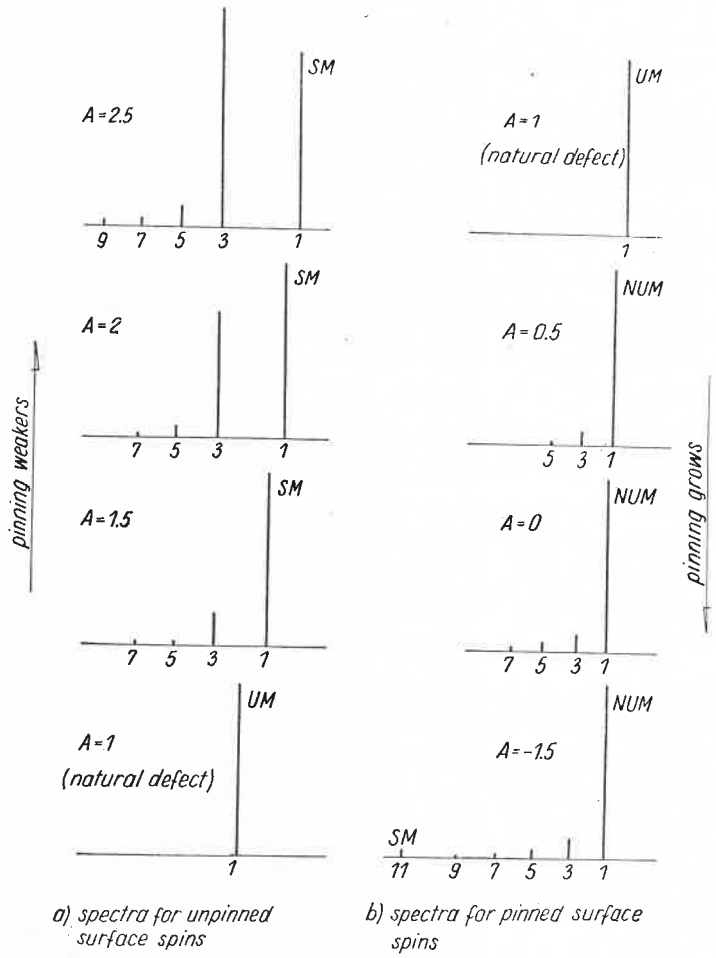

Fig. 2. SWR spectra calculated for various values of the surface parameter A *i.e.* for various pinning of the surface spins. The spectra exhibit only peaks corresponding to symmetric modes (of odd number $n = 1, 3, \ldots$). The calculations are for the case of $L = 11$ (eleven layers in the film). *UM* — uniform mode, *SM* — surface mode, *NUM* — nonuniform mode. For the case $A = -1.5$, the peak with number $n = 11$ corresponds to an "optical" surface mode ($\tau' = \pi + it$); in all other cases *SM* denotes an "acoustical" surface mode ($\tau' = it$). Units on the horizontal axis are proportional to the wave number k (for space peaks) or to t (for surface peaks)

$A = 2.5$, A = 2, $A = 1.5$, $A = 1, A = 0.5, A = 0$, and $A = -1.5$. Analysis of these spectra resorting to Figs 2 and 9 (Part I) and Conclusions 1–4 of the preceding subsection leads to the following, additional conclusions:

5) For values $|A| \leqslant 1$, the SWR spectrum consists only of the space peaks, whereas for $|A| > 1$ it moreover contains one *surface peak* (which then is the first or last peak of the spectrum).

6) The space peak intensities decrease less and less steeply as A moves away from unity. In the limiting case $|A| = \infty$ we have $k_n = n\pi/(L-1)$, and Eq. (1.8) yields in the large L approximation $P(k_n) \sim n^{-2}$, in agreement with the result of Kittel [3]. Thus, *the space peak intensities decrease according to an* n^{-r} *law, with* $r \geqslant 2$ *depending on the pinning of the surface spins.*

7) The intensity of the accoustic surface peak decreases with increasing A. At sufficiently large values of A, it can become equal to or even smaller than the intensity of the first space peak (see, *e.g.* case $A = 2.5$).

8) *No peak corresponding to the uniform mode* ($k = 0$) *appears in multi-peak SWR.* This peak appears only in one-peak resonance (in the case of natural defect ($A = 1$)).

9) According to the surface spin pinning, the first peak of the spectrum is of the nature of a surface mode (at pinning weaker than natural), or of a nonuniform space mode (at pinning stronger than natural).

In Part I, we derived the following expression for the surface parameter A (see, Eq. (I.3.15)):

$$A = 1 - \frac{g\mu_B}{2Sz_1J_1}(\vec{\gamma} \cdot \boldsymbol{K}_{surf}) = 1 - \frac{g\mu_B}{2Sz_1J_1}|\boldsymbol{K}_{surf}| \cos(\alpha_0 + \varphi), \tag{1.12}$$

whence A is seen to depend on certain physical factors by way of the surface anisotropy field $\boldsymbol{K}_{surf}$ as well as on the configuration and value of the static field $\boldsymbol{H}$ by way of the versor $\vec{\gamma}$. This provides the possibility of comparing the preceding results with experiment (see, Section II).

4. Dependence of surface pinning on the number of the resonance mode

We recall the substitution applicable to space modes (I.3.26):

$$k_n = (n - \delta)\frac{\pi}{L-1}. \tag{1.13}$$

δ defines the deviations of resonance peaks from the $H_n \sim n^2$ law, and consequently is accessible to measurement. In theory, the boundary conditions quantizing the wave number k require that δ shall fulfil an appropriate equation (see, Eqs I. 3.26a). In compliance with this equation, δ has to depend on the number n of the mode, the surface parameter A and the film thickness L, thus $\delta = \delta(n, A, L)$. We now proceed to a discussion of this dependence.

Fig. 3*a*, *b* shows curves of $\delta(n)$ for various values of the parameters A and L. One notes that, for sufficiently thick films, δ is linear in n and is a growing function if surface spin pinning

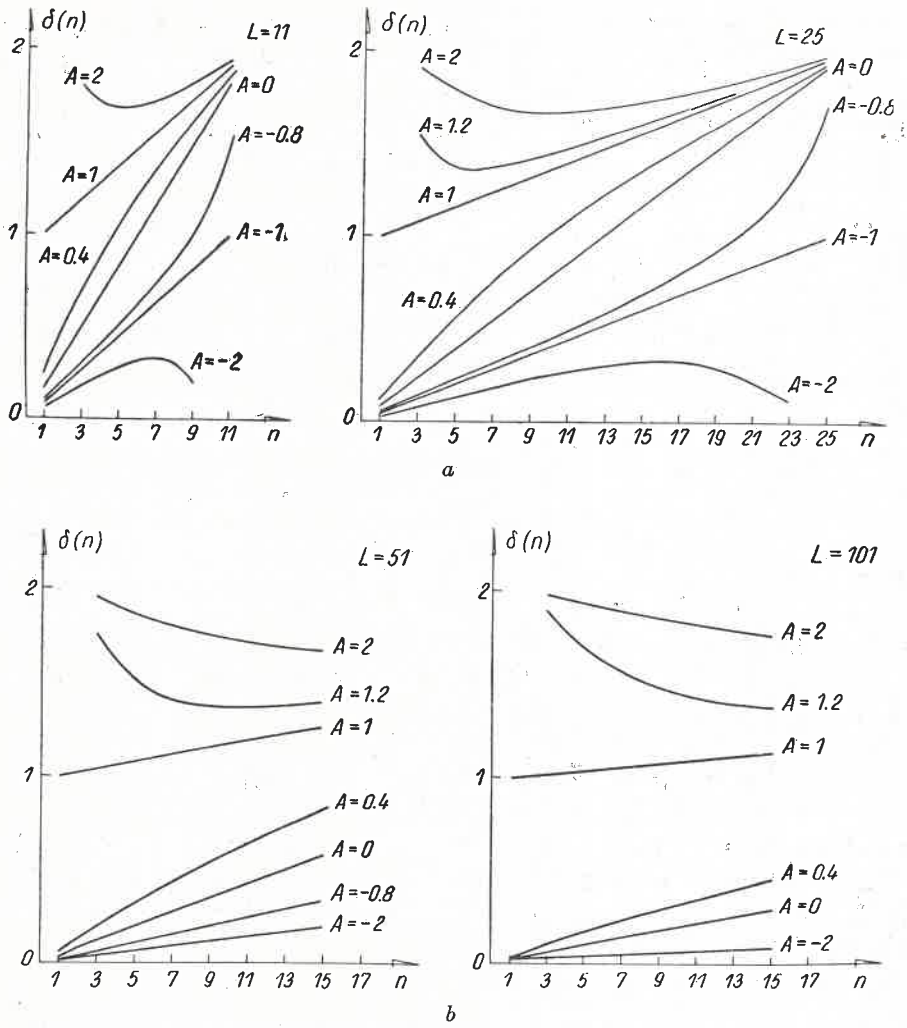

Fig. 3. δ (as defined by Eq. (1.13)) *vs.* the number of the space mode, at various values of the surface parameter A and film thickness L (from Eqs I.3.26a)

is stronger than natural and a decreasing function if pinning is weaker. Hence, obviously, by measuring δ *versus* n one can gain some information on the surface pinning. Fig. 4 shows $\delta(A)$ for various values of the parameters n and L. It is in all cases a growing function; for modes of lowest numbers n, the steepness is particularly large close to natural pinning ($A \gtrsim 1$).

When measuring δ from the SWR spectrum, care has to be given to yet another essential detail. To each resonance mode with the number n, there corresponds a distinct resonance field H_n; this circumstance, with regard to Eq. (I.3.2), means that to each mode there corresponds a distinct direction of the magnetisation vector $\vec{\gamma}_n = \vec{\gamma}(H_n)$ and, consequently, a distinct value of the surface parameter $A_n = A(\vec{\gamma}_n)$ (see, Eq. 1.12). As a consequence,

the pinning of surface spins depends on the number of the resonance mode under consideration. The shape of $A = A(n)$ is defined essentially by the direction of the surface anisotropy field $\boldsymbol{K}_{surf}$; one notes immediately that this dependence can be increasing or decreasing. This circumstance, when properly taken into account, leads to a relationship $\delta = \delta(n, A_n, L)$ in place of that of Eq. (1.11). Thus, the shape of $\delta = \delta(n)$ obtained from SWR spectrum measurements can differ from the curves of Fig. 3. Nevertheless, a confrontation of these theoretical curves and the experimental results can yield qualitative information regarding the type of surface pinning in a given sample.

Fig. 5 represents the results of measurements of the quantity δ by Searle *et al.* [4]. The general tendency of δ to increase with n points to the presence of pinning stronger than natural in their samples. The lack of surface peak in the SWR spectrum additionally

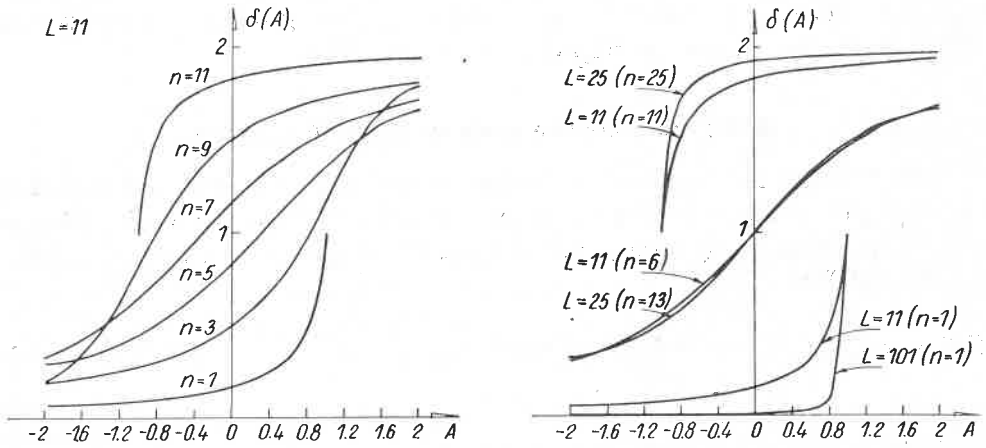

Fig. 4. δ *vs.* the surface parameter A for space modes of given n, at various values of the film thickness L (from Eqs I.3.26a)

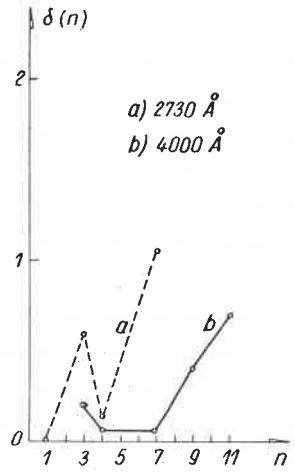

Fig. 5. δ *vs.* the mode number from SWR spectrum measurements by Searle *et al.* (from Ref. [4], Table II)

confirms this conclusion (*cf.* 145). The relatively large variations in δ apparent in Fig. 5 for the first several peaks (*i.e.* the variations in surface pinning) show (*cf.* Fig. 4) this pinning to be but slightly in excess of natural.

The preceding mechanism of the dependence of surface pinning on the number of the resonance mode permits the statement that the change in δ for any two consecutive modes is the larger the thinner is the film, since $H_n - H_{n+1} \approx \left(\frac{\pi}{L-1}\right)^2$. This is corroborated by the already cited measurements (Searle *et al.* [4]). Thus, for very thick films, the influence of the resonance field on pinning is very small, and with respect to such films one is justified in applying *the approximation of constant surface pinning for all modes.* In the present paper (with the exception of this subsection), all considerations are within the framework of such an approximation, the surface parameter occurring everywhere having attributed to it a value $A \equiv A_0$, with A_0 defining a surface spin pinning which corresponds to the uniform mode *i.e.* to the value H_{unif} of the resonance field.

5. *Surface spin-wave mode in the SWR spectrum*

In the present subsection, we shall propose a method of identifying the surface peak and of measuring the surface anisotropy energy by resorting to this peak. Our considerations will deal with the symmetric (acoustic) surface mode. Its energy can be expressed as follows (see, Eq. (I.4.9)):

$$E(A) = g\mu_B[H\cos(\Phi-\varphi) - 4\pi M\cos^2\varphi] + 2Sz_1J_1\left[\left(2-A-\frac{1}{A}\right) - A^{-(L-2)}(1-A^{-2})^2\right]. \tag{1.14}$$

Eq. (1.14) shows that the energy of the surface mode is practically insensitive to changes in film thickness L, provided L is sufficiently large; on the other hand, for thinner films, the energy decreases with diminishing L. For the sake of comparison, it may be worth stating that the energy of space modes increases with diminishing L (*cf.* Part I, subsection IV.3), with the exception of the uniform mode, the energy of which does not depend on the film thickness. With regard to the fact that the uniform mode does not occur in multi-peak resonance, the following identification criterion can be enounced: *a resonance peak which does not change its position in the multi-peak SWR spectrum or shifts towards stronger fields with decreasing film thickness corresponds to the surface mode*[2]. In experiments, absorption peaks behaving in this way have indeed been observed, but have erroneously been interpreted as the uniform mode peak (see further on, Section II).

The presence (or absence) of the surface peak in the SWR spectrum informs us whether the surface spins are unpinned (or pinned); moreover, measurements of the shift ΔH in surface peak when the film thickness is varied by the amount ΔL permit calculations of the surface anisotropy energy. We shall restrict our considerations to the cases of perpendicular

[2] The critical film thickness below which the shift in surface peak becomes perceptible depends on the surface parameter, and can be evaluated at between 10^2 and 10^3 Å.

configuration ($\varphi \approx \Phi = 0$) and parallel configuration $\left(\varphi \approx \Phi = \frac{\pi}{2}\right)$. By Eq. (1.14) and the condition (1.4), we derive the following approximate expression for ΔH at these configurations:

$$\Delta H \equiv H_{L-\Delta L} - H_L = \frac{2Sz_1J_1}{g\mu_B}(1-A^{-2})^2 A^{2-L}(A^{\Delta L}-1) \tag{1.15}$$

(L and ΔL in lattice units). On calculating A by means of Eq. (1.15) we can calculate the surface anisotropy energy E_s by resorting to Eq. (I.5.2). Fig. 6 shows $\Delta L = \Delta L(A)$ from Eq. (1.15), with ΔH as parameter.

In thin films, the spontaneous magnetisation is directed either parallel or perpendicularly to the film surface (Corciovei and Vamanu [5]). Consequently, the method proposed here

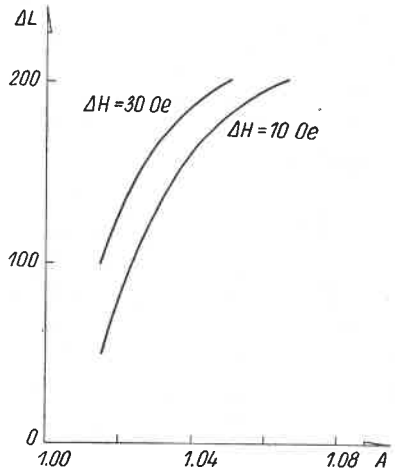

Fig. 6. The amount ΔL (in lattice units) by which the film thickness L has to be reduced in order to shift the resonance surface mode peak towards stronger fields by the amount ΔH, as a function of the surface parameter A. From Eq. (1.15), for $L = 300$, $\Delta H = 10$ Oe and 30 Oe

permits to determine the surface anisotropy energy at spontaneous equilibrium of the system.

The present method of measuring the surface parameter A involves the use of samples of various thickness prepared in identical physical conditions. Wedge-shaped thin films, obtained by the technology proposed recently by Okochi and Nosé [6], are particularly well adapted to this aim.

II. COMPARISON WITH EXPERIMENT

1. Square law

In a well-defined SWR spectrum, all peaks correspond in general to space spin-wave modes; the present theory predicts that their spacing shall fulfil an $(n-\delta)^2$ law, where $\delta = \delta(n)$. Searle *et al.* [4], by fitting δ to the experimental data, concluded that $\delta = \delta(n)$ grows with the mode number n thus pointing to a surface spin pinning in excess of natural (see subsection I.4) in their samples.

2. *Critical phenomenon*

Many authors (Wigen *et al.* [7, 8], Rossing [9], Suran [10], Nisenoff and Terhune [11]) state that, for certain samples, there exists a critical angle Φ_{crit} (between the static field and the normal to the film) at which only one resonance peak appears; at angles $\Phi < \Phi_{crit}$ a well-defined SWR spectrum is obtained, whereas at angles $\Phi > \Phi_{crit}$ the spectrum consists in general of only two absorption peaks.

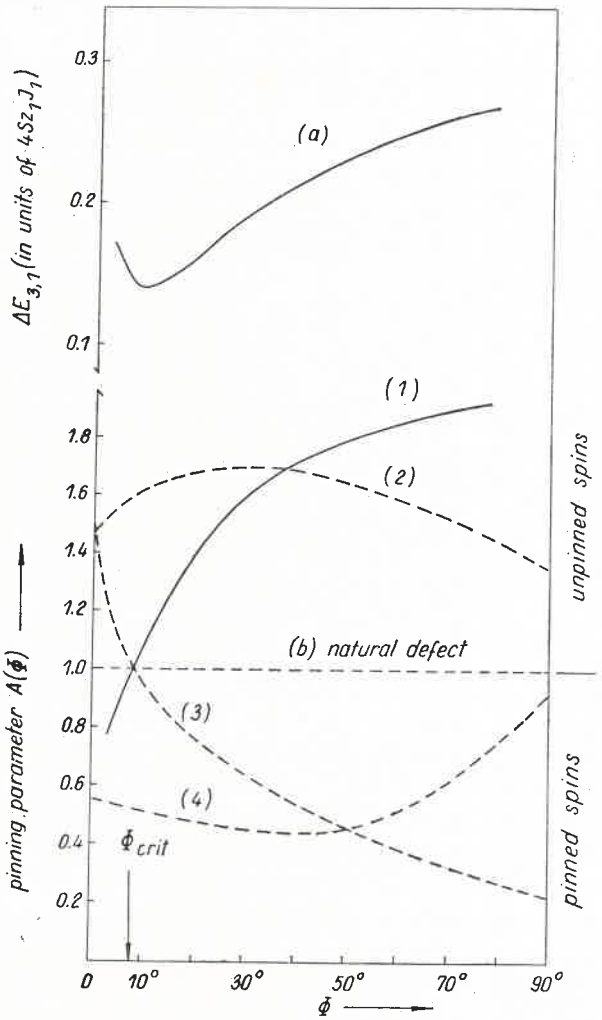

Fig. 7. The parameter A of surface spin pinning varies with Φ, the angle defining the configuration of the static field with respect to the normal to the film surface; the corresponding changes in SWR spectrum can be followed in Fig. 2. The curves (*1*), (*2*), (*3*) and (*4*) correspond to four orientations of the surface anisotropy field $\boldsymbol{K}_{surf}$ as shown in Fig. 8 (curve (*1*) was *calculated* from Eq. (1.12) assuming $\alpha_0 = 70°$, $g\mu_B|\boldsymbol{K}_{surf}| = 2Sz_1J_1$, $\omega/4\pi M\gamma = 0.6$; curves (*2*), (*3*) and (*4*) are intended primarily to *illustrate* the analytical behaviour of $A(\Phi)$). The straight line (*b*) represents the case of natural surface defect ($\boldsymbol{K}_{surf} = 0$). Curve (*a*) shows the separation of peaks $n = 1$ and $n = 3$ in its dependence on Φ for the case (*1*). Noteworthy is the occurrence in cases (*1*) and (*3*) of the critical configuration Φ_{crit}, for which $A = 1$

This critical effect is accessible to an interpretation within the framework of our theory by resorting to the dependence of the surface parameter A on the magnetisation direction $\vec{\gamma}$ (Eq. (1.12)).

In Fig. 7, curve (1) shows the shape of $A = A(\Phi)$ calculated for $\alpha_0 = 70°$ (corresponding to orientation (1) of the field $\boldsymbol{K}_{surf}$ in Fig. 8) from Eq. (1.12) and the equilibrium condition (I.3.2) (for $H = H_{unif}$). The changes in SWR spectrum accompanying changes in configura-

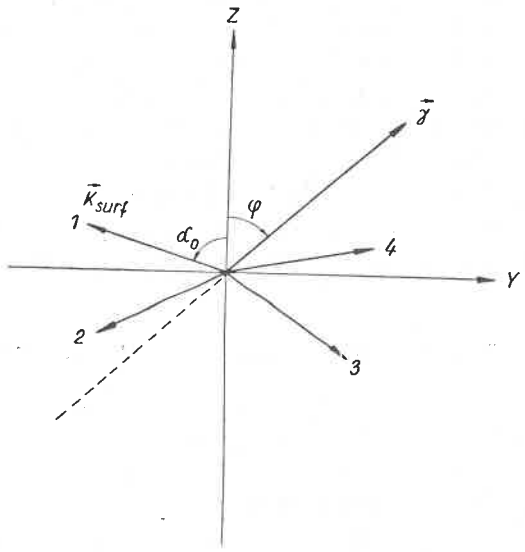

Fig. 8. Graph illustrating typical orientations of the surface anisotropy field $\boldsymbol{K}_{surf}$ with respect to the direction of magnetisation $\vec{\gamma}$. The z-axis is normal to the film surface

tion of the static field can easily be followed from Fig. 2. One sees that at a well-defined angle $\Phi = \Phi_{crit}$ the parameter A is equal to 1; this means that at the configuration in question only one resonance line appears. We moreover note that at critical configuration, by Eq. (1.12), the magnetisation of the sample is perpendicular to the surface anisotropy field:

$$\boldsymbol{M} \perp \boldsymbol{K}_{surf}. \tag{2.1}$$

The condition (2.1) can be referred to as *the condition for the critical effect*. At angles $\Phi < \Phi_{crit}$, the surface parameter A is less than unity (pinned spins) and the spectrum is well-defined, whereas at angles $\Phi > \Phi_{crit}$ the parameter A is larger than unity (unpinned spins) and the spectrum reveals the surface peak and one significant space peak, of comparable intensities (see Fig. 2, cases $A = 0$ and $A = 2$). The calculated Φ-dependence of the separation between the first two resonance peaks (curve (a) of Fig. 7) has a minimum at the angle $\Phi = \Phi_{crit}$; this result, too, is in agreement with experiment [7, 8].

The experiments where no critical phenomenon was observed are also accessible to an interpretation by the present theory.

Curves (2), (3) and (4) of Fig. 7 show the trend of $A = A(\Phi)$ for some typical cases represented in Fig. 8. In the case of curve (4), there is no critical effect, and all the peaks of

the spectrum correspond to space modes. This case requires very strong pinning of the surface spins; it is that of Okochi and Nosé 's experiment [6]. These authors, when investigating SWR spectra in three 76% Ni permalloy films of thicknesses amounting to about 1400, 1650 and 1900 Å respectively, found neither a critical effect nor a peak with position independent of the thickness or dependent thereon in a manner characteristic of the surface peak. Strong surface spin pinning in those films may have been due to the high Fe content of the alloy causing the surface stratum to be enriched in "pinning enhancing" oxide Fe_2O_3.

Neither does the case of curve (2) involve a critical effect; here, at all configurations Φ, the spectrum contains the surface peak. In the case of curve (3), the critical effect is present; the surface peak occurs only at configurations $\Phi < \Phi_{crit}$. Cases like (2) and (3) should be observable in permalloy films coated on either surface with a Fe or Ni stratum, since these metals act in a manner to "unpin" the surface spins.

From the theory, in the case of the straight line (b) only one resonance peak corresponding to the uniform mode appears at all configurations. This case is that of natural surface defect *i.e.* of the absence of a surface anisotropy field ($\boldsymbol{K}_{surf} = 0$).

3. Influence of film thickness. Uniform mode and surface mode

Experiments reveal the existence of absorption peaks which, as the film thickness decreases, shift position towards weaker fields or towards stronger fields or fail to shift altogether. From the theory, the first correspond to space modes and the second to surface modes, wheras a peak with thickness-independent position occurring at the critical configuration is the uniform peak or, occurring at a non-critical configuration (in very thick films) is the surface mode peak. The occurrence of the surrface peak in SWR has been confirmed experimentally by various authors, thus by Frait and Mitchell [12] and Searle *et al.* [4] at parallel configuration and by Holzer *et al.* [13] at configurations $\Phi_{crit} < \Phi \leqslant \frac{\pi}{2}$ who interpreted it incorrectly as the uniform mode peak. Their experiments can rather be thought to correspond to the case of curve (1) (Fig. 7). The nature of the theoretically uniform peak occurring at critical configuration has hitherto not been put to test by varying the film thickness.

4. Surface anisotropy

The SWR spectrum depends on the kind of strata present on the surfaces (Kooi *et al.* [14], Searle *et al.* [4], Stankoff [15], Waksmann [16]). The fact of such a dependence can be explained by the assumption that these strata give rise to a magnetic field $\boldsymbol{K}_{surf}$ of surface anisotropy. Thus *e.g.* it has been shown experimentally that oxidation of the surface of a permalloy film causes an increase in intensity of the peaks $n = 3, 5, \ldots$ with regard to that of the peak $n = 1$. The present theory leads to the same result, on the assumption that oxidation strengthens the pinning of surface spins, thus lowering the surface parameter A (*cf.* Fig. 2) *i.e.* modifying the field $\boldsymbol{K}_{surf}$.

The sign of the surface anisotropy energy E_s can be decided by investigating whether the SWR spectrum contains the surface peak; if it does, then $A > 1$ and by Eq. (I.5.2) E_s

is positive (the surface spins are unpinned), whereas if it does not then $A<1$ and E_s is negative (the surface spins are pinned). If $E_s > 0$, the value of E_s can be assessed from the shift in surface peak due to a change in film thickness. With the data of Searle *et al.* [4] (parallel configuration, 80% Ni permalloy), we evaluate $A = 1.05$ from Eq. (1.15). This yields values of $E_s = 0.5$ erg.cm^{-2} and $|\boldsymbol{K}_{surf}| \approx 10^4$ Oe.

Information as to the angle α_0 can be derived from measurements of the critical angle in conjunction with the condition (2.1). When the critical effect is present, the field $\boldsymbol{K}_{surf}$ is directed as in cases (*1*) and (*3*) (see Figs. 8 and 9); if it is absent, $\boldsymbol{K}_{surf}$ has the direction of cases (*2*) and (*4*). The critical effect moreover permits to decide whether $\boldsymbol{K}_{surf}$ changes its direction as a result of a chemical or mechanical treatment of the film surface; if it does, then by condition (2.1) this should entail a change in critical angle.

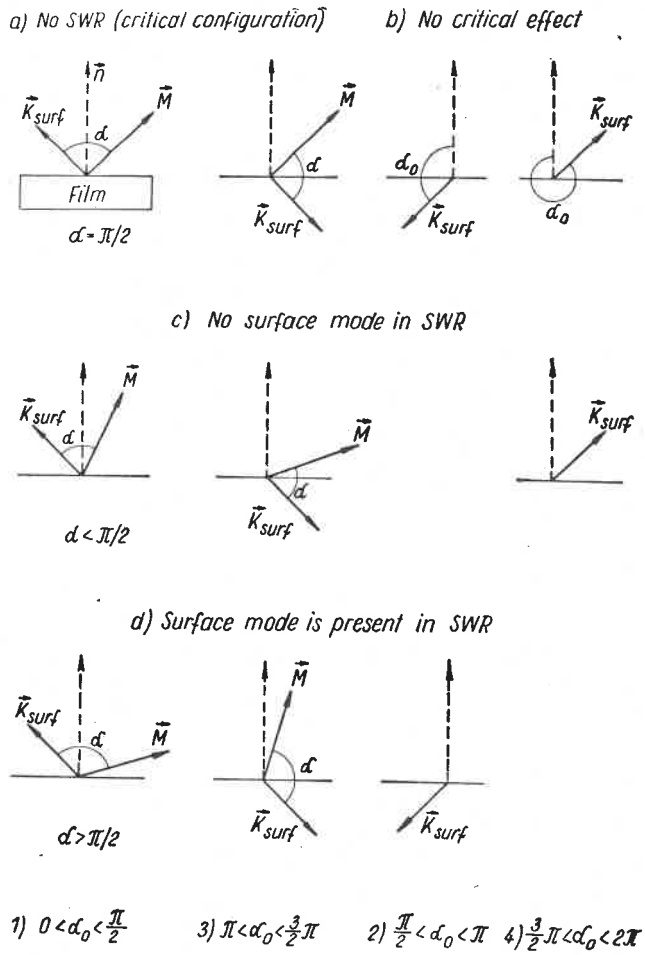

Fig. 9. Set of graphs illustrating the conditions for the vanishing of SWR absorption as well as for the occurrence of the surface peak in the SWR spectrum. $\boldsymbol{M}$ — magnetisation of the sample $\boldsymbol{K}_{surf}$ — surface anisotropy field. The graphs should be viewed column-wise, each column corresponding to an orientation of $\boldsymbol{K}_{surf}$ given by an angle α_0 from one of the four quadrants (*cf.* Fig. 8)

Fig. 9 provides an illustration of our interpretation of the critical effect and renders apparent the conditions at which the surface peak occurs in the SWR spectrum. In this interpretation, the essential role belongs to the angle α between the magnetisation direction $\boldsymbol{M}$ and the surface anisotropy field $\boldsymbol{K}_{anis}$ as well as to the angle α_0 defining the orientation of $\boldsymbol{K}_{anis}$ with respect to the normal. All considerations of subsections 2–4 can be easily read from Fig. 9.

5. *Influence of other factors*

The theory proposed here explains the experimentally found dependence of the SWR spectrum on the AC field frequency (Wolf [17], Nisenhoff and Terhune [18]) as follows: a rise in frequency involves an increase in resonance field strength; this, by the equilibrium condition (I.3.2), leads to a change in $\vec{\gamma}$-direction *i.e.* to a change in surface parameter A.

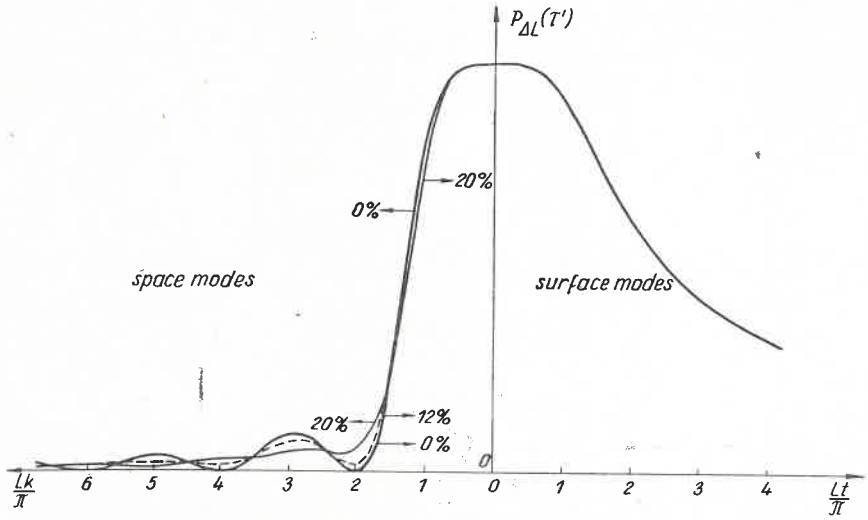

Fig. 10. Influence of surface roughness (per cents give the ratio $\Delta L/L$) on the shape of the probability curve $P(\tau')$. Significantly, the probability curve for the surface mode is insensitive to variations in film thickness

From the theory, the SWR spectrum has to depend on the structure of the body and on the orientation of its surface. By Eq. (1.12), the smaller is Sz_1J_1 the better defined should be the spectrum. However, no experiments on the subject are as yet available.

Searle *et al.* [4] studied (in 80% Ni permalloy films) the influence of surface roughness on SWR. They found that at perpendicular configuration high roughness led to a vanishing of resonance but at parallel configuration (with one peak occurring) to no more than a slight broadening of the line. This has its theoretical justification. Inasmuch as concerns space modes, roughness entails: (1) the occurrence of certain allowed (non-contiguous) "intervals" Δk_n in place of the discrete values k_n (see, Part I, Fig. 2) and (2) a "smoothening" of the probability curve $P(k)$ (Fig. 10). The result is a broadening of the lines, so that if the roughness ΔL becomes very considerable SWR can vanish completely. With regard to surface peaks, roughness ΔL affects their position and intensity but slightly (Fig. 10). On the assumption that, in the experiments cited, the surface peak appeared at parallel configuration and space

peaks at perpendicular configuration, these considerations provide an explanation of the results of Searle *et al.* [4] concerning surface roughness.

Inhomogeneity of surface anisotropy in the plane of the film (thus, inhomogeneity of surface oxidation) affects SWR similarly entailing some "variation" δA of the surface parameter; as seen from Fig. 2 of Part I, this leads to allowed "intervals" $\Delta\tau_n'$ and thus to a broadening of the resonance lines. It is worth noting that, in this case, the space peaks as well as the surface peak are affected.

III. CONCLUSIONS

Spin-wave resonance absorption of the energy of an electromagnetic field by a thin ferromagnetic film can take place only if the film exhibits inhomogeneities. In particular, the inhomogeneity represented by an effective surface anisotropy field $\boldsymbol{K}_{surf}$ is sufficient for resonance absorption. From our evaluations, it results that $|\boldsymbol{K}_{surf}| \approx 10^4$ Oe. Were one to assume, in accordance with the ideas underlying the "dynamic pinning" model (Wigen *et al.* [7, 8]), that the source of surface anisotropy resides exclusively in the fall in magnetisation ΔM on the surface, then one would have to write $\boldsymbol{K}_{surf} = \boldsymbol{n}_z 4\pi\Delta M \cos\varphi$, whence $\Delta M \approx 10^3$ Oe would be of the order M, which seems incredible. A number of experiments can be interpreted on the assumption that $\boldsymbol{K}_{surf}$ is due chiefly to the presence of strata of various chemical compounds on the film surfaces. In particular, we have shown that oxide strata "unpin" the surface spins (in a static field perpendicular to the surface).

In the specific conditions referred to as the critical configuration of the static field, the SWR spectrum vanishes, showing that the inhomogeneity on the film surface has undergone compensation (compensation condition: $\boldsymbol{M} \perp \boldsymbol{K}_{surf}$). In this configuration, the sample in "homogeneous" with respect to SWR. Consequently, the critical configuration appears to be the configuration best adapted to investigations concerning the influence of various artificially produced inhomogeneities on the SWR spectrum of films. However, the theory predicts that there can exist samples where the critical effect cannot take place.

The peak corresponding to the surface spin-wave mode appears in the SWR spectrum when conditions are such that the surface spins are unpinned. In the literature, this peak has hitherto been interpreted as being the uniform spin-wave mode peak. According to the kind of film and chemical compound forming a stratum on its surface, the surface peak can appear in SWR at various configurations of the static field. The uniform mode peak appears only at critical configuration of the static field, as the sole absorption peak.

In concluding, we note that the quantum theory of SWR resorting to the SI model explains satisfactorily the majority of the experimental facts. In this paper, we have not considered effects resulting from asymmetry of the boundary conditions, but hope soon to extend the theory to this category of facts too.

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