# ZERO-POINT OSCILLATIONS AND THE SHAPE OF NUCLEI IN TRANSITION REGIONS

BY E. YE. BERLOVITCH

A. F. Ioffe Physical-Technical Institute of the USSR, Academy of Sciences, Leningrad*,

*(Received April 1, 1970)*

The measured curves of the relationship between potential energy of even-even nuclei in transition regions as the neutron number varies shows the existence of a distinct correlation with the change in mode of quasi-rotational bands and their conversion to rotational bands in strongly deformed nuclei. The existence of zero-point oscillations at an amplitude comparable to or larger than the deformation parameter and the asymmetry of potential energy curves result in a difference between the "centre of oscillations" and the deformation conforming to the minimum curve. The position of this "centre of oscillations" detemines the true value of the nuclear deformation parameter.

In papers [1,2] the author developed the concepts according to which, among odd-mass nuclei located within transition regions between near-magic spherical and strongly deformed nuclei, both ground state spherical and low equilibrium deformation nuclei with the deformation parameter close to $\beta_0 \approx 0.1$ can exist. In particular, odd isotopes of europium ($^{147}$Eu, $^{149}$Eu, $^{151}$Eu), promethium ($^{145}$Pr, $^{147}$Pr, $^{149}$Pr) gold ($^{193}$Au, $^{195}$Au, $^{197}$Au) and iridium ($^{191}$Ir, $^{193}$Ir) fall in the last-mentioned group. At the same time, odd-neutron isotopes of neodymium, samarium and gadolinium take on an equilibrium shape close to the spherical one. In this case, the same nucleus in different states can accept both spherical and non-spherical shapes.

These concepts are based on a detailed analysis of various properties of the aforesaid nuclei, *i. e.* gamma- and beta-transition probabilities, quadrupole and magnetic moments, and effective deformation.

The above-mentioned distinguishing features of nuclei in transition regions were related to the average field level properties (Nilsson diagrams) [3] close to the Fermi surface as well as to the pairing effects. As the neutron number varies from 84 to 88, deformation of odd-proton nuclei increases slowly; the author relates a severe discontinuity in the nucleus shape with variation of the neutron number from 88 to 90 to a sinking of the Nilsson diagram proton orbitals leaving the node point $h_{11/2}$ (orbitals 1/2 — [550], 3/2 — [541] and 5/2 — [532]) below the Fermi surface.

---

* Address: USSR Academy of Sciences, A. F. Ioffe Physical Technical Institute, Leningrad, K–21, USSR.

The distinctive feature of all nuclei considered in [1,2] is their "softness" in relation to beta- and gamma-oscillations, and this is why the value of equilibrium deformation $\beta_0^2$ turns out to be comparable to or even less than the amplitude of zero point oscillations, $\langle\beta^2\rangle$. In the case of "hard" strongly deformed nuclei, the amplitude of zero-point oscillations is small compared to the value of equilibrium deformation ($\beta_0 \approx 0.3$).

Simultaneously with the publication of papers [1,2] concerning transient odd-mass nuclei, it was found that in the case of even-even nuclei in the transition regions there is an analogy of the properties of these nuclei with those of strongly deformed even-even nuclei [4,6].

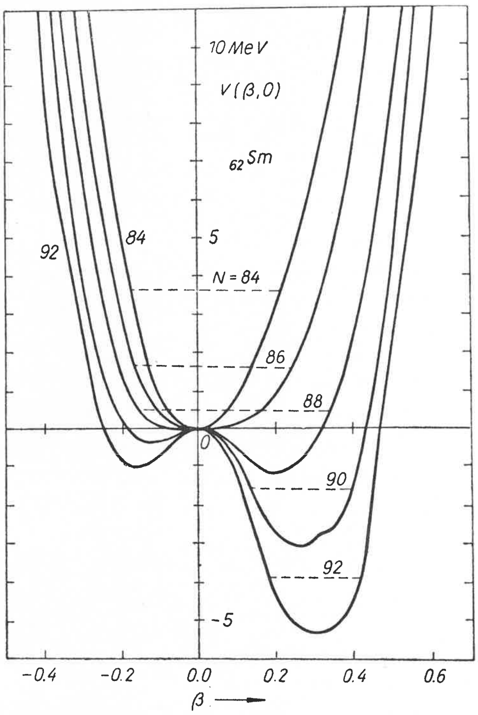


**Fig. 1.** Potential energy *versus* deformation for even-even isotopes of samarium

Specifically, the so-called "quasi-rotational" bands with a spin sequence similar to that of the levels in rotational bands of strongly deformed nuclei were detected.

We wish to show that a gradual change in the properties of nuclei in transition regions, just as the relatively abrupt change in nuclear properties with the variation of the neutron number from $N = 88$ to $N = 90$, correlates well with the nature of the change in the shape of the nuclear potential energy curve as a function of deformation $V(\beta)$.

Fig. 1 shows potential energy *versus* deformation for even-even isotopes of samarium according to the calibrations carried out by Baranger and Kumar [7]. Dashed horizontal lines show energy values for zero-point oscillations.

The curves for the $^{152}$Sm ($N = 90$) and $^{154}$Sm ($N = 92$) are characterized by deep minima (3–5 MeV) conforming to the values of the deformation parameter equal to approximately 0.25 and 0.30, the energies of zero-point oscillations in both cases being much less than the depths of the minima. At the same time, both these nuclei have well-marked rotational bands, and the values of deformation determined from the reduced probabilities $B(E2)\uparrow$ of the transition $0^+ \rightarrow 2^+$ agree with the minima positions specified above.

In the case of the $^{150}$Sm nucleus containing 88 neutrons, there is also a defined minimum in the potential energy curve conforming to the deformation parameter of $\beta_0 \approx 0.2$, the energy of zero-point oscillations being 1.7 MeV, which is 0.5 MeV over the depth of the minimum (1.2 MeV). The authors [7] believe that the $^{150}$Sm nucleus, like the lighter isotopes of $^{148}$Sm and $^{146}$Sm, is unstable with respect to deformation. However, due to a sharp asymmetry in the shape of the $V(\beta)$ curve for $^{150}$Sm, it is readily apparent that in this case, too, there are similar grounds for asserting that this nucleus is unstable with respect to the spherical shape.

It seems to us that in this case (as in the case of strongly deformed nuclei) the equilibrium nuclear shape is determined by the shape of the surface about which oscillations occur. Taking account of the area limited by the curve below the dashed line which conforms to the energy of zero-point oscillations, the deformation parameter in the mid-equilibrium position can be estimated at $\beta_0 \approx 0.15 \div 0.16$.

In this connection the fact that the values of the r clear deformation parameter for $^{150}$Sm calculated from the value of $B(E2)\uparrow$ which was obta ed in the experiments on Coulomb excitation [8] as well as from direct measurements c the quadrupole moment $Q_2$ for level $2^+$ of Sm$^{150}$ (and the internal quadrupole moment ( , respectively) [9] are the same and equal to $\beta_0 = 0.16$, which agrees with the estimate m le above, seems to be very important.

In our opinion, a good agreement between the values of the deformation parameter obtained by three independent methods is a sufficie y objective proof of the existence of substantial equilibrium deformation of the $^{150}$Sm nucleus, despite a considerable departure of the level energy ratios $\frac{E_4}{E_2} = 2.34$ from a purely rotational value $\frac{E_4}{E_2} = 3.33$.

A perfectly analogous situation occurs, too, with $^{152}$Gd and $^{148}$Nd nuclei also containing 88 neutrons each.

As to even-even nuclei with the neutron number $N < 88$, *e. g.* the $^{148}$Sm and $^{146}$Sm isotopes, their deformation values in the equilibrium position are even smaller, but also different from zero, as can be seen from the asymmetry of the potential energy curves for these nuclei (Fig. 1). The "centre of oscillations" here is also shifted towards positive values of $\beta_0$, although not to such an extent as in $^{150}$Sm.

Fig. 2 presents level schemes for even-even isotopes of samarium [6]. As is apparent from Fig. 2, so called "quasi-rotational" bands evolve gradually into purely rotational bands with an increase in the mass number. A strong connection between rotation and oscillation for nuclei with $N < 90$ is obvious. The wave functions of excited states $2^+$, $4^+$, *etc.*, contain both the rotational and phonon components. This results in a distortion of the rotational spectrum: the ratio of level energies differs substantially from those in purely rotational bands of strongly deformed nuclei and the values of reduced probabilities $B(E2)\uparrow$ for the transition $0^+ \rightarrow 2^+$ are less than for the latter. The whole evolution of "quasi-rotational" bands into rotational ones as the neutron number increases becomes clear in the light of the aforesaid increase in the value of deformation conforming to the mid-equilibrium position. It is related to the increase in the contribution of the rotational component to the nuclear wave function. After a jump in deformation with the transition to $^{152}$Sm related to rebuilding of proton orbits, the rotational motion is more marked and the spectrum acquires typical properties of a rotational one.

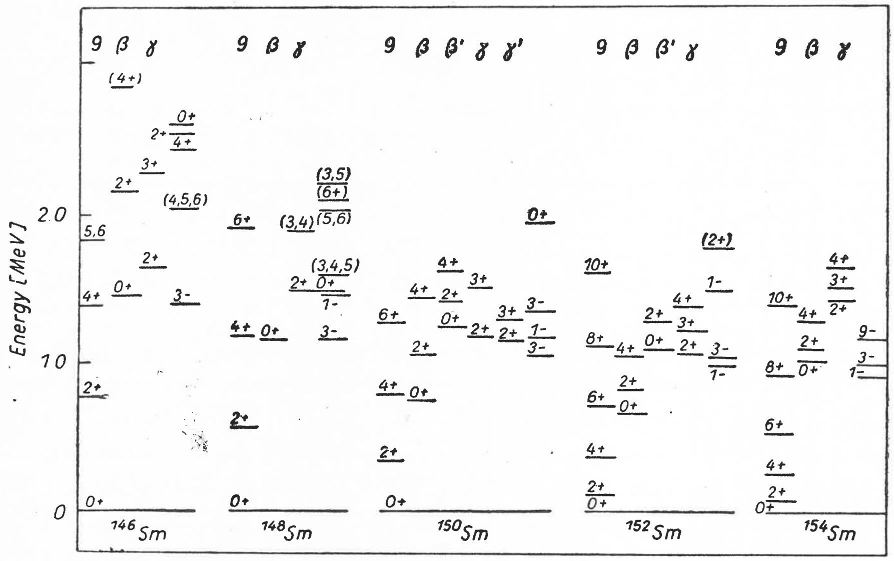


**Fig. 2.** Level structure of even-even isotopes of samarium, $g$ – quasi-rotational band of ground state, $\beta$ – quasi-beta-vibrational band, $\beta'$ – two phonon quasi-beta-vibrational band, $\gamma$ – quasi-gamma-vibrational band

It should be noted that the transition from $^{148}$Sm to $^{150}$Sm is accompanied by a more abrupt change in properties than that from $^{146}$Sm to $^{148}$Sm. In particular, this also applies to the relative rate of energy decrease for the $2^+$ level and the change in the trend of the

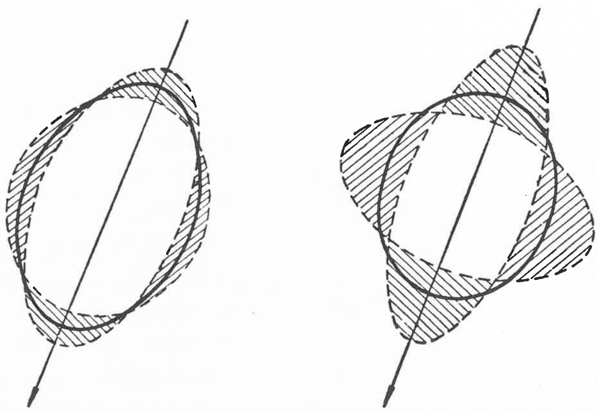

Fig. 3. Definition of the nuclear shape concept. Left — small amplitude of oscillations. Right — large amplitude of oscillations

potential energy curve $V(\beta)$; with deformation different from zero, a minimum appears in the curve for $^{150}$Sm only. The latter circumstance is probably due to the fact that the three descending orbitals leaving the node point $h_{11/2}$ actually sink below the Fermi surface not simultaneously, but in two stages, that is, orbitals $^1/_2$ — [550] and $^3/_2$ — [541] located on the Nilsson diagram particularly close to one another sink with the 87th and 88th neutrons ($^{150}$Sm) added and the third orbital $^5/_2$ — [532] ($^{152}$Sm) spaced apart slightly further from the first two orbitals sinks with the 89th and 90th neutrons added.

The importance of the rearrangement of the single-particle levels near the Fermi surface in the origin of the sharp change in the nuclear shape close by $N = 90$ was noted by the author earlier, when considering the properties of odd isotopes of europium and promethium [1,2]. Certainly, the odd-proton nuclei in transition regions are also complex in nature. Despite the fact that, as an analysis of their properties and data on quadrupole momenta shows, they feature small equilibrium deformations, single-particle levels are not purely Nilssonian due to a strong interaction of single-particle motion with rotation and nuclear surface oscillations.

In conclusion, Fig. 3, which explains the concept of the nuclear surface shape, is given. The left-hand part of Fig. 3 shows the case when the amplitude of zero-point oscillations is small compared to the value of equilibrium deformation: $\langle\beta^2\rangle < \beta_0^2$ (a "hard", strongly deformed nucleus). The right hand part presents the opposite case, when the amplitude of zero-point oscillations is large compared to the value of equilibrium deformation: $\langle\beta^2\rangle > \beta_0^2$ (a "soft" nucleus). Heavy lines in each case indicate the "equilibrium" position of the nuclear surface about which the oscillations occur. For simplicity, only beta-oscillations are considered.

It is evident that adoption of such a natural definition of the nuclear shape as a shape of equilibrium surface about which oscillations take place involves no radical difference between the two cases: the oscillations can be large or small, simple or complex, but in all cases there is a particular equilibrium position of the surface around which oscillations occur.

## REFERENCES


[1] E. Ye. Berlovitch, *Izv. Akad. Nauk SSSR, Ser. Fiz.*, **29**, 2177 (1965); *Yadernaya Fizika*, **5**, 560 (1967).

[2] E. Ye. Berlovitch, *Proceedings of the Conf. on the Electron Capture and Higher Order Processes in Nuclear Decays*, Debrecen, Hungary, July 15–18, 1968. Eotvos Lorand Physical Society, Budapest, vol. 3, 444, 1968.

[3] S. F. Nilsson, *Mat. Fys. Medd. Dan. Vid. Selsk*, **29**, 16 (1955).

[4] F. S. Stephens, N. L. Lark, R. M. Diamond, *Nuclear Phys.*, **63**, 82 (1965).

[5] N. L. Lark, H. Morinaga, *Nuclear Phys.*, **63**, 466 (1965); **67**, 315 (1965).

[6] M. Sakai, *Nuclear Phys.*, **A104**, 301 (1967).

[7] M. Baranger, K. Kumar, *Nuclear Phys.*, **62**, 113 (1965); **A110**, 490 (1968); **A110**, 29 (1968).

[8] R. J. Keddy, Y. Yoshizawa, B. Elbek, B. Herskind, M. C. Olesen, *Nuclear Phys.*, **A113**, 676 (1968).

[9] P. H. Stockstand *et. al.*, *Nuclear Phys.*, **A92**, 319 (1967).