

BACK SCATTERING OF ALPHA PARTICLES

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Angular distributions in the elastic scattering of alpha particles on atomic nuclei exhibit a complicated behaviour. Some possible explanations are discussed.

In the early Rutherford time of nuclear physics, the elastic scattering of alpha particles led to the discovery of the atomic nucleus and was used as a basic tool for the determination of nuclear dimensions.

The work of Blair [1] Drozdov [2] and Inopin [3] has shown that alpha particles can serve as nuclear probes not only for measuring nuclear radii but also for more detailed studies of nuclear structure. Further development of the adiabatic approximation [4], the optical model [5], DWBA [6] and coupled channels [7] approximations have made the elastic and inelastic scattering of alpha particles a powerful source of information on collective excitations of atomic nuclei.

All this information can be extracted from the angular distributions in the forward direction, but subsequent measurements¹ at the extreme backward scattering angles have revealed some new phenomena.

The most common feature of differential cross-sections at backward angles is the appearance of a maximum at 180°. This maximum is observed for light and medium weight target nuclei (Fig. 1) and for alpha particles with energies exceeding the Coulomb barrier. It can also be seen on Fig. 2 where data on the elastic scattering of alpha particles from ⁴⁰Ca nuclei are presented.

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¹ The list of references can be found in the paper [8].

The second feature characteristic of some nuclei is a substantial rise of the angular distribution for angles larger than about 90° . This behaviour, found for example in ^{28}Si , ^{39}K and ^{40}Ca nuclei, differs significantly from the regular overall decrease of the angular distribution curve for heavier nuclei.

The third feature observed for ^{39}K [8] and ^{40}Ca [9] nuclei are broad maxima in the excitation functions for energies from about 20 to 30 MeV.

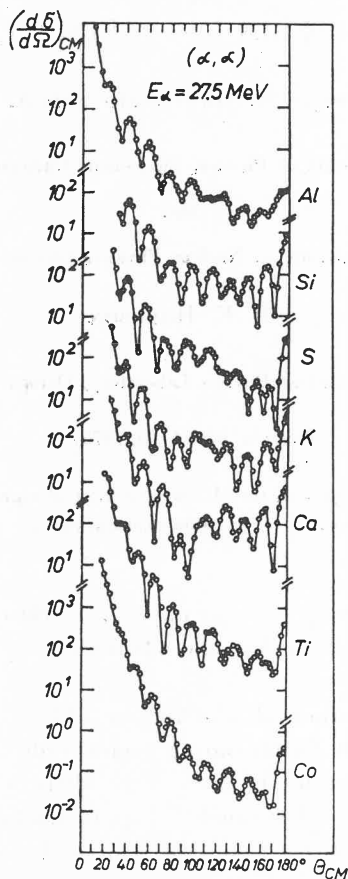


Fig. 1. Experimental angular distributions for the elastic scattering of alpha particles at the incident energy around 27.5 MeV [8], [9], [10]

In the region of backward angles one can expect contributions from many interaction mechanisms, and some of them may add coherently to produce complicated interference patterns. The aim of the present paper is an attempt to analyse on the basis of different models of scattering some of the experimental data obtained in Cracow and elsewhere.

The backward maximum in the elastic scattering of alpha particles on ^{28}Si , ^{39}K and ^{40}Ca nuclei is part of a diffraction-like pattern centered at 180° . This picture resembles the glory effect known in the meteorology for the backward scattering of light by water droplets

[12]. Bryant and Jarmie [13] following the suggestion of Ford and Wheeler [14], have used a model based on the glory effect as a possible explanation of the scattering of alpha-particles at backward angles. According to this model in the case of zero spin bombarding particles and neglecting the influence of the target spin, the differential cross-section from about 140° to 180° is described by the simple formula:

$$\frac{d\sigma}{d\Omega} = AJ_0^2(u) \quad (1)$$

where $J_0(u)$ is the zero order Bessel function of the argument $u = kR_g \sin(\pi - \theta)$, R_g is the interaction radius, k is the wave number of the bombarding particles in the CM system, θ is the scattering angle and A is a normalization factor that is not given by the theory.

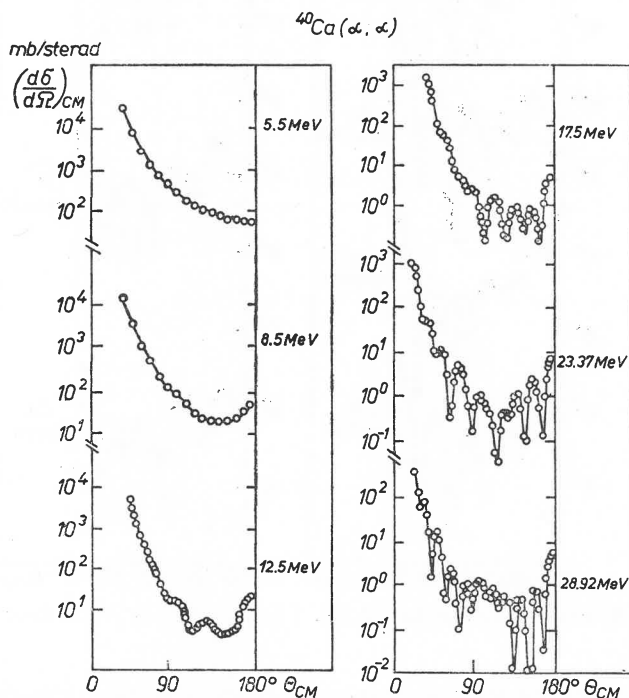


Fig. 2. Energy dependence of the experimental angular distributions for the elastic scattering of alpha particles from ^{40}Ca nuclei. 5.5 MeV, 8.5 MeV, 12.5 MeV and 17.5 MeV data were taken from Reference [11], 23.37 MeV and 28.92 MeV data from Reference [9]

The glory model fits to some (α, α) elastic scattering data are presented on Figs 3 and 4. In the case of well-developed diffraction-like structure of backward scattering (^{28}Si , ^{39}K and ^{40}Ca) the model reproduces satisfactorily the shape of last three maxima in the angular distribution.

Although the 180° maxima are also well fitted by the model for ^{27}Al , ^{32}S , Ti and ^{59}Co the structure of the angular distributions for these nuclei in the backward region is so irregular that it cannot be approximated by the simple Bessel function.

While this simple model is partially successful it is inadequate due to its essentially classical nature, and its inability to give more than the angular variation of the differential cross-sections in the extreme backward direction. One therefore looks for a quantum-mechanical model that will give the complete angular distribution in absolute terms, and this is provided by the optical model in its single-channel approximations.

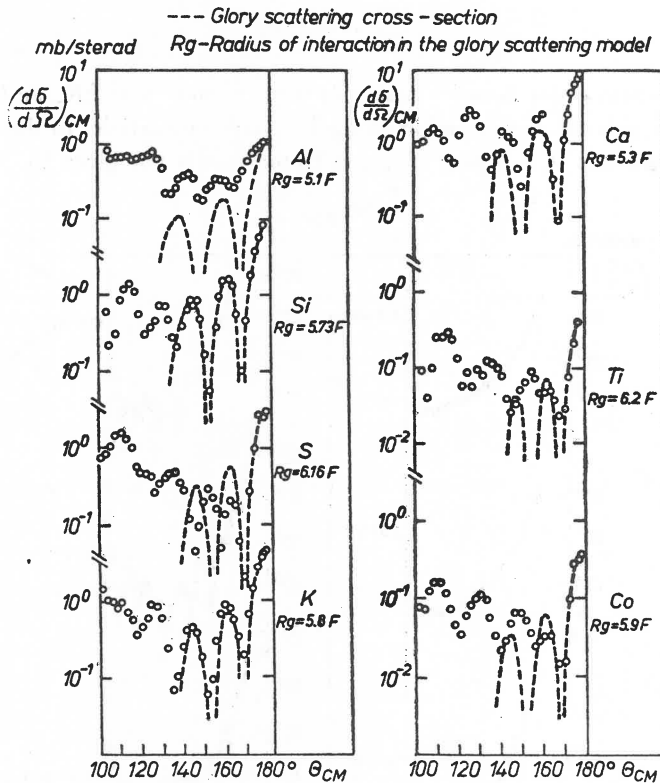


Fig. 3. Backward scattering data of Fig. 1 with predictions of the glory model

The simplest optical model analysis is made by adjusting the strengths and radial form-factors of the real and imaginary potentials to optimise the fit to the experimental data, and Fig. 5 shows the result of such a calculation for the elastic scattering of alpha particles by ^{59}Co . Several discrete phase-equivalent potentials were found, and the fit shown corresponds to the minimum value of χ^2 . As can be seen the agreement is quite satisfactory and the values of the potential parameters are reasonable. The best fit corresponds to the potential with the depth parameter of the real part of ~ 70 MeV. It is worth mentioning that the microscopic theory of the optical model can in fact give depth of the real part of the optical potential as shallow as 70 MeV by suitable adjustment of the range parameter in the nucleon-nucleon potential.

It is notable that the optical model is able to account quite naturally for the magnitude and the width of the sharp rise in the cross-section around 180° (the glory effect) and for

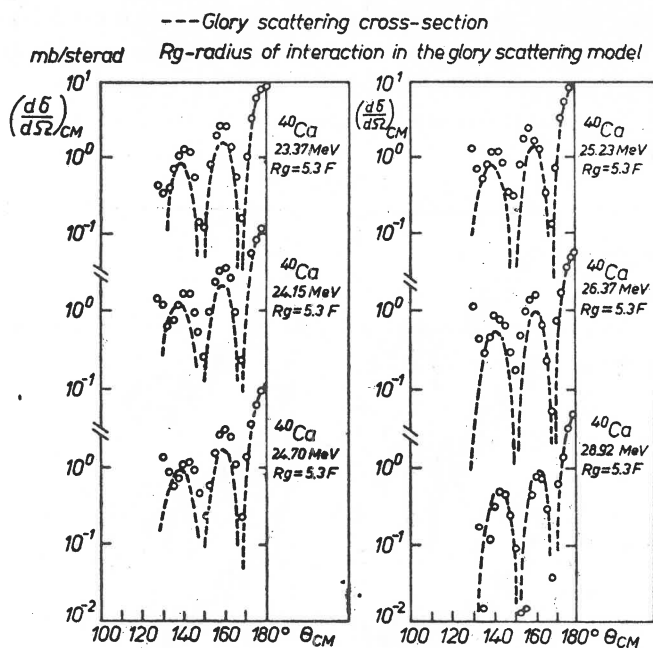


Fig. 4. Nuclear glory model fits to the $^{40}\text{Ca}(\alpha, \alpha)$ backward scattering data

TABLE I

The optical model parameters corresponding to the fits shown in Fig. 5

U MeV	W MeV	r_0 fm	a fm
43.92	11.62	1.635	0.493
63.93	11.01	1.643	0.486
104.58	19.18	1.519	0.489
137.57	22.75	1.487	0.485
175.34	26.01	1.458	0.484
216.09	29.03	1.434	0.483
266.29	32.74	1.406	0.483
313.72	36.58	1.388	0.482
362.88	40.41	1.370	0.481
423.19	46.26	1.350	0.481

the general rise for angles greater than 90° . However this has been achieved by optimising the parameters to fit this particular set of data, whereas the optical model implies that it should be possible to fit a wide range of data, for different nuclei and at different energies, with the potential having a smooth dependence on E , A and Z . Unless this is done, then the fit to a particular set of data may be due mainly to the flexibility of the model, and significant physical effects may thereby be obscured. To investigate this, additional calculations were made for a number of other alpha particle interactions using an optical model potential with

the depth parameter U and the geometrical parameters r_0 and a fixed. The values of these parameters were obtained by averaging over the values of the best fit parameters from 4 parameter searches. The depth of the imaginary part W was adjusted in each case in order to obtain the best fit to the experimental data. Some of the results are shown in Fig. 6. It is

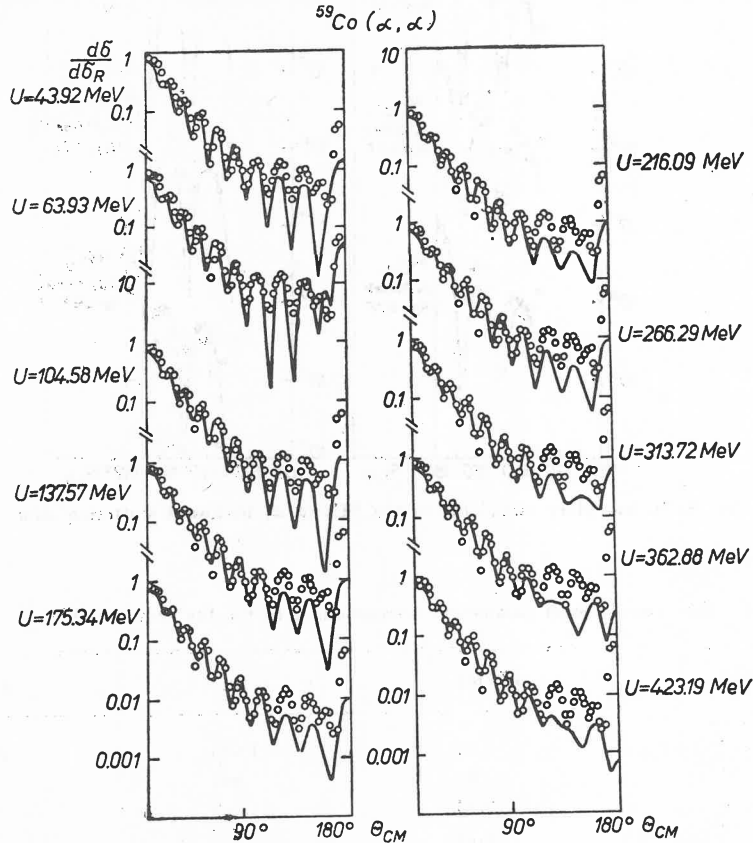


Fig. 5. Optical model fits [15] to 27.5 MeV $^{59}\text{Co}(\alpha, \alpha)$ scattering data corresponding to different depths of the real part of the potential

apparent that while the optical model is able to give the observed maximum in the cross-section at 180° , it often fails to give its magnitude correctly in the angular region from about 90° to 160° .

It is possible that this failure of the simple optical model is due to the neglect of the coupling to the inelastic channels [16]. Since the allowed channels, and strengths of the coupling to them, differ from nucleus to nucleus this could account for differences in the cross-sections, even though the optical potential remains the same [17]. This possibility was investigated by making a number of coupled-channels calculations with different values of the coupling strength. The reaction chosen was $^{40}\text{Ca}(\alpha, \alpha)$ at 24.7 MeV, and calculations were made with the same optical potential for three cases: (1) elastic channel only (2) elastic +

coupling to 3^- state at 3.73 MeV, (3) elastic + coupling to 3^- state at 3.73 MeV and 5^- state at 4.483 MeV. The results are given in Fig. 7 and are compared with glory model curves calculated from expression (1), in which R is adjusted in turn to give the same width of the peak at 180° of each of the calculated curves. All curves are normalized to the same cross-section at 180° . It is apparent that the optical model in each case agrees quite closely with

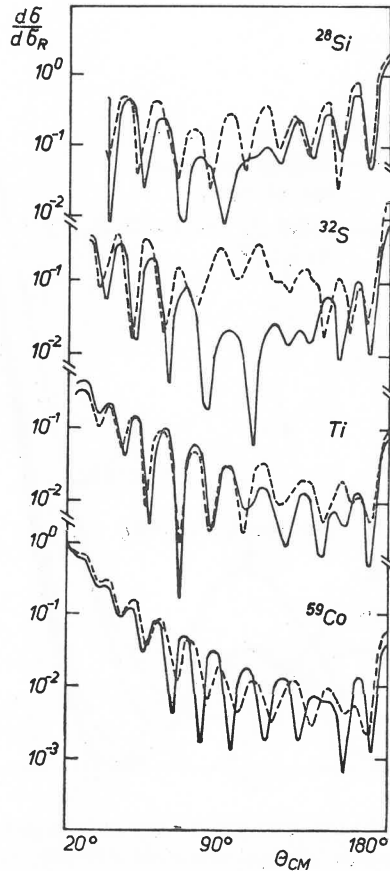


Fig. 6. Optical model fits to 27.5 MeV (α, α) scattering data $U = 52.05$ MeV, $r_0 = 1.6$ fm, $a = 0.556$ fm, $W = 8.96, 10.76, 10.78$ and 10.78 MeV for ^{28}Si , ^{32}S , Ti and ^{59}Co respectively. (Dotted curve — experimental angular distributions)

the simple formula (1) in the range 160° – 180° . The values of the radius parameter R are similar to those used in optical model calculations. It is also notable that both the shape and the magnitude of the backward peak is quite sensitive to the coupling used. This strengthens the previous conclusion that the glory effect can be accounted for by the optical model. It would be interesting to see how well this model is able to fit the extreme backward scattering of alpha particles by many nuclei over a range of energies, and in particular to investigate whether the data imposes any restrictions on the parameters of the model. Such a study

cannot, however, be undertaken until the backward scattering as a whole is understood, for if a particular mechanism must be invoked to account for it, this same mechanism will also affect the extreme backward scattering.

The backward scattering from 90° to 160° has recently been studied in detail by the Heidelberg group [18], who have found that in some nuclei the cross-section in this region

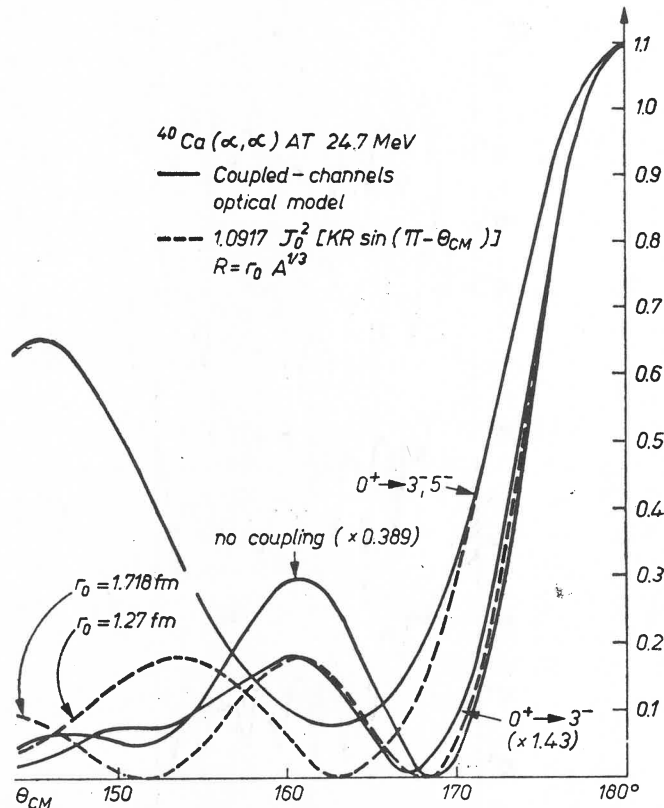


Fig. 7. Optical model calculations of backward scattering $^{40}\text{Ca} (\alpha, \alpha)$ angular distributions including coupling to one or two inelastic channels compared with glory model predictions

is notably enhanced. Each individual angular distribution can be fairly well fitted by the optical model, but if one requires that the parameters do not change appreciably from one nucleus to the next then the cross-section calculated from the optical model with parameters obtained by fitting the data that do not show the enhancement fall short of the experimental values in the anomalous cases by as much as an order of magnitude. This cannot be due to the effect of coupling to inelastic channels because as shown in Fig. 7 this coupling reduces the backward cross-section, and in addition the enhancement is greatest for just those nuclei like ^{40}Ca with a stable structure for which the coupling is weakest. Furthermore, the enhanced cross-sections are not observed for other incident particles like protons and deuterons, which suggests that the effect is in some way particularly associated with the incident particles

being alpha particles. The enhancement cannot therefore be accounted for satisfactorily by the optical model.

The optical model, even in its coupled channels version, is essentially a rather simple way of replacing all the complicated participating nucleon-nucleon interactions by an averaged potential. It is likely that a more detailed microscopic model that takes these interactions into account will be able to give a more accurate representation of the experimental data. Such a model has recently been proposed by Greenlees, Pyle and Tang [19], who have shown how the nucleon-nucleus potential can be obtained from an nucleon-nucleon interaction and a Fermi parametrization of the nuclear matter distribution. In order to calculate α -nucleus potential we have applied the GPT model in the following way: [20]

1) The nucleon — α potential has been derived by folding an effective nucleon-nucleon potential of the Yukawa or Gaussian form with the density distribution of nuclear matter in the alpha particle.

2) The α -nucleon potential obtained in this way was folded with the density distribution of nuclear matter in the target nucleus.

3) The calculated real potential was supplemented by a phenomenological imaginary part of the Saxon-Woods form.

Fig. 8 shows a typical fit to the $^{59}\text{Co}(\alpha, \alpha)$ scattering data obtained by this procedure.

Table II shows the depths of the real Saxon-Woods potential equivalent to the potential calculated for Co (α, α) in the above way as a function of the range parameter μ .

The nucleon-nucleon interaction was assumed of the Yukawa form $V_0 \exp(-\mu r)/\mu r$ with the depth parameter V_0 being kept at the constant value 46 MeV. The density distributions of nuclear matter in the alpha particle and in the Co-nucleus were obtained from the electron scattering data as the difference in the neutron and proton distributions is negligible [21]. It is clear from Table II that it is possible to obtain potentials with depths varying in a broad region of values properly matching the range of the nucleon-nucleon interaction.

For some nuclei such as ^{39}K and ^{40}Ca the shallow phenomenological potential does not reproduce cross-sections in the region of backward scattering angles. With deeper optical potentials quite good fits were obtained for ^{39}K nuclei in the whole region of angles [22]. However, the calculated cross-sections are slightly too low at forward scattering angles, the imaginary part of the optical potential is smaller than for other nuclei and the values of the geometrical parameters are rather unusual ($r_0 \approx 1.1$ fm, $a \approx 0.8$ fm).

These difficulties suggest that there is an additional physical process that takes place for some nuclei but not for others. One possibility is an exchange interaction, and this naturally enhances the cross-section in the backward direction. Some detailed calculations have been made by Honda *et al.*, [23] and they find that such an interaction is indeed important, but that, as might be expected its magnitude is greatest for alpha particle scattering by light nuclei, and it decreases so rapidly for the heavier nuclei that it is unable to account for enhanced maxima for the nuclei under consideration here.

A new model to account for these effects has recently been proposed by Schmeing [24]. According to this model the amplitude for alpha particle scattering can be written as a sum

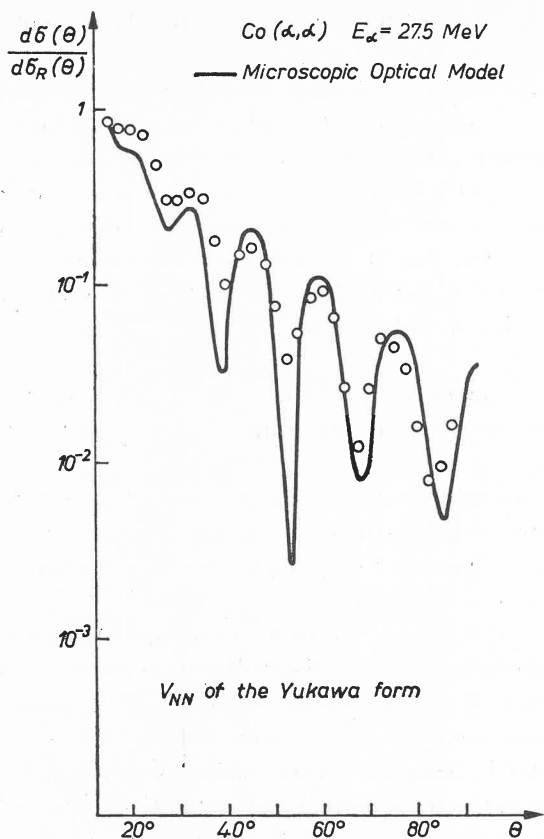


Fig. 8. Microscopic optical model calculations for elastic scattering of 27.5 MeV alpha particles on Co nuclei with the Yukawa form of the nucleon-nucleon potential. Matter density distribution of the Yukawa form with $\langle r^2 \rangle_{\text{He}}^{1/2} = 1.40$ fm and $\langle r^2 \rangle_{\text{Co}}^{1/2} = 4.66$ fm. Nucleon-nucleon potential parameters: $V_0 = 44.4$ MeV and $\mu = 1.4$ fm $^{-1}$. Imaginary part of the optical potential: $W = 7.37$ MeV, $r_w = 1.432$ fm and $a_w = 0.815$ fm

of two parts, one representing the scattering from the normal state of the nucleus (the usual optical model amplitude), and the second one corresponding to the scattering on the partly-clustered nuclear state. This clustering is not taken into account by the optical model, and this could be the reason for its failure. Schmeing has calculated this second amplitude in the framework of the DWBA formalism. The scattering by the alpha clusters located in the surface nuclear region is treated as a direct reaction while the optical potential for the interaction with the whole nucleus is taken as a distorting potential. The radius parameter of this distorting potential is assumed smaller than usual what can be explained by some sort of shrinkage of the nucleus in the cluster phase.

The enhancement of the backward cross-section [18] that one seeks to explain by this model is strongly present for ^{36}Ar , ^{40}Ca and ^{39}K , is weak for ^{42}Ca and ^{41}K , and is absent for ^{40}Ar , ^{44}Ca and ^{48}Ca . Thus one has to explain why it is present for $N-Z = 0$ or -1 , is weak for $N-Z = 2$ or 3 and is absent for $N-Z = 4$ or 8 . According to Schmeing this

can be understood qualitatively by considering the availability of phase space, since nuclei with even, equal numbers of protons and neutrons have marked alpha correlations [24]. Removing one proton from ^{40}Ca does not destroy the alpha particle correlations because the exclusion principle inhibits the clusters at worst no more than in ^{40}Ca . Adding neutrons does destroy the correlations because they occupy the phase space needed by the clusters

TABLE II

The depth of the real part of the optical model potential as a function of the range parameter of the Yukawa potential

μ	$V_{\text{eff}}(\text{MeV})$
1.6	68
1.4	128
1.3	158
1.2	197
1.1	250
1.0	324

since a cluster formed from four particles normally in the $2s-1d$ shell has components in many higher shells as well. The extra neutrons thus perturb the clusters, making them energetically less probable.

Additional more detailed calculations are required to confirm this model, but it certainly appears to provide a way of understanding the enhancement of the extreme backward cross-section in some nuclei above that expected from the optical model.

It remains to consider the broad maxima in the excitation functions for energies from 20 to 30 MeV. Such maxima are predicted by the simple classical glory effect model as a consequence of the orbiting of the alpha particle around the nucleus, which occurs at particular energies [8], [9]. The equivalent quantum-mechanical description interprets the maxima as resonances in the potential well formed by the superposition of its nuclear, Coulomb and centrifugal components, and thus the effect arises quite naturally from the optical model (Gruhn and Wall [26], Berry [25]). The dip in the potential in which these resonances occur is, however, within the nucleus, and so it is essential for the radius of the imaginary potential to be even smaller for otherwise the particles would be absorbed before they reached this part of the potential. A characteristic feature of optical model analyses of the elastic scattering of alpha-particles, and indeed of most particles particularly those that are strongly absorbed, is that the imaginary potential has a radius substantially greater than the real potential. In order to maintain the above explanation it is thus necessary to suppose that the imaginary potential is particularly small for the partial wave in which the resonance occurs, and this introduces the possibility of l -dependent potentials.

Some insight into this phenomenon may be obtained by considering that in the elastic channel there are waves of high angular momentum with appreciable transmission coefficients, so that these high angular momenta are given to the compound nucleus. However,

the final states energetically accessible for the decay of the compound nucleus may not have sufficiently high spins or the outgoing particles may not have sufficiently high energies to allow these high angular momentum states to decay into non-elastic channels so that the corresponding flux is returned to the elastic channels. Such an effect will only occur for a particular value of l over a limited energy range, because at lower energies it does not appreciably contribute while at higher energies more final states rapidly become available. This resonance-like behaviour can be represented by adding a Breit-Wigner amplitude to the optical model amplitude in the partial wave concerned. This method has been used by Kim [27] to account for the marked $l = 10$ resonance in the elastic scattering of 20 MeV alpha-particles by ^{24}Mg . Similar resonant behaviour has been observed in the elastic scattering of 20–23 MeV alpha particles by ^{26}Mg [28]. The partial wave in which the resonance occurs can be identified particularly easily by the Regge-pole model [29]. Another method is to allow the imaginary part of the optical potential to depend on l , in particular allowing it to become very small for higher l . Such a potential has been shown by Bisson and Davis [30] to improve the fit to the elastic scattering of 5.5–17.5 MeV alpha particles by ^{40}Ca .

An investigation of this effect was made by analysing the broad maximum in the backward scattering of alpha particles by ^{39}K and ^{40}Ca around 24 MeV. Starting from an averaged optical potential with U around 100 MeV and r_0 about 1.5 fm it was found that a modification of the $l = 10$ reflection coefficient is necessary in order to improve the fit in the region of energy from 23 MeV to 25 MeV [22], [31]. A similar result was obtained by the Louvain group [29] on the basis of the Regge-pole model. It was found that the same procedure can be extended to other energies by modifying η_l , where l increases with energy.

There are thus many interesting phenomena connected with the backward scattering of alphaparticles by nuclei that are only partially understood, and their further investigation promises to be a fruitful field of research in the future.

REFERENCES

- [1] J. S. Blair, *Phys. Rev.* **115**, 928 (1959).
- [2] S. I. Drozdov, *Zh. Eksper. Teor. Fiz.*, **28**, 734, 736 (1955).
- [3] E. V. Inopin, *Zh. Eksper. Teor. Fiz.*, **31**, 901 (1956).
- [4] N. Austern, J. S. Blair, *Ann. Phys.* **33**, 15 (1965)
- [5] P. E. Hodgson, *The Optical Model of Elastic Scattering*, Oxford Clarendon Press 1963.
- [6] R. H. Bassel, G. R. Satchler, R. M. Drisko, E. Rost, *Phys. Rev.*, **128**, 2693 (1962).
- [7] B. Buck, *Phys. Rev.*, **127**, 940 (1962), **130**, 712 (1963); B. Buck, A. P. Stamp, P. E. Hodgson, *Phil. Mag.*, **8**, 1805 (1963), T. Tamura, *Rev. Mod. Phys.*, **37**, 679 (1965).
- [8] A. Bobrowska, A. Budzanowski, K. Grotowski, L. Jarczyk, S. Micek, H. Niewodniczański, A. Strzałkowski, Z. Wróbel, *Nuclear Phys.*, **A126**, 361 (1969).
- [9] A. Budzanowski, K. Grotowski, L. Jarczyk, B. Łazarska, S. Micek, H. Niewodniczański, A. Strzałkowski, Z. Wróbel, *Phys. Letters*, **16**, 135 (1965).
- [10] A. Bobrowska, A. Budzanowski, K. Grotowski, L. Jarczyk, S. Micek, H. Niewodniczański, A. Strzałkowski, Z. Wróbel, *Proc. Int. Conf. on Nucl. Structure*, Dubna (1968) 116; *IFJ Report No 624/PL* (1968).
- [11] K. A. Eberhard, R. H. Davis (preprint).
- [12] H. C. Van de Hulst, *Light Scattering by Small Particles*, John Wiley, 1967.
- [13] H. C. Bryant, N. Jarmie, *Ann. Phys.*, **47**, 127 (1968).

- [14] K. W. Ford, J. A. Wheeler, *Ann. Phys.*, **7**, 259 (1959).
- [15] *Part of an extensive optical model analysis* (to be published).
- [16] A. Budzanowski, A. D. Hill, P. E. Hodgson, *Nuclear Phys.*, **A117**, 509 (1968).
- [17] F. G. Perey, ANL — 6848, 114 (1964).
- [18] G. Gaul, H. Lüdecke, R. Santo, H. Schmeing, R. Stock, *Nuclear Phys.*, **A137**, 177 (1969).
- [19] G. W. Greenlees, G. J. Pyle, Y. C. Tang, *Phys. Rev.*, **171**, 1115 (1968).
- [20] A. Budzanowski, A. Dudek, K. Grotowski, A. Strzałkowski, *Proceedings of the International Symposium on Nuclear Reaction Mechanism and Polarization Phenomena*, Université Laval, Quebec, 1–2 September (1969).
- [21] G. W. Greenlees, W. Makofske, G. J. Pyle (preprint).
- [22] A. Budzanowski, A. Dudek, R. Dymarz, K. Grotowski, L. Jarczyk, H. Niewodniczański, A. Strzałkowski, *Nuclear Phys.*, **A126**, 369 (1969).
- [23] T. Honda, Y. Kudo, H. Ui, *Nuclear Phys.*, **44**, 472 (1963).
- [24] N. C. Schmeing, *Nuclear Phys.*, **A142**, 449 (1970).
- [25] C. R. Gruhn, N. S. Wall, *Nuclear Phys.*, **81**, 161 (1966).
- [26] M. V. Berry, *Proc. Phys. Soc. (London)*, **88**, 285 (1966).
- [27] H. J. Kim, *Phys. Letters*, **19**, 296 (1965).
- [28] P. P. Singh, B. A. Watson, J. J. Kroepfel, T. P. Marvin, *Phys. Rev. Letters*, **17**, 968 (1966).
- [29] E. Lakie, I. Lega, P. C. Macq, *Nuclear Phys.*, **A135**, 145 (1969).
- [30] A. E. Bisson, R. H. Davis, *Phys. Rev. Letters*, **22**, 542 (1969).
- [31] A. Budzanowski, K. Grotowski, L. Jarczyk, H. Niewodniczański, A. Strzałkowski, *International Nuclear Physics Conference*, Gatlinburg 1966, p. 81, Academic Press 1967.