Vol. A38 (1970) ACTA PHYSICA POLONICA Fasc. 1

MAGNONS IN THIN FILMS AT LOW TEMPERATURES

By L. Wojtczak

Department of Theoretical Physics, University of Łodź*

A. Sukiennicki

Institute of Physics, Technical University, Warsaw**

A. Corciovei and D. Vamanu

Institute of Atomic Physics, Bucharest, Roumania***

(Received January 24, 1970)

Spin wave procedure is adapted to ferromagnetic thin films with sufficiently strong uniaxial anisotropy parallel to the film surfaces by using the Heisenberg Hamiltonian for Valenta's model of sublattices. The demagnetizing effects are included by means of the effective demagnetizing factors. The conditions determining the occurrence of single domain films are given. They lead to the conclusion that the single domain films occur below some critical thickness and the domain structure appears above it. The magnetization is directed along an axis parallel with the film surface. The magnon frequencies are determined.

1. *Introduction*

Low temperature theoretical studies of spontaneous magnetization in ferromagnetic thin films were made by various authors who, however, did not take into account the angular distribution of magnetization [1].

The distribution of magnetization directions in a film is one of the important problems in thin film physics. Such a distribution can change the energy eigenvalues and spin wave amplitudes [2] and so leads to the dependence of properties of a film on the distribution of magnetization directions. Up to the present the theoretical considerations are mainly connected with two problems:
1) the direction of magnetization in single domain films [3] and
2) the appearance of magnetic domain structure [4].

* Address: Katedra Fizyki Teoretycznej Uniwersytetu Łódzkiego, Łódź, Kościuszki 21, Polska.
** Address: Instytut Fizyki Politechniki Warszawskiej, Warszawa, Koszykowa 75, Polska.
*** Address: Institutul de Fizica Atomica, Bucuresti, Casuta Postale 35, Roumania.

The aim of this paper is to discuss the spin wave spectrum of thin films in which the distribution of magnetization directions is taken into account. Conditions of the homogeneous magnetization or of the appearance of domain structure are also considered.

We assume Valenta's model of sublattices [5]. A thin film is considered as a superposition of n monoatomic square layers labelled by ν in the direction of the X-axis of the coordinate system connected with the film. The position of an atom in the plane (yz) is given by the vector $\boldsymbol{j}$. In the direction y or z there are N atoms. The magnetization direction is determined by the angle $\varphi_{\nu j}$ which describes the rotation of the quantization axis around an axis with direction cosines $e^{\alpha}_{\nu j}$ $(\alpha = x, y, z)$ [4]. The distribution function of this angle can be obtained by minimization of the free energy of the system. This free energy can be calculated by means of the usual thermodynamic procedure from the partition function which is the trace of the density matrix diagonalized in the class of the eigenstates in which the Hamiltonian is diagonal.

For the sake of simplicity of calculation we consider films with simple cubic crystallographic structure and with uniaxial anisotropy, sufficiently strong in the plane of the film. This allows us to discuss the stripe domain structure. The demagnetizing field is assumed to be an effective field expressed by the classical demagnetizing factors.

2. *Magnetization direction in single domain films*

Let us consider single domain thin films. Their magnetic properties can be described by Hamiltonian of the form

$$H = -J \sum_{\langle \nu j, \nu' j' \rangle} \sum_{\alpha} S^{\alpha}_{\nu j} S^{\alpha}_{\nu' j'} - \sum_{\nu j} K(\nu) S^{z}_{\nu j} S^{z}_{\nu j} + \sum_{\nu j} \sum_{\alpha} N^{\alpha} S^{\alpha}_{\nu j} S^{\alpha}_{\nu j} \tag{1}$$

where $S^{\alpha}_{\nu j}$ denote the spin operator components $(\alpha = x, y, z)$ in the lattice site $(\nu \boldsymbol{j})$. The brackets $\langle\rangle$ indicate that summation is carried out only on pairs of nearest neighbours.

The first term of (1) represents the Heisenberg term with the exchange integral J. The second term corresponds to uniaxial anisotropy including the surface anisotropy effects.

The anisotropy constant $K(\nu)$ is of the form

$$K(\nu) = K + K'(\delta_{1\nu} + \delta_{n\nu}). \tag{2}$$

The third term describes the demagnetizing field Hamiltonian. By analogy to the classical treatment of this problem the demagnetizing field is assumed to be proportional to the magnetization of the film. In terms of quantum theory this corresponds to the proportionality to the spin operator with the proportionality factor given by classical considerations. Taking into account that $L_x \ll L_z$, $L_y \approx L_z$ (L_α denotes the dimensions of a sample) we can approximately put $N^x \simeq 2\pi \mathfrak{z}$, $N^y = 0$, $N^z = 0$, $\mathfrak{z} = (g\beta)^2/V_0$; g is the gyromagnetic factor, β—Bohr's magneton, V_0 denotes the volume of an elementary cell.

We make the transformation (used in the theory of the screw structures of bulk bodies [6]) to the coordinate system in which the z-axis is directed along the magnetization of the considered atom in the $(\nu \boldsymbol{j})$ position. This transformation is of the form

$$S^{\alpha}_{\nu j} = \sum_{\beta} R^{\alpha\beta}_{\nu j} S^{\beta}_{\nu j}, \tag{3}$$

where

$$R_{\nu j}^{\alpha\beta} = \sum_{\gamma} \varepsilon^{\alpha\gamma\beta} e_{\nu j}^{\gamma} \sin \varphi_{\nu j} + \delta^{\alpha\beta} \cos \varphi_{\nu j} + e_{\nu j}^{\alpha} e_{\nu j}^{\beta} (1 - \cos \varphi_{\nu j})$$

and $\varepsilon^{\alpha\gamma\beta}$ denotes a completely antisymmetric tensor. The direction of the magnetization can be determined by the angle φ with respect to the plane of a film. This angle is taken to be the same for each spin of a film. It varies from 0 (magnetization parallel with the surfaces of the film) to $\pi/2$ (magnetization perpendicular to the film surface). Thus the transformation (3) is reduced to $R_{\nu j}^{\alpha\beta} = R^{\alpha\beta}$ for $e_y = 1$, $e_x = e_z = 0$.

Next we apply the diagonalization procedure discussed in [7]. The main steps of this procedure are the following:

a) We introduce the creation and annihilation operators of spin waves in the Holstein–Primakoff approximation by means of the relations

$$S_{\nu j}^{\pm} = \sqrt{2S}\, a_{\nu j}^{\mp}, \qquad S_{\nu j}^{z} = S - a_{\nu j}^{+} a_{\nu j}^{-}. \tag{4}$$

b) We transform the operators $a_{\nu j}^{\pm}$ into $a_{\nu h}^{\pm}$ by means of Fourier transformation in the plane of a layer; h denotes the wave vector in the plane of the film.

c) To diagonalize the terms of the type $a_{\nu h}^{\pm} \cdot a_{\nu,-h}^{\pm}$ we divide the region of propagation vectors in the film plane considered into two regions such that if $\boldsymbol{h}$ belongs to the first region $(\boldsymbol{h})$ then $-\boldsymbol{h}$ belongs to the second one $(\bar{\boldsymbol{h}})$. The point $\boldsymbol{h} = 0$ is excluded and will be treated separately.

d) We introduce the canonical conjugated operators $q_{\nu h}$, $p_{\nu h}$ by means of the usual relations $a_{\nu h}^{\pm} = \frac{1}{\sqrt{2}} (q_{\nu h} \pm i p_{\nu h})$.

e) We transform the operators $q_{\nu h}$, $p_{\nu h}$ into the operators $q_{\tau h}$, $p_{\tau h}$ representing independent oscillators. This transformation is of the form

$$q_{\nu h} = \sum_{\tau} T_{\nu\tau}^{qh} q_{\tau h}, \tag{5}$$

$$p_{\nu h} = \sum_{\tau} T_{\nu\tau}^{ph} p_{\tau h}.$$

The coefficients $T_{\nu\tau}^{qh}$, $T_{\nu\tau}^{ph}$ are determined by means of the difference equations [8] (s.c. structure is assumed with orientation (100))

$$-x_{\tau}^{kh} T_{\nu\tau}^{kh} + T_{\nu+1,\tau}^{kh} + T_{\nu-1,\tau}^{kh} = 0 \tag{6}$$

with the boundary conditions

$$(r^{kh} - x_{\tau}^{kh}) T_{1\tau}^{kh} + T_{2\tau}^{kh} = 0, \tag{6a}$$

$$(r^{kh} - x_{\tau}^{kh}) T_{n\tau}^{kh} + T_{n-1,\tau}^{kh} = 0, \tag{6b}$$

$$\sum_{\nu} T_{\nu\tau}^{kh} T_{\nu\tau'}^{kh} = \delta_{\tau\tau'}. \tag{6c}$$

The index k represents q or p respectively; the parameter r^{kh} is determined by the surface anisotropy K' and the angle φ as follows

$$r^{qh} = r^{p\bar{h}} = 1 + \frac{K'}{J} \cos^2 \varphi,$$

$$r^{q\bar{h}} = r^{ph} = 1 + \frac{K'}{J} \cos^2 \varphi. \tag{6d}$$

The point $\boldsymbol{h} = 0$ is now included into the first $(\boldsymbol{h})$ region.

The solution of (6) with respect (6a), (6b) and (6c) is given by

$$T_{\nu\tau}^{kh} = A_{\tau}^{kh} \cos \left[\alpha_{\tau}^{kh} \left(\nu - \frac{n+1}{2} \right) - \frac{\pi(\tau - 1)}{2} \right],$$

$$x_{\tau}^{kh} = 2 \cos \alpha_{\tau}^{kh}, \tag{7}$$

where α_{τ}^{kh} play the role of x-components of a spin wave vector. The solution for α_{τ}^{kh} depends on the surface anisotropy constant K'. For $K' > 0$ we obtain the surface modes (*cf.* [9]). The important role of this parameter K' in the resonance effects, especially, in the appearance of the surface resonance modes, is discussed in [10]. The various approaches to the resonance problem (*cf.* [11]) show that the surface anisotropy is responsible for the appearance of some of the resonance peaks which are observed in experiments. The surface parameter K' also influences the variation of magnetization across a film [12].

Eventually we obtain the diagonal form of the Hamiltonian (1) as follows:

$$H = H_0 + H_0' + \sum_{\tau h} E_{\tau h} \left(n_{\tau h} + \frac{1}{2} \right), \tag{8}$$

where

$$H_0 = -JS(S+1)\, N^2 n \left[\left(3 - \frac{1}{n} \right) + \frac{K}{J} \cos^2 \varphi - \frac{N^x}{J} \sin^2 \varphi + \frac{2}{n} \frac{K'}{J} \cos^2 \varphi \right],$$

$$H_0' = -JS^2 N^2 n \left\{ \frac{\sin^2 \varphi}{4} \frac{1}{n} \sum_{\tau} \frac{\left[\sum_{\nu} T_{\nu\tau}^{q0} (K + N^x) \frac{1}{J} + \frac{K'}{J} (T_{1\tau}^{q0} + T_{n\tau}^{q0}) \right]^2}{1 - \cos \alpha_{\tau}^{q0} + \frac{K + N^x}{J} \cos 2\varphi} \right\}$$

$$E_{\tau h} = JS \left[\varepsilon_{\tau h} + 2 \frac{K + N^x}{J} \cos 2\varphi \right]^{1/2} \left[\varepsilon_{\tau h} + 2 \frac{K + N^x}{J} \cos^2 \varphi - 2 \frac{N^x}{J} \right]^{1/2},$$

$$\varepsilon_{\tau h} = \sin^2 \frac{h_y a}{2} + \sin^2 \frac{h_z a}{2} + \sin^2 \frac{\alpha_{\tau}^{qh}}{2}.$$

In the above formula H_0' originates from the shift transformation reducing the linear terms. $n_{\tau h}$ denotes the mean number of spin waves.

The direction of magnetization can be obtained by minimizing the free energy of a sample with respect to the angle φ. At absolute zero temperature this corresponds to the minimization of (8) for $n_{\tau h} = 0$. The calculations lead to the following results:

The direction of magnetization depends on the anisotropy constants, the demagnetizing field and the film thickness. The conditions for occurrence of magnetization parallel or perpendicular to the film surfaces are found to be analogous to those discussed in [3]. However, in our theory, an additional condition should be taken into account, *i.e.* the spin wave spectrum should be positive; then $K > N^x \operatorname{tg}^2 \varphi$.

In our case of films with sufficiently strong uniaxial anisotropy in the film plane the above conditions are fulfilled if the magnetization is parallel with the surface of the film. This case (strong anisotropy) is especially interesting in investigations of domain structure in thin films; the domain in the form of stripes then appear and such structure is relatively simple to describe. The considerations of domain structure in the present paper are connected with the conditions in which the appearance of single domain films is possible.

3. *Domain structure of thin films*

In the following we assume there is a domain structure, which appears in thin films with sufficiently strong uniaxial anisotropy lying in the plane of film. The domains are stripes parallel to the magnetization. The width of the domain is Δ. The neighbouring domains with antiparallel magnetization are separated by a wall of width δ. We would like to notice here that the domain structure examined by us is different from that called in literature "stripe domain structure" in which the magnetization points out from the plane of a film due to strong anisotropy perpendicular to the film surfaces.

In the present paper the domain structure is considered as a screw structure with the angle $\varphi_{\nu j}$. In the case of our domains the angle distribution can be assumed, for the sake of simplicity, in the Néel aproximation: $\cos^2 \varphi_{\nu j} = 1$, inside the domains and $\varphi_{\nu j} = \frac{\pi}{\delta} y + \frac{\pi}{2}$ in the domain walls, where y is calculated from the wall midplane. This means that the domain structure can be considered as the screw structure in which the rotation angle of the quantization axis is a constant inside the domains and changes linearly inside the walls. Now, the coefficients $R^{\alpha\beta}_{\nu j}$ in (3) are functions of the y-component of the vector $\boldsymbol{j}(\varphi_{\nu j} = \varphi_y)$. We consider the Néel ($e^x_{\nu j} = 1, e^y_{\nu j} = e^z_{\nu j} = 0$) and the Bloch ($e^x_{\nu j} = e^z_{\nu j} = 0, e^y_{\nu j} = 1$) walls. Taking into account the mentioned Néel approximation for a distribution of the angle φ_y we can assume by analogy to Ziętek's calculations [6] that the angle φ_y changes very slowly between two neighbouring positions of spins. Then the following approximations are valid

1) $$\cos(\varphi_{y'} - \varphi_y) = \begin{cases} 1 & \text{inside the domains,} \\ 1 - \frac{1}{2} a^2 v_y^2 (\nu' \boldsymbol{j}' - \nu \boldsymbol{j}) \pi^2/\delta^2 & \text{inside the walls,} \end{cases}$$

2) $$\sin(\varphi_{y'} - \varphi_y) = \begin{cases} 0 & \text{inside the domains,} \\ a v_y (\nu' \boldsymbol{j}' - \nu \boldsymbol{j}) \pi/\delta & \text{inside the walls,} \end{cases}$$

3) $$\sin \varphi_y = 0 \qquad \text{inside the domains.}$$

Here a is the lattice constant and $v_y(\nu'j'-\nu j)$ denotes the direction cosine determining the position $(\nu'j')$ of the nearest neighbour to the atom (νj) with respect to the axis y of the coordinate system. In the case of the simple cubic structure we have 6 nearest neighbours with the coefficients $v_y(\nu'j'-\nu j) = >(0, 0, 1, -1, 0, 0)$.

We start from the Hamiltonian (1). Now the demagnetizing factors are different for spins in the domains and those in the domain walls. Their values can be determined by assuming approximately that the walls are ellipsoids with the axes δ, L_x, L_z, whereas the domains are ellipsoids with the axes Δ, L_x, L_z [13].

Substituting (4) and (3) into (1) and taking into consideration the approximations described in this chapter we can express the Hamiltonian (1) in terms of the creation and annihilation operators, neglecting the products of more than two operators, as follows

$$H = H_0 + H_1^1 + H_2^{+1} + H_2^{-1} + H_2^{+2} + H_2^{-2} + H_2^1 + H_2^2, \tag{9}$$

where

$$H_0 = -J \sum_{\langle \nu j, \nu' j' \rangle} S(S+1) \cos(\varphi_{y'} - \varphi_y) - \sum_{\nu j} S(S+1) K \cos^2 \varphi_y +$$

$$+ S(S+1) \sum_{\nu j} [N^z(y) \cos^2 \varphi_y + N^y(y) \sin^2 \varphi_y],$$

$$H_1^1 = i\sqrt{S/2}\, S \sum_{\nu j} [(K + N^y(y) - N^z(y)) \sin 2\varphi_y + 2J \sum_{\nu' j' \in \nu j} \sin(\varphi_y - \varphi_{y'})](a_{\nu j}^+ - a_{\nu j}^-),$$

$$H_2^{\pm 1} = \frac{S}{2} \sum_{\nu j} \{[N^x(y) - N^y(y)] \sin^2 \varphi_y + [K - N^z(y)] \sin^2 \varphi_y\} a_{\nu j}^{\pm} a_{\nu j}^{\pm},$$

$$H_2^{\pm 2} = -\frac{S}{2} J \sum_{\langle \nu j, \nu' j' \rangle} (1 - \cos(\varphi_{y'} - \varphi_y))\, a_{\nu j}^{\pm} a_{\nu' j'}^{\pm},$$

$$H_2^1 = S \sum_{\nu j} L^*(y)\, (a_{\nu j}^+ a_{\nu j}^- + a_{\nu j}^- a_{\nu j}^+),$$

$$H_2^2 = -\frac{S}{2} J \sum_{\langle \nu j, \nu' j' \rangle} (1 + \cos(\varphi_{y'} - \varphi_y))\, (a_{\nu j}^+ a_{\nu' j'}^- + a_{\nu j}^- a_{\nu' j'}^+),$$

$$L^*(y) = L(y) + \frac{1}{2} J \sum_{\nu' j' \in \nu j} \cos(\varphi_{y'} - \varphi_y),$$

$$L(y) = \left[K - N^z(y) + \frac{1}{2} N^y(y)\right] \cos^2 \varphi_y + \frac{N^x}{2} + \left[-\frac{1}{2} K + \frac{1}{2} N^z(y) - N^y(y)\right] \sin^2 \varphi_y.$$

Next we can apply the usual steps of diagonalization procedure described in the previous chapter and applied to films with domain structure in [4]. However, taking such procedure into account in the case of films with hexagonal structure, it was possible to diagonalize the

Hamiltonian only approximately [4]. In the case of films with the simple cubic structure and with $K' = 0$ the complete diagonalization can be obtained. At first there is to be made the transformation

$$a^{\pm}_{\nu j} = \sum_{\tau} T_{\nu\tau} a^{\pm}_{\tau j}, \tag{10}$$

where the coefficients $T_{\nu\tau}$ satisfy the conditions

$$\sum_{\nu} T_{\nu\tau} T_{\nu\tau'} = \delta_{\tau\tau'},$$

$$(1-x_\tau)T_{1\tau}+T_{2\tau} = 0,$$

$$(1-x_\tau)T_{n\nu}+T_{n-1,\tau} = 0,$$

$$T_{\nu+1,\tau}+T_{\nu-1,\tau}-x_\tau T_{\nu\tau} = 0.$$

They have the usual form [14]

$$T_{\nu\tau} = \sqrt{\frac{1-\delta_{1\tau}}{n}} \cos\left(\frac{\pi(\tau-1)}{n}\left(\nu-\frac{1}{2}\right)\right)$$

and

$$x_\tau = 2\cos\left(\frac{\pi(\tau-1)}{n}\right).$$

The introduction of the transformation (10) as the first step is possible for the Hamiltonian for which the coefficients $T_{\nu\tau}$ are independent of $\boldsymbol{h}$. This is satisfied, in particular, for films with the simple cubic structure and without surface anisotropy.

Next we transform the Hamiltonian in the plane of the film by two steps. We apply first the standard Fourier transformation in the direction of the magnetization

$$a^{\pm}_{\tau jyjz} = (1/N) \sum_{hz} e^{ihzjz} a^{\pm}_{\tau hzjy}. \tag{11}$$

By analogy to the calculations in the previous chapter (transformation steps c) and d) we isvide the region of propagation vectors in the film plane into two regions such that if h_z dilongs to the first region (h_z) then $-h_z$ belongs to the second one $(\bar{h}_z)$. The point $h_z = 0$ be excluded and treated separately [7]. Next we introduce the canonical conjugated operators $q_{\tau hzjy}$, $p_{\tau hzjy}$.

In the second step we introduce new transformation in the direction perpendicular to the domain walls:

$$q_{\tau hzjy} = \sum_{hy} \Gamma^{q}_{hy,y}(q_{\tau h}+R^{q}_{00hy}), \tag{12}$$

$$p_{\tau hzjy} = \sum_{hy} \Gamma^{p}_{hy,y}(p_{\tau h}+R^{p}_{00hy}).$$

In this manner the diagonalized Hamiltonian (1) becomes

$$H = H_0 + H_0' + \sum_{\tau h} E_{\tau h}\left(n_{\tau h} + \frac{1}{2}\right),$$

$$H_0' = -\frac{1}{4}\sum_{hy}[(R^q_{00hy})^2(E^q_{00hy})^{-1} + (R^p_{00hy})^2(E^p_{00hy})^{-1}],$$

$$E_{\tau h} = 2(E^q_{\tau h}E^p_{\tau h})^{1/2};\ R^{q,p}_{00hy} = \frac{S\sqrt{S}N\sqrt{n}}{2E^{q,p}_{00hy}}[K+N^y]\int_{-\delta/2}^{+\delta/2}\sin\frac{2\pi y}{\delta}\,\Gamma^{q,p}_{hy,y}\,dy \tag{13}$$

where H_0 is given by (9). The values $E^p_{\tau h}$, $E^q_{\tau h}$ and the transformation coefficients $\Gamma^q_{hy,y}$, $\Gamma^p_{hy,y}$ satisfy the following difference equations for h_z belonging to the first region

$$A^s \cdot JS[\Gamma^s_{hy,y-1} + \Gamma^s_{hy,y+1}] + [E^s_{\tau h} - Y^s_{\tau h}]\Gamma^s_{hy,y} = 0, \tag{14}$$

where $s = q$ or p and $A^q = 1$, $A^p = 1$ inside the domains and $A^p = 1 - \frac{\pi^2}{2}\left(\frac{a}{\delta}\right)^2$ inside the domain walls;

$$Y^s_{\tau h} = E_{\tau h_z} + L^s(y), \quad E_{\tau h_z} = 2JS\left[\sin^2\frac{h_z a}{2} + \sin^2\frac{\pi(\tau-1)}{2n}\right] + JSA^p,$$

$$L^q(y) = S\left\{\left[K - N^z(y) + \frac{1}{2}N^x(y)\right]\cdot\cos^2\varphi_y - N^y(y)\cdot\sin^2\varphi_y\right\},$$

$$L^p(y) = S\left\{\left[K - N^z(y) + N^y(y) - \frac{1}{2}N^x(y)\right]\cos^2\varphi_y + [N^z(y) - K - N^y(y)]\sin^2\varphi_y\right\}.$$

The coefficients $\overline{\Gamma}^s_{hy,y}$ for h_z belonging to the second region have the following properties

$$\overline{\Gamma}^q_{hy,y} = \Gamma^p_{hy,y},\ \overline{\Gamma}^p_{hy,y} = \Gamma^q_{hy,y} \tag{15}$$

and for $h_z = 0$ these coefficients are the same as for the first region. We remember that the described coefficients must fulfill the orthogonality conditions.

$$\sum_y \Gamma^s_{hy,y}\Gamma^s_{h'y,y} = \delta_{hy,h'y}. \tag{16}$$

To determine the spin wave eigenvalues (13) in the films with domain structure the parameters δ and Δ must be known. At absolute zero temperature these parameters can be calculated by minimizing of the energy (13) for $n_{\tau h} = 0$ with respect to δ and Δ. The energy of a sample found in this manner is the energy of the ground state in which the "zero point" fluctuations of spins are included. Taking into account our results obtained by the same method [15] for the case of simple hexagonal structure we can see that the contribution of the "zero point" fluctuations can be neglected for sufficiently thick films. To calculate the domain parameters in their explicit form the factors N^α must be known. For the sake of simplicity we assume for them the simple form used in the classical theory of domain walls: N^x, N^y inside the walls are the demagnetizing factors of an ellipsoid (*cf.* [16]); N^z inside the domains can be approximated by [17]: $N^z = 2\pi\mathfrak{z}\frac{L_x}{L_z^2}(\Delta-\delta)$. Minimizing H_0, we obtain the following equations for the domain parameters

$$\delta_N = \pi a\left(\frac{J}{K}\right)^{1/2}\left[1-4\frac{N^z}{K}+\frac{\pi\mathfrak{z}}{K}\frac{L_x^2}{(L_x+\delta_N)^2}\right]^{-1/2} \tag{17}$$

in the case of Néel walls and

$$\delta_B = \pi a\left(\frac{J}{K}\right)^{1/2}\left[1-4\frac{N^z}{K}+\frac{\pi_\mathfrak{z}}{K}\frac{\delta_B(\delta_B+2L_x)}{(L+\delta_B)^2}\right]^{-1/2} \tag{18}$$

in the case of Bloch walls. The domain width is

$$\Delta = \frac{L}{L_x^{1/2}}\left(\frac{K\delta}{4\pi\mathfrak{z}}\right)^{1/2}\left[1+\frac{J}{K}\frac{a^2\pi^2}{\delta^2}+\frac{4\pi\mathfrak{z}}{K}\frac{\delta L_x}{L_z^2}+\frac{N^\beta}{K}\right]^{1/2} \tag{19}$$

$$\beta = x \text{ or } y.$$

Analysis of the above equations leads to the following conclusions:

1) Domain wall width δ:

a) $N^z = 0$: our equations are reduced to the results of Middelhoek's theory [16];

b) to compare our results with those obtained for the simple hexagonal structure we must neglect the demagnetizing factors ($N^z = 0$, $\mathfrak{z} = 0$). Moreover, we have to remember that our formula for the Néel wall width in the case of simple hexagonal structure was obtained using the Hamiltonian with the pseudodipolar interaction C. Namely,[1]

$$\delta_N = \pi a\left(\frac{J}{C}\right)^{1/2}\left(\frac{1-\frac{1}{3n}}{1+\frac{1}{n}+\frac{2}{nS}}\right)^{1/2} \tag{20}$$

for sufficiently thick films. Looking at the formulae (17) and (20) we can conclude that the role of the anisotropy constants K and C is analogous; the pseudodipolar interactions are responsible for the appearance of the uniaxial anisotropy in hexagonal crystals.

2) Domain width Δ:

a) the demagnetizing field leads to the finite domain width;

b) the domain structure depends on the geometrical form of a sample (on the ratio L_y/L_z^x). Domains appear if $\Delta \leqslant L_y$. In the case of a square sample ($L_y = L_z$) the equation (19) gives

$$L_{x,\,\mathrm{crit}} \approx \frac{a}{\mathfrak{z}}\sqrt{2JK},$$

which determines the critical thickness of a below which only single domain films occur.

---

[1] This formula in [4] is given in an incomplete form. The physical conclusions remain the same as for the correct formula given in [18].

4. *Magnetization of the one domain film*

If we consider a thin film with sufficiently strong uniaxial anisotropy in the plane of a film, we can conclude that a single domain film can be observed in the interval $(0, L_{x,\text{crit}})$ of the film thickness. The critical thickness below which only one domain occurs is greater for films with stronger anisotropy. The magnetization is directed along the anisotropy axis and lies in the plane of a film. To direct the magnetization out from the plane of a film a perpendicular field which compensates the demagnetizing field has to be applied if the parallel anisotropy is small. But, in the case of small anisotropy parallel to the film, domain structures different from these considered in this paper may occur, and the conditions for magnetization direction in the single domain film become very complicated as they are a superposition of many concurrent processes.

Returning to our case of a single domain film with sufficiently strong anisotropy, we can get from (8) the spin wave spectrum in the following form

$$E_{\tau h} = JS\left[\varepsilon_{\tau h} + 2\,\frac{K+N^x}{J}\right]^{1/2}\left[\varepsilon_{\tau h} + \frac{2K}{J}\right]^{1/2} \tag{21}$$

and $\varepsilon_{\tau h}$ is the solution of (6) and (7) for $\varphi = 0$.

Magnetization M of a single domain film equivalent to spontaneous magnetization can be calculated by means of the standard procedure *i.e.* by derivating of free energy. This method applied to thin films is described in detail in [7]. Taking into account the spectrum (21) we obtain

$$\frac{M}{M_0} = 1 - \frac{k_B T}{4\pi JS^2}\frac{1}{n}\sum_{\tau=1}^{n} \ln \frac{1-\exp\left[-\dfrac{2JS}{k_B T}D(2\pi)\right]}{1-\exp\left[-\dfrac{2JS}{k_B T}D(0)\right]}, \tag{22}$$

where

$$D(x) = \left[\left(x + \sin^2\frac{\alpha_\tau^q}{2} + \frac{K+N^x}{J}\right)\left(x + \sin^2\frac{\alpha_\tau^q}{2} + \frac{K}{J}\right)\right]^{1/2},$$

M_0 is the value of magnetization at absolute zero temperature, T denotes the temperature of the film, and k_B is Boltzmann's constant.

The expression (22) shows the general dependence of the spontaneous magnetization on the temperature, on the thickness, the uniaxial anisotropy and the demagnetizing field. The dependence on the surface anisotropy is given by the terms α_τ^q which depend on the constant K' (Eq. (6)).

Taking into account the numerical results obtained in [7] for $\eta_{\|} = \dfrac{K}{J}$, $\eta_{\perp} = \dfrac{N^x}{J}$ we can conclude that the dependence of the magnetizartion on T at low temperatures is linear. The decrease of magnetization with temperature is the more pronounced the thinner the film. The anisotropy constant assures the convergence of the expression (12) and the strong anisotropy leads to smaller differences between the magnetization of a few layers and that of bulk bodies. The influence of the surface constant can be studied by taking into account the solutions for α_τ^q given in [9]. We find thus that the appearance of surface anisotropy leads to lowering of spontaneous magnetization if the other parameters remain the same.

The authors are grateful to Professor S. Szczeniowski for his interest in this paper.

REFERENCES

[1] A. Corciovei, *IEEE Trans. Mag.* MAG-4, 6 (1968).
[2] L. Wojtczak, *Acta Phys. Polon.*, **28**, 25 (1965); **31**, 441 (1967).
[3] A. Corciovei, D. Vamanu, *J. Appl. Phys.*, **39**, 1381 (1968); *Rev. Roumaine Phys.*, **7**, 629 (1967).
[4] L. Wojtczak, A. Sukiennicki, *Acta Phys. Polon.*, **34**, 355 (1968).
[5] L. Valenta, *Czech. J. Phys.*, **7**, 127 (1957).
[6] W. Ziętek, *Acta Phys. Polon.*, **21**, 175 (1962).
[7] A. Corciovei, *Acta Phys. Polon.*, **24**, 91 (1963).
[8] A. Corciovei, *Phys. Rev.*, **130**, 2223 (1963); L. Valenta, L. Wojtczak, *Proc. Conf. Magnetism*, Nottingham 1964.
[9] L. Wojtczak, *Rev. Roumaine Phys.*, **12**, 577 (1967).
[10] H. Puszkarski, *Acta Phys. Polon.*, **33**, 769 (1968).
[11] R. Soohoo, *Magnetic Thin Films*, Harper and Row, New York 1965.
[12] A. R. Ferchmin, *Acta Phys. Polon.*, **32**, 289 (1967); A. Corciovei, D. Vamanu, Communication to the 1969 INTERMAG, Conference in Amsterdam.
[13] A. Sukiennicki, L. Wojtczak, *Rev. Roumaine Phys.*, **15**, 163 (1970).
[14] W. Brodkorb, W. Haubenreisser, *Phys. Status Solidi*, **8**, K21 (1965).
[15] L. Wojtczak, A. Sukiennicki, *Acta Phys. Polon.*, **33**, 1013 (1968).
[16] S. Middelhoek, *J. Appl. Phys.*, **34**, 1054 (1963).
[17] A. Sukiennicki, L. Wojtczak, *Phys. Status Solidi*, **25**, K19 (1968).
[18] A. Sukiennicki, *Prace Instytutu Fizyki*, Politechnika Warszawska, Nr 2 (1969), in Polish.