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CROSS-SECTION FOR ELECTRON-DEUTERON ELASTIC SCATTERING AT VERY LARGE MOMENTUM-TRANSFER

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The cross-section of electron-deuteron elastic scattering is evaluated at very large momentum-transfer, assuming that the internal nonrelativistic wave function of deuteron is an absolute, *i. e.* independent of the Lorentz frames of reference, characteristic of deuteron. This assumption is a consequence of the geometrical hypothesis — called the phase geometry — which is briefly presented in the first part of the paper.

1. *Introduction*

Large momentum-transfer scattering of electrons from deuterons is one of the processes where the relativistic dynamical theory of composite systems — here of deuteron — becomes especially urgent. This is so, because the structure of deuteron (*i.e.* its ground state wave function) is relatively well known, as well as the electron-deuteron interaction which is dominated by pure electromagnetic forces. Therefore electron-deuteron scattering is relatively well suited for measurement of the deuteron vertex function. The dynamical theory should just determine this vertex function or, which is equivalent, the relativistically invariant form factors of deuteron.

It is remarkable that the same problem when the incident particle is strongly interacting is much more involved, because its interaction is poorly known (at high incident energy), hence the separation of the interaction from the structure of the interacting particles is not well defined. Thus at present we can prove or disprove some hypothesis concerning the structure of the form factors at very large momentum transfers mostly with the help of the electromagnetic interactions.

From general symmetries, such as the Lorentz invariance, time reversal, and the gauge invariance, combined with the assumption of the Born approximation — which is very well justified in electron collisions with light nuclei [1] — one can draw a general expression for the electron-deuteron scattering amplitude which is the analogue of the Rosenbluth formula for particles of spin one [2], [3]. Since the corresponding cross-section deals with

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relativistically invariant form factors, it certainly does not impose any restriction upon the magnitude of the momentum transfer. However, the shape of these form factors remains undetermined, much like in the case of the Rosenbluth cross-section, as long as we have no satisfactory dynamical, relativistic theory of composite systems.

In low energy physics, the Schroedinger equation gives a complete account for the dynamics of the collision process together with the structure of the interacting entities. Even in high-energy collisions we still have a reliable theory as long, as the recoil of the target particle remains "nonrelativistic", *i.e.* $|t| \ll M^2$, where M is the mass of the target particle. For example, the "eikonal" approximation proposed by Glauber [4] gives powerful a tool in describing such collisions. This approximation can be even applied to high-energy collisions of strongly interacting particles when the Born approximation is not sufficient [5]. The fundamental difficulty always arises when the parameter t/M^2 cannot be neglected, *i.e.* when the recoil of the target particle — in our case of deuteron — becomes "relativistic".

The attempts at evaluating, in some approximation, the relativistic form factors of deuteron follow different ways. One of them is mostly based on dispersion relations [6]. However, this method has a tendency to mix up the internal structure of the interacting particles with the over-all interaction. This implies that any theory or model capable of predicting the internal structure of interacting systems (particles) is hardly translatable into the dispersion relation language.

One of the most interesting approaches to the problem in question is based on the so called "relativistic wave functions". The idea of relativistic generalization of the concept of the wave function has been proposed by Blankenbeckler and Cook, and corroborated by Cutkosky [7], and different other people [8]. In particular, it has been applied by Gross [9] to loosely bound sytems. The Lorentz-covariant generalization of the wave function notion can be, in principle, done on the basis of the field theory (*e.g.* making use of the Bethe--Salpeter equation) or dispersion relations. The relativistic wave function, much like its nonrelativistic (NR) counterpart in low energy physics, should describe all the properties of the interacting particles which are necessary to an interpretation of the experiment. Thus this scheme parallels that of elementary wave mechanics. Of course, the covariant wave function must account for the relativistic effects, in particular, for the most fundamental kinematical effect of the Lorentz–FitzGerald contraction of the system under description. Basing on this idea, Gross [10] has evaluated the electron-deuteron elastic cross-section in the first-order approximation of the parameter t/M^2, where M denotes the deuteron mass.

In a series of papers [11] we have proposed some geometrical framework — called the phase geometry — which, in contrast to the relativity theory, implies that the internal wave function of a composite system is its absolute characteristic, similarly as its invariant mass. If so, the very idea of the Lorentz-covariant internal wave function is wrong. In Chapter 2 a brief survey of the phase geometry framework is presented. In particular, it provides us with the recipe of how to construct the deuteron vertex function. It will become apparent that the modifications implied by the phase geometry (PG) framework come together with the "relativistic recoil" of deuteron.

The final formula for electron-deuteron cross-section requires the following assumptions to be fulfilled. *1°* The deuteron structure is described within the PG framework. This point will be elucidated in Chapter 2, and assumption *1°* will be formulated at the end of this Chapter. *2°*. The Born approximation which, as we have already mentioned [1], is very well justified because of the small electromagnetic coupling constant. *3°*. The one-photon exchange as responsible for the interaction between electron and the nucleons in deuteron. Moreover, we use the nucleon form factors from the Rosenbluth cross-section for electron-nucleon elastic cross-section. *4°*. The interaction between electron and deuteron is assumed to be equal to the sum of the interactions between electron and proton, and electron and -neutron. This is the basic assumption of the impulse approximation which can be found *e. g.* in the classical paper on electron-deuteron collision of Jankus [12], and which we call the additivity assumption. Thus all "collective" interactions due to the deuteron as a whole are neglected. In particular, we neglect all corrections due to the meson exchange currents in deuteron. They have been discussed by several authors [13], but the quantitative estimation of these corrections are rather vague. For the momentum-transfer not too large, $|t| \lesssim 10\ F^{-2}$, they seem to be negligible [14], nevertheless it is an open question whether all "collective" interactions remain negligible for large $|t|$. If not, they should make the verification of any hypothesis concerning the deuteron vertex function, very difficult. The situation then should become similar to that of strongly interacting particles. Thus assumption *4°* is to be regarded as a hypothesis which, it is claimed, is plausible for ultra-high momentum transfer ($|t| \gtrsim M^2$), when we are very far from the diffraction peak.

These four assumptions enable us to evaluate the cross-section for elastic electron-deuteron collisions at arbitrary large $|t|$.

The electron-deuteron cross-section determines, although indirectly, the neutron form factors at any value of t. Apart from the technical difficulties of this procedure, due mainly to the small deuteron form factor, there is one advantage which results from simple kinematics. The maximum value of the momentum-transfer $|t|$ is equal to

$$(-t)_{\max} = \frac{2ME}{1+M/2E},$$

where E is the laboratory energy of electron (whose mass is always neglected), and M is the mass of the targer particle. If $E \gg M$, then $(-t)_{\max}$ is, at fixed value of E, almost twice larger for electron-deuteron than for electron-nucleon collision. As t is a common argument of all form factors which enter into the cross-section, one obtains the nucleon form factors for $|t|$ almost twice larger from electron-deuteron than from electron-free-nucleon collision.

We do not discuss in this paper the polarization effects of deuterons or electrons, although they can be easily evaluated within our approach. The summation procedure over all polarization states of electron and deuteron, which deserves some caution, leads us to the differential cross-section for the angular distribution of electrons.

Finally, in Chapter 4 a brief comment is given concerning a very important problem of the relationship between the space-like and the time-like behaviour of the relativistic form factors. In connection with this, some simple explanation of the so called "dipole" fit of the nucleon form factors is proposed which is based on the phase geometry framework.

2. Phase Geometry

2A. An idea of new parametrization

Let us consider an isolated system S consisting of N loosely bound particles which fact justifies to treat S nonrelativistically. The total hamiltonian $\hat{H}^G$ (the superscript "G" means "Galilean") takes the following general form

$$\hat{H}^G = (\hat{\boldsymbol{P}}^G)^2/2m + \hat{h}^G(\hat{\boldsymbol{y}}_1^G, \ldots, \hat{\boldsymbol{y}}_{N-1}^G; \hat{\boldsymbol{q}}_1^G, \ldots, \hat{\boldsymbol{q}}_{N-1}^G), \tag{2A.1}$$

where $m = \sum_{A/1}^{N} m_A$ is the total mass of S, $\hat{\boldsymbol{y}}_B^G$ and $\hat{\boldsymbol{q}}_B^G$ $(B = 1, \ldots, N-1)$ are some relative coordinates of the constituents of S, and their canonically conjugate momenta, respectively, and $\hat{\boldsymbol{P}}^G$ is the total momentum operator of S [1]. The whole parametrical dependence of $\hat{H}^G$ on the external (here Galilean) reference frames is manifested through the external hamiltonian $(\hat{\boldsymbol{P}}^G)^2/2m$, whereas the internal hamiltonian $\hat{h}^G$ is a Galilean-invariant operator, as it is parametrized in terms of the Galilean-absolute relative phase coordinates. Thus the Schroedinger equation of the internal motion of S

$$\hat{h}^G \psi_{wG}^G = w^G \psi_{wG}^G \tag{2A.2}$$

is Galilean-invariant, hence it provides us with the set of the Galilean-invariant eigenvalues w^G_k, $w^G = G$-inv.

Let us suppose now that the same system S is considered in a reference frame where its velocity is comparable with that of the light. Then we must apply the relativistic kinematics to the external motion of S. Let P_μ be the four-momentum of S as a whole, $P_\mu = \left(\boldsymbol{P}, \frac{i}{c} E\right)$. The Lorentz-invariant mass M of S then is equal to

$$M = \frac{1}{c}(-P_\mu^2)^{1/2} = \frac{1}{c}(E^2/c^2 - \boldsymbol{P}^2)^{1/2} = L\text{-inv.}. \tag{2A.3}$$

Since S is a loosely bound system, we have that

$$M = m + \frac{w^G}{c^2} + O\left(\frac{1}{c^4}\right), \tag{2A.4}$$

However, M and m are Lorentz-invariants, and $O\left(\frac{1}{c^4}\right)$ is a negligible correction, therefore (2A.4) proves that w^G is an absolute quantity not only of the Galilean, but also of the Lorentz geometry, *i. e.*

$$w^G = w = L\text{-inv.} \tag{2A.5}$$

In fact, the external motion of S as a whole does not affect O, independently of the magni-

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[1] By $\hat{a}$ we always denote the q-number, while by a its c-number eigenvalue or the corresponding classical quantity.

tude of O. Consequently, the internal NR eigenfunctions ψ^G_{wG} must be the characteristics of S which are also insensitive (as the mathematical functions) not only to the Galilean, but also to the Lorentz frames of reference, much like the eigenvalues $w^G = w$. Thus, according to the completeness of the eigenstates ψ^G_{wG} of $\hat{h}^G$, for any internal wave function ψ^G we have that

$$\psi^G = \psi = \text{Lorentz-absolute quantity.} \tag{2A.6}$$

By omitting the superscript "G" we mean that the corresponding quantity is considered within the theory which accounts for the finitness of c. We deliberately do not call such a theory the relativity theory, because, as we try to show, the statement (2A.6) conflicts with the Lorentz geometry of the relativity theory.

Let us illustrate this question in the most simple example when S is a system composed of two spinless particles in the S-state. Then the internal wave function of S is a scalar function of spherical symmetry, $\psi^G(r^G)=\psi(r)$. The meaning of r^G is clear, as we have $r^G=|\boldsymbol{x}^G_2-\boldsymbol{x}^G_1|$, where $\boldsymbol{x}^G_{1,2}$ are the Galilean space coordinates of the constituents of S. The meaning of r which is the argument of the Lorentz-absolute function ψ will become clear from further considerations. Note that within the Lorentz geometry, the Lorentz-absolute nature of ψ implies that ψ must be generalized into a Lorentz-invariant function $\Psi(x^2_\mu)$ $(x^2_\mu=(\Delta\boldsymbol{x})^2-c^2(\Delta t)^2)$, as x^2_μ is the only Lorentz-invariant interval which generalizes the Galilean-invariant interval r^G. Let us postpone to Chapter $2C$ the question of the relationship between ψ and Ψ. The point is that independently of the details of this relation, $\Psi(x^2_\mu)$, in contrast to the NR internal wave function $\psi^G(r^G)$, does not account for the fact that S is a system which is stable in time. This is due to the indefinite metric of the Lorentz geometry.

On the other hand, we have the Lorentz-covariant generalization of the NR internal wave functions [7, 8, 9] which accounts for the aforementioned stability of the described system, but these functions cease to be Lorentz-absolute. Indeed, the relativistic wave functions, as the solutions of some relativistically covariant equation — *e. g.* the Bethe-Salpeter equation — account automatically for the Lorentz contraction of the described system S. Thus they are not Lorentz-invariant (-absolute) characteristics of S any more.

In order to reconcile both requirements: A) the stability of the described system, and B) the Lorentz-absolute nature of ψ, *i. e.* the parametrical independence of ψ on the external Lorentz frames of reference, one is forced to introduce a new geometry which we call the phase geometry. The continuum of the phase geometry will be regarded henceforth as the first physical continuum which means that the space-time continuum ceases to be regarded as the fundamental physical continuum. We shall show that the latter is the limiting case of the x-continuum of the phase geometry conditioned by the structure of the physical system under consideration. In particular, the Lorentz space-time continuum governs all directly measurable quantities.

The phase continuum of the phase geometry (PG) is parametrized by six q-number coordinates, and the internal time τ satisfying the following four postulates:

1. The Cartezian representation of the phase coordinates $\hat{y}_j$, $\hat{q}_j$ $(j, k, \ldots = 1, 2, 3)$ fulfil the canonical commutation relations

$$[\hat{y}_j, \hat{y}_k] = [\hat{q}_j, \hat{q}_k] = 0, \quad [\hat{y}_j, \hat{q}_k] = i\hbar\, \delta_{jk}, \tag{2A.7}$$

where the square of the x-interval $\hat{r}^2$ is given by the positive-definite form $\hat{\boldsymbol{y}}^2 = \hat{y}_j \hat{y}_j$ (sum over j).

2. Let us assume for the moment that the Lorentz geometry of the relativity theory rules all directly measurable — we call them external — quantities. Let $\hat{H}$, $\hat{P}_j$, $\hat{J}_j$ and $\hat{K}_j$ be the ten generators of the Lorentz group. For any complete theory, *i. e.* for any theory which does not introduce any external field, the eigenvalues of these ten generators lead us to ten integrals of motion which implies that the system under consideration is completely isolated. The general idea of the PG consists in that that it provides us with the parametrization of the internal degrees of freedom of all isolated systems, and that this parametrization is *a priori* cut-off from the Lorentz parametrization of the external, directly measurable quantities. Let us explain this in more details. Note first that any isolated system can be enlarged by adding to it another isolated system. However, independently of how involved is this composite system, any measurement, as such, remains "external" with regard to all isolated systems [15]. Measurement means namely an irreversible, actualized process of the reduction of the wave packet which is of entirely different nature than the unitary, hence time-reversible, development of an isolated system [16]. Thus there is no inconsistency in that that the same object when measured from the outside (by heavy apparatus) is contracted, but when, as a constituent of an isolated system, it collides with another constituent of the same system, exhibits its absolute, internal structure. These are two entirely different processes. In the first, we must ascertain two coincidences of the edges of this body with the external rods, and these are irreversible processes which result in the determination of the usual length [17]. In the second case we measure the invariant momentum transfer of the recoilled entity, and indirectly we determine the absolute internal shape of this body [17]. Thus the consistency of the PG picture is "protected" by the quantum difference between what Heisenberg calls the "actual" and "potential" [15].

The above considerations can be formulated as follows: The phase geometry coordinates (operators!) $\hat{y}_j$ and $\hat{q}_j$ commute with all generators of the Lorentz-Poincaré group

$$[\hat{y}_j, \hat{H}] = [\hat{q}_j, \hat{H}] = [\hat{y}_j, \hat{P}_k] = [\hat{q}_j, \hat{P}_k] = 0$$

$$[\hat{y}_j, \hat{J}_k] = [\hat{q}_j, \hat{J}_k] = [\hat{y}_j, \hat{K}_k] = [\hat{q}_j, \hat{K}_k] = 0 \tag{2A.8}$$

Eq. (2A.8) implies that all scalars constructed from the three-dimensional vectors of the PG, like the fundamental scalars $\hat{y}_j\hat{y}_j$, $\hat{y}_j\hat{q}_j$, $\hat{q}_j\hat{q}_j$, are the Lorentz-absolute quantities, as they commute with all Lorentz generators. We shall show that these scalars cannot be *a priori* expressed by the Lorentz geometry objects, and therefore the phase geometry parametrization is not isomorphic with that of the Lorentz geometry.

Let us emphasize that according to the commutation relations (2A.8) the rotation generators $\hat{J}_k$ do not generate the PG internal rotations. The point is that *a priori* there is no connection between the two, external and internal continua, in particular, between the space orientations of the internal (PG) and the external (Lorentz geometry) vectors. We show in the next Chapter 2B that such connection exists but *a posteriori* and it is established *via* the real asymptotic states.

3. The internal, absolute time τ can be defined by the internal Schroedinger equation

in the PG continuum, namely

$$i\hbar \frac{\partial \psi}{\partial \tau} = \hat{h}\psi, \qquad (2A.9)$$

where $\hat{h}$ is the internal hamiltonian parametrized by the PG coordinates. This hamiltonian generalizes the NR internal hamiltonian $\hat{h}^G$ — *cf.* Eq. (2A.1). The eigenvalues W of $\hat{h}$ are the rest-energies of the isolated system under description, *i. e.* $W = Mc^2$, where M is the Lorentz-invariant mass of that system.

4. In the case of free-particle systems, laws of motion which result from the PG framework are equivalent with these based on the classical (Lorentz) space-time continuum. In other words, the PG framework maintains the relativistic kinematics.

2B. Lorentz and phase geometries

Basing on four assumptions formulated in the previous Chapter 2A one can prove that in the NR limit ($c \to \infty$) the PG coordinate $\boldsymbol{y}$ becomes isomorphic with the Galilean relative space coordinate $\boldsymbol{y}^G$, and the internal time τ becomes isomorphic with the absolute Newtonian time t^G [11]. Thus the NR phase geometry is isomorphic with the Galilean geometry of the classical space-time. This is due to the absolute character of the Newtonian space and time, each separately, which implies that we deal with two invariant (Galilean-absolute) intervals

$$r^G = |\boldsymbol{y}^G| = G\text{-inv.}, \text{ and } \Delta t^G = G\text{-inv.}, \qquad (2B.1)$$

corresponding to the Lorentz-absolute intervals $r = |\boldsymbol{y}|$, and $\Delta\tau$, respectively, of the PG. In the relativity theory the situation is quite different, as we have only one invariant interval x_μ^2.

Another limiting case of the PG coincides with the Lorentz geometry. We shall show this in a simple example which at the same time will illustrate the PG approach to the problem of motion. Let us consider the two-body system S of spin-less particles. The internal PG hamiltonian $\hat{h}$ of S can be taken in the following form ($c = 1$)

$$\hat{h} = (m_1^2 + \hat{\boldsymbol{q}}^2)^{1/2} + (m_2^2 + \hat{\boldsymbol{q}}^2)^{1/2} + \hat{U}(\hat{r}), \qquad (2B.2)$$

where $\hat{\boldsymbol{q}}$ is the PG momentum canonically conjugate to $\hat{\boldsymbol{y}}$, $r = |\hat{\boldsymbol{y}}|$, and $\hat{U}(\hat{r})$ is a scalar function of $\hat{r}$ which vanishes when $r \to \infty$. This function generalizes the NR potential. The Schroedinger equation (2A.9) results in the eigenproblem for the invariant mass M of S

$$\hat{h}\psi = M\psi. \qquad (2B.3)$$

For the sake of simplicity (in order not do deal with the infinite representation of the rotation group) let us assume that ψ is also the eigenstate of the internal angular-momentum square $\hat{\boldsymbol{l}}^2$, and $\hat{l}_3$, where

$$\hat{l}_j = e_{jks}\hat{y}_k\hat{q}_s \qquad ([\hat{h}, \hat{l}_j] = 0). \qquad (2B.4)$$

Then ψ takes the following form ($\hbar = 1$)

$$\psi(\boldsymbol{y}, \tau) = R_{nl}(r)\, Y_{lm}(\theta, \varphi) \exp(-iM\tau). \qquad (2B.5)$$

Now, *i. e. a posteriori* when the internal structure of S is realized, we construct the corresponding ten generators of the Lorentz-Poincaré group. We have namely

$$\hat{H} = (M^2+\hat{\boldsymbol{P}}^2)^{1/2},\ \hat{P}_j,\ \hat{J}_j = e_{jks}\hat{X}_k\hat{P}_s + L_j^{(l)},\ \hat{K}_j = \hat{X}_j\,(M^2+\hat{\boldsymbol{P}}^2)^{1/2}. \tag{2B.6}$$

Here $\hat{P}_j$ is the total momentum operator of S, and $\hat{X}_j$ is the coordinate canocinally conjugate to $\hat{P}_j$, *i. e.*

$$[\hat{X}_j, \hat{X}_k] = [\hat{P}_j, \hat{P}_k] = 0, \quad [\hat{X}_j, \hat{P}_k] = i\hbar\,\delta_{jk}. \tag{2B.7}$$

Since $\hat{\boldsymbol{l}}^2\psi = l(l+1)\,\psi$, we have that $(L^{(l)})^2 = l(l+1)$, where $L_j^{(l)}$ are three $(2l+1)$-dimensional matrices which represent the rotation group. The essential point is that that the quantities M and $L_j^{(l)}$ are c-numbers, as they describe a definite eigenstate ψ of the internal motion of S. Therefore the generators (2B.6) commute with the internal PG coordinates $\hat{y}_j$ and $\hat{q}_j$ which stands in accordance with the commutation relations (2A.8). Note that exactly in the same way, *i. e* a posteriori, one constructs the Lorentz-Poincaré generators for a particle of mass M and spin l, both taken from an experiment.

On having these generators we have the "external" equations of motion for the composite particle S, namely

$$i\hbar\frac{\partial}{\partial t}\,\psi(\boldsymbol{X}, t) = \hat{H}\psi(\boldsymbol{X}, t). \tag{2B.8}$$

This equation is of course Lorentz-covariant, hence $\boldsymbol{X}$, t denote the Lorentz four-point representing S as a whole, from the outside of S.

In the usual picture which is based on the universal space-time continuum there is no room for the presented above "hierarchic" description of the motion of S. Here we must start from the two-body hamiltonian parametrized by the Lorentz variables

$$\hat{H}^L = (m_1^2+\hat{\boldsymbol{p}}_1^2)^{1/2} + (m_2^2+\hat{\boldsymbol{p}}_2^2)^{1/2} + \hat{U}(\hat{\boldsymbol{x}}_2-\hat{\boldsymbol{x}}_1). \tag{2B.9}$$

Therefore $\hat{H}^L$ never commutes with the relative Lorentz coordinate $\hat{\boldsymbol{x}} = \hat{\boldsymbol{x}}_2 - \hat{\boldsymbol{x}}_1$

$$[\hat{H}^L, \hat{\boldsymbol{x}}^L] \neq 0. \tag{2B.10}$$

On the other hand, by comparing the hamiltonians (2B.2) with (2B.9) we infer the following c-number equalities

$$D_{jk}q_k = q_j^L \underset{\mathrm{df}}{=} p_{2j}|_{\boldsymbol{P}=0}, \qquad \text{and } D_{jk}y_k = y_j^L \underset{\mathrm{df}}{=} (x_{2j}-x_{1j})|_{\boldsymbol{P}=0}, \tag{2B.11}$$

where D is an arbitrary orthogonal matrix. Since from Eq. (2A.8)we have that $[\hat{H}, \hat{y}_j] = 0$, we see from Eq. (2B.10) that in spite of the numerical equalities (2B.11) the geometrical nature of the PG coordinate $\boldsymbol{y}$ is different from that of the space part of the Lorentz four-vector $x_\mu = (\boldsymbol{x}_2 - \boldsymbol{x}_1, i(t_2-t_1))$.

In the scattering problem the asymptotic states of S are directly measurable, hence simultaneously parametrizable in terms of both, the phase and the Lorentz geometries. Therefore they establish *a posteriori* (*i. e.*, when the asymptotic state of S is given) a definite relation between the space orientations of the external (Lorentz) and the internal (PG) reference frames. Then we can put $D = 1$, and we have

$$q_j = q_j^L \text{ and } y_j = y_j^L. \tag{2B.11'}$$

Now, the rotation of external devices (Lorentz frames) performed by some orthogonal matrix induces the same rotation of the internal (PG) reference frames. However, this relation is due to the boundary conditions which are the asymptotic states. As such, it is given *a posteriori,* and it is consistent with the commutation relations (2A.10) which tell only that the Lorentz generators J_1 do not induce *a priori* the internal (PG) rotations. Thus the quantities such as $(\boldsymbol{y}^L)^2$, $\boldsymbol{y}^L\boldsymbol{q}^L$ or $(\boldsymbol{q}^L)^2$ are the Lorentz-invariants, but *a posteriori,* as the c-number condition $\boldsymbol{P}=0$ is unanvoidable in determining y_j^L and q_j^L. At the same time, the corresponding scalars of the PG, namely $\boldsymbol{y}^2$, $\boldsymbol{yq}$ and $\boldsymbol{q}^2$ are also the Lorentz-absolute quantities, but *a priori.* Similarly, the quantities $(\boldsymbol{y}^G)^2$, $\boldsymbol{y}^G\boldsymbol{q}^G$ and $(\boldsymbol{q}^G)^2$ are Galilean-invariants *a priori,* because the boundary condition $\boldsymbol{P}=0$ is superfluous in determining $\boldsymbol{y}^G$ and $\boldsymbol{q}^G$ from the Galilean coordinates. This reflects the already well known fact that in the NR limit the PG is isomorphic with the Galilean geometry of space-time continuum.

Eq. (2B.11′) clearly shows that within the classical framework ($\hbar=0$) there is no room for other fundamental continuum than the space-time. Indeed, here the positions and momenta are *a priori* sharply determined ($\hbar=0$), and therefore the notion of the relative coordinate must be secondary to the notion of the cooridnates $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ which are the ends of $\boldsymbol{y}$. Since $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are the Lorentz coordinates, the coordinate $\boldsymbol{y}$, as a geometrical object, must be identified with $\boldsymbol{y}^L$, *i. e.* with space part of the Lorentz four-vector $x_\mu = x_{2\mu}-x_{1\mu}$ in the CM-system. Within the quantum physics the situation is quite different. The boundary condition $\boldsymbol{P}=0$ which is necessary in determining $\boldsymbol{y}^L$ implies that the uncertainty of the localization $\boldsymbol{X}$ is infinite, $\Delta\boldsymbol{X}=\infty$. Thus the interval $r=|\boldsymbol{y}|$ of the PG, and similarly the interval $\Delta\tau$ of the PG time, both are *a priori* completely unlocalized with respect to the external (Lorentz) measuring rods and clocks, respectively. If so, there is no necessity to identify the geometrical natures of $\boldsymbol{y}$ and $\boldsymbol{y}^L$; $\boldsymbol{y}$ is absolute *a priori,* while $\boldsymbol{y}^L$ is absolute *a posteriori,* as the Lorentz relative coordinate in a given reference frame. Eq. (2B.11′) tells only that the numerical values of $\boldsymbol{y}$ and $\boldsymbol{y}^L$ coincide.

From the aforementioned we see that the modifications implied by the PG come with finite c, but the very possibility of the PG hypothesis is due to the second universal constant $\hbar$.

Let us show now that the Lorentz geometry can be regarded as the limiting case of the PG. For this purpose we consider the PG hamiltonian (2B.2), and we perform the following limiting procedure

$$\hat{h}^L = \lim_{m_1\to\infty}(\hat{h}-m_1) = (\hat{\boldsymbol{q}}^2+m_2^2)^{1/2}+U(\hat{\boldsymbol{y}}). \tag{2B.12}$$

The limiting hamiltonian $\hat{h}^L$ is identical with the relativistic one-body hamiltonian, and so it results in equations of motion for particle "m_2" which are covariant under the Lorentz group. Thus the heretofore absolute quantities of the PG, like $\boldsymbol{y}$ or U, can be relativized according to the symmetry group of the equations of motion, *i. e.* according to the Lorentz group. In particular, U must be identified with the fourth component of the four-vector U_μ. The four-potential U_μ then becomes an external field which is time-independent in the rest-frame Σ_0 of infinitely heavy particle "m_1", and in Σ_0 it takes the form $U_\mu|_{\Sigma_0}=(\boldsymbol{O},\, iU)$. The infinitely heavy particle "m_1" has dropped from the equations of motion, but it has generated the Lorentz space-time continuum. Thus in this limit, the x-continuum of the PG becomes isomorphic with the Lorentz space-time.

From the viewpoint of the PG upon the relativity theory, the relativity together with the calssical continuum of events x_μ is secondary to the absolute continuum of the PG. In particular, the measuring devices play the role of such infinitely heavy bodies which generate the classical space-time, hence the relativity theory.

Let us emphasize that within the physical framework which takes into account the finitness of $\hbar$ the procedure of the determination of a four-point x_μ requires infinite energy-momentum; $\Delta t = 0$, hence $\Delta E = \infty$. On the other side, the finitness of c implies that any isolated system is of finite energy:, $W = Mc^2 < \infty$. Therefore it seems to us that within the theory which accounts for the finitness of $\hbar/c$ the four-points x_μ are wrong variables for the parametrization of internal structure of any isolated system. The parametrization given by the phase continuum of the PG eliminates this dilemma, and consequently modifies the physical description of these processes where the universal constant $\hbar/c$ cannot be neglected.

2C. Absolute functions

Let us now consider the question of the relationship between the Lorentz-absolute functions in the PG continuum, and the Lorentz-absolute functions in space-time, *i. e.* the Lorentz-invariant functions. For the sake of simplicity, we shall discuss the PG Lorentz-absolute functions which depend on $r = |\boldsymbol{y}|$, and the corresponding Lorentz-invariant functions which depend on x_μ^2.

Let us denote by $\Psi(x_\mu^2)$ the Lorentz-invariant function. Its four-dimensional Fourier transform $\Psi(p_\mu^2)$ is given by

$$\Psi(p_\mu^2) = \int d^4x \Psi(x_\mu^2) \exp(-ip_\mu x_\mu).$$

By putting $p_4 = 0$ we define the three-dimensional absolute momentum space $\boldsymbol{q}$ for the space-like four-vectors p_μ; $p_\mu^2 = \boldsymbol{q}^2 \geqslant 0$. This three-dimensional absolute momentum space $\boldsymbol{q}$ is identified with the momentum continuum of the PG, and according to this fact we can determine the Lorentz-absolute function of the PG in the momentum representation

$$\psi(\boldsymbol{q}^2) = \Psi(p_\mu^2 = \boldsymbol{q}^2).$$

By virtue of the canonical commutation relations (2A.7) the x-representation of the Lorentz-absolute function ψ is given by the three-dimensional Fourier transform

$$\psi(\boldsymbol{y}) = \psi(r) = (2\pi)^{-3} \int d^3q \psi(\boldsymbol{q}^2) \exp(i\boldsymbol{q}\boldsymbol{y}) \quad (r = |\boldsymbol{y}|). \tag{2C.1}$$

Vice-versa, on having $\psi(r)$ we find its three-dimensional Fourier transform $\psi(\boldsymbol{q}^2)$, and we extend it analytically for all p_μ (*i. e.* for $p_\mu^2 < 0$) by inserting p_μ^2 in place of $\boldsymbol{q}^2$. In this way we obtain the Lorentz-invariant function in the momentum representation, namely

$$\Psi(p_\mu^2) = \psi(\boldsymbol{q}^2 = p_\mu^2).$$

Finally, the four-dimensional Fourier transform of $\Psi(p_\mu^2)$ determines the corresponding Lorentz-invariant function in the x-representation, *i. e.* in the Lorentz space-time,

$$\Psi(x_\mu^2) = (2\pi)^{-4} \int d^4p \Psi(p_\mu^2) \exp(ip_\mu x_\mu). \tag{2C.2}$$

In high-energy physics one very often encounters with the presented here procedure of determining the Lorentz-absolute function $\psi(r)$ from the Lorentz-invariant function $\Psi(x_\mu^2)$. In particular, in electron-proton elastic collision one deals with the Lorentz-invariant electirc form factor G_E which determines the proper charge distribution of proton $e(r)$, exactly in the way indicated in (2C.1) [18].

Let us consider two examples: *i.* First we take the Lorentz-invariant function $\Psi(x_\mu^2)$ which is the Feynman propagator (distribution). Then $\Psi(p_\mu^2) = -(p_\mu^2+m^2-iO)^{-1}(\hbar = c = 1)$ and from Eq. (2C.1) we determine $\psi(\boldsymbol{y})$,

$$\psi(\boldsymbol{y}) = \frac{1}{4\pi r} \exp(-mr). \tag{2C.3}$$

This is the Yukawa potential in the x-representation of the PG. In particular, in the Lorentz limit (2B.12), when the particle "m_1" connected with the Feynman propagator becomes infinitely heavy, the static field (2C.3) becomes identical with the external Lorentz field, where r is the Lorentz distance from infinitely heavy particle "m_1" in its rest-frame. *ii.* Now let us assume some Lorentz-absolute function $\psi(r)$ in the x-continuum of the PG, *e. g.* $\psi(r) = \exp(-r^2/2a^2)$. The corresponding Lorentz invariant function in the space-time is then given by Eq. (2C.2),

$$\Psi(x_\mu^2) = (2\pi)^{-4} (\sqrt{2\pi}\, a)^3 \int d^4p \exp(-a^2p_\mu^2/2) \exp(ip_\mu x_\mu). \tag{2C.4}$$

However, this integral is unreasonable.

It turns out that the class of the Lorentz-absolute functions in the continuum of the PG is much larger than that of the Lorentz-invariant functions in the space-time. The NR internal wave functions regarded as the Lorentz-absolute functions are of such a mathematical structure that their Lorentz-invariant counterparts are usually given by unreasonable integrals, much like it takes place in the example *ii.* The so called "dipole" fit of the nucleon form factors represents also an absolute function, whose time-like behaviour is unreasonable because of the dipole divergency [19].

It seems to us that here is the answer to the general question, namely why the relativity theory is so exceedingly restrictive for any dynamical theory — like is the field theory — which is formulated in the space-time continuum [20], and at the same time, why the same relativity theory does not impose any essential restrictions upon a theory — like is the S-matrix theory — formulated from the very beginning in terms of the momentum invariants [21]. The answer is that the momentum invariants of the Lorentz geometry (as the only Lorentz invariants!) are at the same time the objects of the PG. For example, in the elastic scattering of two particles, the full set of the momentum invariants s, t is given "from the outside" of that system by the Lorentz invariants, namely

$$s = -(p_{1\mu}+p_{2\mu})^2, \qquad t = -(p_{1\mu}-p'_{1\mu})^2, \tag{2C.5}$$

($(p_{1,2})_\mu$, $(p'_{1,2})_\mu$ are the asymptotic four-momenta of the colliding particles) and "from the inside" of this system, by the Lorentz-absolute quantities of the PG, as we have

$$s = [(\boldsymbol{q}^2+m_1^2)^{1/2}+(\boldsymbol{q}^2+m_2^2)^{1/2}]^2$$
$$t = (\boldsymbol{q}-\boldsymbol{q}')^2 \quad (|\boldsymbol{q}| = |\boldsymbol{q}'|). \tag{2C.6}$$

Here $\boldsymbol{q}$ and $\boldsymbol{q}'$ are two asymptotic eigenvalues of $\hat{\boldsymbol{q}}$, and as we know, their relation to the external (measurable) asymptotic Lorentz momenta is given *a posteriori*, as we have $\boldsymbol{q}=p_2|_{\boldsymbol{P}=0}$, $\boldsymbol{q}'=p_2'|_{\boldsymbol{P}=0}$, where $\boldsymbol{P}$ is the total momentum of the whole isolated two-body system. Thus the S-matrix theory does not imply the universal validity of the Lorentz geometry.

We see then that the momentum-, not x-representation provides us with parametrization which enables the measurement of the internal structure of isolated systems. This is not accidental, because sharp determination of momenta eliminates the space-time localization of the interaction process, which fact is the basis of the PG hypothesis.

According to the PG viewpoint, the geometrical nature of $\psi(r)$ is identical with that of the charge distribution $e(r)$ of proton, and speaking generally, of all quantities which characterize the internal structure of any isolated system. In the PG picture no difficulty arises when we interpret $e(r)$ as the proper charge distribution of proton, as $e(r)$ is a Lorentz-absolute internal characteristic of isolated system (proton). Within the Lorentz geometry, the argument r of the function e is identified with $r^L=|\boldsymbol{y}^L|$ — *cf.* (2B.11′) — which coincides with the space-interval in the CM-system of the colliding proton and electron. However, if $e(r^L)$ has to represent the proper charge distribution of proton, then in the CM-system such a shape is contracted by the Lorentz factor of proton. This makes that the relation of $e(r^L)$ to the proper charge distribution of proton is quite obscure [18], the more so that the recoil itself parametrizes the shape of the charge distribution [17].

Let us emphasize that the x-continuum of the PG is not directly measurable, hence $e(r)$ is not directly measurable either. The function $e(r)$ is evaluated from Eq. (2C.1), whereas the directly measurable is its momentum representation, as we have that $e(\boldsymbol{q}^2) = G_E(p_\mu^2=\boldsymbol{q}^2)$. Due to this fact, the PG framework can maintain the energy-mass relation — which is the momentum representation relation — without at the same time identifying its x-continuum with the Lorentz space-time.

This short survey of the PG framework enables us to formulate our fundamental assumption 1° concerning the description of deuteron namely that: 1° the internal non-relativistic wave function of deuteron is a Lorentz-absolute characteristic of deuteron in the continuum of the phase geometry.

3. *Cross-section for elastic e–d scattering*

The NR internal wave function of the ground state of deuteron, which according to the assumption 1° is regarded as the Lorentz-absolute characteristic of deuteron, takes the following well known form

$$\psi_m = (4\pi)^{-1/2}\,\frac{1}{r_1}\left[u(r_1)+\frac{1}{\sqrt{8}}\,w(r_1)S(\boldsymbol{n}_1)\right]X_m. \tag{3.1}$$

Here X_m $(m=0,\ \pm 1)$ are the triplet spin wave functions, $S(\boldsymbol{n}_1)=3(\boldsymbol{\sigma}^{(p)}\boldsymbol{n}_1)(\boldsymbol{\sigma}^{(n)}\boldsymbol{n}_1)-\boldsymbol{\sigma}^{(p)}\boldsymbol{\sigma}^{(n)}$, where $\boldsymbol{\sigma}^{(p,n)}$ are the Pauli spin matrices of proton and neuteron, and $u(r_1)$, $w(r_1)$ are the radial wave functions of the S- and D-states of deuteron. The coordinate $\boldsymbol{y}_1$ is the PG (relative) coordinate of neutron with respect to proton, and $\boldsymbol{n}_1=\boldsymbol{y}_1/r_1$, where $r_1=|\boldsymbol{y}_1|$. [2] The radial

[2] Since we are working in the Schroedinger representation, the coordinates "y" are c-numbers.

functions u and w are normalized as usually, namely

$$\int_0^\infty dr_1\,[u^2(r_1)+w^2(r_1)] = 1, \qquad \text{hence } \int d^3y_1\, \psi^+_{m'}(\boldsymbol{y}_1)\, \psi_m(\boldsymbol{y}_1) = \delta_{mm'}.$$

Assumption 1° enables us to generalize the expression of the scattering amplitude for high-energy, and large momentum transfer (*i. e.* "relativistic") collisions of composite particles. The essential point is that that the absolute wave function (3.1) determines directly the vertex function of deuteron.

Making use of the Born approximation (assumption 2°) and of the assumption 1°, the matrix element for electron-deuteron elastic collision can be written as follows

$$S_{\substack{\alpha\,\alpha'\\ m,m'}} = ie(2\pi)^4\, \delta(P_\nu - P'_\nu)\bar{u}_{\alpha'}(\boldsymbol{p}')\gamma_\mu u_\alpha(\boldsymbol{p}) \int d^3y_1\, d^3y \psi^+_{m'}(\boldsymbol{y}_1)$$
$$\exp\,[i(\boldsymbol{q}-\boldsymbol{q}')\boldsymbol{y}]\, V_\mu\,(\boldsymbol{y}_1, \boldsymbol{y};\, b)\, \psi_m(\boldsymbol{y}_1).^{3} \tag{3.2}$$

By unprimed (primed) letters we always denote the corresponding quantities before (after) the collision. Here P_ν, P'_ν are the four-momenta of the whole (isolated) system of electron and deuteron. The Dirac spinors $u_\alpha(\boldsymbol{p})$, $u_{\alpha'}(\boldsymbol{p}')$ describe electron of momenta $\boldsymbol{p}, \boldsymbol{p}'$, and the polarizations α, $\alpha' = \pm 1/2$. The asymptotic momenta $\boldsymbol{q}$ and $\boldsymbol{q}'$ are the eigenvalues of the PG absolute momentum $\hat{\boldsymbol{q}}$, where $\hat{\boldsymbol{q}}$ is the momentum canonically conjugate to $\hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{y}}$ is the PG coordinate of electron with respect to the centre of gravity of deuteron. The four-potential V_μ depends on $\boldsymbol{y}_p$ and $\boldsymbol{y}_n$ which are the PG coordinate of electron with respect to proton and neutron, respectively. Besides, V_μ is an operator in the spin spaces of proton and neutron, and it depends on the kinematical factors denoted by "b" which is due to the fact that the interacting particles have spins. This dependence will become clear when we construct V_μ.

The Lorentz-absolute amplitude (3.2) will be evaluated in the CM-system of electron and deuteron, where the kinematical dependence of V_μ on "b" becomes particularly simple. In this system, where the electron momenta amount to: $\boldsymbol{p} = \boldsymbol{q}$, $\boldsymbol{p}' = \boldsymbol{q}'$, we denote "$b$" by "$b_0$".

The additivity assumption 4° implies that V_μ is the sum of two terms, the first responsible for the interaction between electron and proton, hence depending on $\boldsymbol{y}_p$, and the second describing the interaction between electron and neutron, parametrized by $\boldsymbol{y}_n$. Thus the potential V_μ takes the following form

$$V_\mu(\boldsymbol{y}_1, \boldsymbol{y};\, b_0) = V^{(p)}_\mu(\boldsymbol{y}_p;\, b_0) + V^{(n)}_\mu(\boldsymbol{y}_n;\, b_0), \tag{3.3}$$

as we have that $\boldsymbol{y} = \boldsymbol{y}_p - \frac{1}{2}\boldsymbol{y}_1 = \boldsymbol{y}_n + \frac{1}{2}\boldsymbol{y}_1$ (it is assumed that the deuteron mass M is twice larger than the average nucleon mass m). Moreover, making use of the deuteron ground state symmetry which tells that $\psi_m(-\boldsymbol{y}_1) = \psi_m(\boldsymbol{y}_1)$, the matrix element (3.2) can be rewritten in the following form

$$S_{\substack{\alpha\alpha'\\ m,m'}} = ie(2\pi)^4\delta(P_\nu - P'_\nu)\bar{u}_{\alpha'}(\boldsymbol{q}')\gamma_\mu u_\alpha(\boldsymbol{q}) \int d^3y_1 \psi^+_{m'}(\boldsymbol{y}_1)\, \exp\,(i\boldsymbol{k}\boldsymbol{y}_1)$$
$$\{\int d^3y[V^{(p)}_\mu(\boldsymbol{y};\, b_0) + V^{(n)}_\mu(\boldsymbol{y};\, b_0)]\, \exp\,(2i\boldsymbol{k}\boldsymbol{y})\}\psi_m(\boldsymbol{y}_1). \tag{3.4}$$

Here $\boldsymbol{k} = \frac{1}{2}(\boldsymbol{q}-\boldsymbol{q}')$, hence $\boldsymbol{k}^2 = -\frac{1}{4}t$, where t is the invariant momentum transfer.

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[3] We have adopted the conventions used in *Kwantowaja Elektrodynamika*, A. I. Achiezer and W. B. Berestecki, Gosizdat, Moskwa 1959. The only difference is that the Dirac spinors are here normalized in a covariant way, *i. e.* $\bar{u}u = 1$, instead of $u^+u = 1$, as in ***A–B***.

We see that the directly unobservable PG coordinates $\boldsymbol{y}$ and $\boldsymbol{y}_1$ which parametrize the internal structure of the whole isolated system of electron and deuteron are integrated out. In consequence, the cross-section resulting from the amplitude (3.4) will be a function of the PG momentum scalars, and according to the Eqs (2C.5) and (2C.6), the latter can be rewritten in terms of the Lorentz momentum scalars, *e.g.* $k^2 = -{}^1/_4 t$. Therefore, the cross--section, as a function of the Lorentz momentum invariants, fulfils all requirements of the relativity theory, although the corresponding amplitude is based on the PG framework which is not isomorphic with that of the Lorentz geometry. Remember that the cross-section must be compatible with the relativity theory, because the latter rules all directly observable quantities, hence in particular the cross-section.

Before going to determine the four-potential V_μ, let us introduce the Fourier transform of the density matrix of the ground state of deuteron. We have

$$R_{mm'} = \int d^3x \psi_m(\boldsymbol{x}) \exp(i\boldsymbol{k}\boldsymbol{x}) \psi^+_{m'}(\boldsymbol{x})$$

$$= \left(U_0 - \frac{1}{8} W_0\right) X_m X^+_{m'} + \left(\frac{1}{8} W_2 - \frac{1}{\sqrt{8}} V_2\right) [S(\boldsymbol{\nu}) X_m X^+_{m'} + X_m X^+_{m'} S(\boldsymbol{\nu})] +$$

$$+ \left(\frac{3}{40} W_0 + \frac{3}{28} W_2 + \frac{9}{280} W_4\right) I_{mm'} - \frac{9}{56}(W_2 + W_4) K_{mm'} +$$

$$+ \frac{9}{8} W_4 (\boldsymbol{\sigma}^{(p)}\boldsymbol{\nu})(\boldsymbol{\sigma}^{(n)}\boldsymbol{\nu}) X_m X^+_{m'} (\boldsymbol{\sigma}^{(p)}\boldsymbol{\nu})(\boldsymbol{\sigma}^{(n)}\boldsymbol{\nu}), \qquad \boldsymbol{\nu} = \boldsymbol{k}/k,$$

$$U_n(k) = \int_0^\infty dx\, u^2(x) j_n(kx), \qquad W_n(k) = \int_0^\infty dx\, w^2(x) j_n(kx),$$

$$V_n(k) = \int_0^\infty dx\, u(x) w(x) j_n(kx); \quad j_n(z) = (-z)^n \left(\frac{d}{zdz}\right)^n \left(\frac{\sin z}{z}\right),$$

$$I_{mm'} = X_m X^+_{m'} + \sigma^{(p)}_j \sigma^{(n)}_k X_m X^+_{m'} \sigma^{(p)}_j \sigma^{(n)}_k + \sigma^{(p)}_j \sigma^{(n)}_k X_m X^+_{m'} \sigma^{(n)}_j \sigma^{(p)}_k,$$

$$K_{mm'} = X_m X^+_{m'} (\boldsymbol{\sigma}^{(p)}\boldsymbol{\nu})(\boldsymbol{\sigma}^{(n)}\boldsymbol{\nu}) + \sigma^{(p)}_j (\boldsymbol{\sigma}^{(n)}\boldsymbol{\nu}) X_m X^+_{m'} (\boldsymbol{\sigma}^{(n)}\boldsymbol{\nu}) \sigma^{(p)}_j +$$

$$+ \sigma^{(p)}_j (\boldsymbol{\sigma}^{(n)}\boldsymbol{\nu}) X_m X^+_{m'} (\boldsymbol{\sigma}^{(p)}\boldsymbol{\nu}) \sigma^{(n)}_j + \sigma^{(n)}_j (\boldsymbol{\sigma}^{(p)}\boldsymbol{\nu}) X_m X^+_{m'} (\boldsymbol{\sigma}^{(n)}\boldsymbol{\nu}) \sigma^{(p)}_j +$$

$$+ \sigma^{(n)}_j (\boldsymbol{\sigma}^{(p)}\boldsymbol{\nu}) X_m X^+_{m'} (\boldsymbol{\sigma}^{(p)}\boldsymbol{\nu}) \sigma^{(n)}_j + (\boldsymbol{\sigma}^{(p)}\boldsymbol{\nu})(\boldsymbol{\sigma}^{(n)}\boldsymbol{\nu}) X_m X^+_{m'} \tag{3.5}$$

(summation over $j, k = 1, 2, 3$).

In order to determine V_μ, let us begin from the following remark. Spins of proton and neutron in deuteron are described by the Pauli spinors, therefore the potential V_μ must also be expressed in terms of the Pauli spin operators of both nucleons. On the other hand, we know the nucleon currents, hence the interactions between electron and nucleons, but expressed in terms of the Dirac operators. So we must translate the nucleon currents onto the language of the Pauli operators. For this purpose, let us consider the one-particle amplitude $\bar{u}' O_D u$, where u and u' are the Dirac spinors describing a spin one-half particle with given momenta and energies, and O_D is a 4×4 operator of the Dirac algebra. We introduce the following notation:

$$O_D = \begin{pmatrix} O^{11} & O^{12} \\ O^{21} & O^{22} \end{pmatrix}, \quad \text{and} \quad u = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} v \\ \dfrac{\boldsymbol{P\sigma}}{E+M} v \end{pmatrix}.$$

Here O^{jk} $(j, k = 1, 2)$ are the Pauli operators, and v is the Pauli spinor normalized to unity, $v^{+}v = 1(\bar{u}u = 1)$. We define the Pauli operator O attached to the Dirac operator O_D (without index "D" we always denote the corresponding Pauli operator) in such a way that the following identity takes place

$$\bar{u}' O_D u = v'^{+} O v.$$

This identity dictates the following relation between the operators O_D and O, and it explains the origin of the kinematical dependence "b" of V_μ

$$O = \sqrt{\frac{E+M}{2M}} \sqrt{\frac{E'+M}{2M}} \left[O^{11} + O^{12} \frac{\boldsymbol{P\sigma}}{E+M} - \frac{\boldsymbol{P'\sigma}}{E'+M} O^{21} - \frac{\boldsymbol{P'\sigma}}{E'+M} O^{22} \frac{\boldsymbol{P\sigma}}{E+M} \right]. \tag{3.6}$$

Note that in the NR limit we have $O = O^{11}$. In order to determine the current operator of deuteron, and consequently the four-potential V_μ, let us assume for the moment that we deal with two free nucleons, proton and neutron which move with the same momenta and energies. According to the additivity assumption $4°$ the current amplitude of such an "unbound deuteron" amounts to

$$\mathscr{J}_\mu = (\bar{u}'^{(p)} J^{(p)}_{D\mu} u^{(p)}) (\bar{u}'^{(n)} u^{(n)}) + (\bar{u}'^{(p)} u^{(p)}) (\bar{u}'^{(n)} J^{(n)}_{D\mu} u^{(n)}), \tag{3.7}$$

where $J^{(p)}_{D\mu}$ and $J^{(n)}_{D\mu}$ are the current operators of proton and neutron, respectively. They are known from the electron-nucleon interaction, and in the momentum representation they take the following form

$$J^{(p,n)}_{D\mu} = \gamma^{(p,n)}_\mu F^{(p,n)}_1(t) + i\sigma^{(p,n)}_{\mu\nu} \Delta q_\nu F^{(p,n)}_2(t). \tag{3.8}$$

Here $F^{(p,n)}_1$, and $F^{(p,n)}_2$ are the Dirac and Pauli form factors of proton and neutron [18]. In the CM-system the four-momentum transfer Δq_ν is equal to $\Delta q_{\nu/CM} = (2\boldsymbol{k}, 0)$. Note that the current amplitude (3.7) has the proper geometrical nature of a four-vector, and moreover it accounts for the additivity of the interaction. From (3.7) we see that the deuteron current equals

$$J^{(d)}_{D\mu} = J^{(p)}_{D\mu} \otimes I^{(n)}_D + J^{(n)}_{D\mu} \otimes I^{(p)}_D, \tag{3.9}$$

where $I^{(p,n)}_D$ are the unite operators in the Dirac spin spaces of proton and neutron, respectively. In fact the deuteron current operator (3.9) presumes — which is strongly suggested by the PG framework — that the spin orientation of the non-interacting nucleon remains fixed with respect to the axes of the over-all CM-system. Now, this operator (3.9) must be rewritten in terms of the Pauli operators. The kinematical factors which enter into Eq. (3.6) are assumed to be the same for proton and neutron in deuteron, and equal to the corresponding kinematical factors of deuteron as a whole. Thus E, E', $\boldsymbol{P}$, $\boldsymbol{P}'$ and M in Eq. (3.6) are identified with the energies, momenta and the mass of deuteron, respectively. This follows from our general assumption, namely that the NR internal wave function (3.1) accunts fully for the internal motion of proton and neutron in deuteron. The internal motion is "nonrelativistic", as deuteron is a loosely bound system, and therefore it does not affect the structure of the current operator $J_{D\mu}^{(d)}$ of deuteron. Finally, in the CM-system where

$$E = E' = W = (M^2+\boldsymbol{q}^2)^{1/2}, \ \boldsymbol{P} = -\ \boldsymbol{q}, \ \boldsymbol{P}' = -\ \boldsymbol{q}'$$

the operator $J_{D\mu}^{(d)}$ rewritten in terms of the Pauli operators takes the following form

$$J_\mu^{(d)} = J_\mu^{(p)} \otimes I^{(n)} + J_\mu^{(n)} \otimes I^{(p)}, \tag{3.10}$$

where

$$J_\mu^{(p,n)} = \frac{W+M}{2M}\left[(J_\mu^{(p,n)})^{11} + (J_\mu^{(p,n)})^{12}\,\frac{\boldsymbol{q}\boldsymbol{\sigma}^{(p,n)}}{W+M} - \frac{\boldsymbol{q}'\boldsymbol{\sigma}^{(p,n)}}{W+M}\,(J_\mu^{(p,n)})^{21} - \right.$$

$$\left. - \frac{\boldsymbol{q}'\boldsymbol{\sigma}^{(p,n)}}{W+M}\,(J_\mu^{(p,n)})^{22}\,\frac{\boldsymbol{q}\boldsymbol{\sigma}^{(p,n)}}{W+M}\right],$$

$$I^{(p,n)} = \frac{W+M}{2M}\left[1 - \frac{(\boldsymbol{q}'\boldsymbol{\sigma}^{(p,n)})\,(\boldsymbol{q}\boldsymbol{\sigma}^{(p,n)})}{(W+M)^2}\right].$$

The form of the operators $(J_\mu^{(p,n)})^{jk}$ can be read off from Eq. (3.8). The amplitude $\mathscr{I}_\mu$ from Eq. (3.7), in the language of the Pauli spinors, takes the following form

$$\mathscr{I}_\mu = (v'^{(p)+}v'^{(n)+})J_\mu^{(d)}(v^{(p)}v^{(n)}).$$

In order to account for the spin orientation of nucleons in deutron one must replace the spinor functions $v^{(p)}v^{(n)}$ and $v'^{(p)}v'^{(n)}$ by the spinor functions X_m and $X_{m'}$, respectively, where X_m and $X_{m'}$ enter into the deuteron wave function (3.1). So, we end up with the following expression of the Fourier transform of the interaction V_μ which entrers into the matrix element given by Eq. (3.4)

$$\int d^3y\,[V_\mu^{(p)}(\boldsymbol{y}; b_0) + V_\mu^{(n)}(\boldsymbol{y};\ b_0)]\ \exp\,(2i\boldsymbol{k}\boldsymbol{y}) = \frac{J_\mu^{(d)}\,(\boldsymbol{k}; b_0)}{(2k)^2}. \tag{3.11}$$

The factor $(2k)^{-2} = (-t)^{-1}$ results from the one-photon exchange interaction (assumption 3°). From the formula (3.10) one can easily read off all kinematical parameters denoted by "b_0" which, as we said, are due to spins of interacting particles.

The differential cross-section for elastically scattered electrons can be written in the form

$$d\sigma = \frac{e^2}{\pi^2}\,\frac{S_{\mu\nu}\,s_{\mu\nu}}{t^2\,(s-M^2)^2}\,\varepsilon'^2\,do', \tag{3.12}$$

where s and t are the Mandelstam variables, ε' is the energy of the scattered electron (whose mass is neglected), and do' is the element of its solid angle. The two four-tensors $S_{\mu\nu}$ and $s_{\mu\nu}$ are the deuteron and electron current tensors, respectively, and they result from the summation over the polarization states of the corresponding bilinear forms of electron and deuteron currents. Formula (3.12) clearly shows that the cross-section is Lorentz-invariant. Indeed, $S_{\mu\nu}\,s_{\mu\nu}$ is a Lorentz scalar which depends on the invariant Mandelstam variables s and t. Also $\varepsilon'^2\,do'$ is Lorentz invariant, and so, the cross-section $d\sigma$ is manifestly Lorentz-invariant.

Inserting the expression (3.11) into the amplitude (3.4), we come to the following expression for the deuteron current tensor $S_{\mu\nu}$

$$S_{\mu\nu} = M^2 \sum_{m,m'} [Sp(R_{mm'} J_\mu^{(d)})\, Sp(R^+_{mm'} J_\mu^{+(d)})],$$

where $J_\mu^{(d)}$ is given in (3.10), and $R_{mm'}$ is the density matrix from Eq. (3.5). Thus we can write that

$$Sp\,(R_{mm'} J_\mu^{(d)}) = X^+_{m'} Q_\mu X_m.$$

We do not write the expression for Q_μ as it is easy to derive from the expressions of $J_\mu^{(d)}$ and $R_{mm'}$. Now we have that

$$S_{\mu\nu} = M^2 \sum_{m,m'} X^+_{m'} Q_\mu X_m X^+_m Q^+_\nu X_{m'}.$$

The summation over all polarization states of the initial and final deuterons can be done with the help of the projection operator

$$\sum_m X_m X_m^+ = \frac{1}{4}\,(3+\boldsymbol{\sigma}^{(p)}\boldsymbol{\sigma}^{(n)}).$$

Finally we come to the following expression of $S_{\mu\nu}$

$$S_{\mu\nu} = \frac{M^2}{16}\, Sp\,[(3+\boldsymbol{\sigma}^{(p)}\,\boldsymbol{\sigma}^{(n)})\, Q_\mu (3+\boldsymbol{\sigma}^{(p)}\,\boldsymbol{\sigma}^{(n)})\, Q^+_\nu], \tag{3.13}$$

where now the trace is to be taken from the corresponding products of the Pauli matrices.

The electron current tensor $s_{\mu\nu}$ is well known, and in the CM-system it takes the form (the electron mass is neglected)

$$\begin{aligned} s_{mn} &= 2(q_m q_n + q_m k_n + q_n k_m + k^2 \delta_{mn}) \quad (m, n = 1, 2, 3) \\ s_{m4} &= -2iq\,(q_m + k_m) = -s_{4m} \\ s_{44} &= 2(q^2 - k^2);\ q = |\boldsymbol{q}|. \end{aligned} \tag{3.14}$$

Inserting $S_{\mu\nu}$ from (3.13) and $s_{\mu\nu}$ from (3.14) into the expression (3.12) for the cross-section, we come after a rather lengthy calculation, to the following formula for the electron-deuteron elastic cross-section

$$\begin{aligned} \frac{d\sigma}{d0'} = \frac{e^2}{4\pi^2}\frac{\varepsilon'^2}{t^2}\, U_0^2(t) \Bigg\{ & F_1^2(t) \left[1 + \frac{st}{(s-M^2)^2} - \frac{1}{6}\, t/M^2 + \right. \\ & \left. + \frac{1}{6}\frac{2-s/M^2}{(s-M^2)^2}\, t^2 - \frac{1}{12}\frac{1}{M^2(s-M^2)^2}\, t^3 \right] + \\ & + MF_1(t)\, F_2(t) \left[\frac{1}{3}\, t/M^2 + \frac{1}{3}\frac{4+s/M^2}{(s-M^2)^2}\, t^2 - \frac{1}{3}\frac{1}{M^2(s-M^2)^2}\, t^3 \right] + \\ & + M^2 F_2^2(t) \left[-\frac{2}{3}\, t/M^2 + \frac{1}{12}\frac{19-14s/M^2+3s^2/M^4}{(s-M^2)^2}\, t^2 + \right. \\ & \left. + \frac{1}{12}\frac{3s/M^2-4}{M^2(s-M^2)^2}\, t^3 \right] \Bigg\} + D(s, t). \end{aligned} \tag{3.15}$$

Here $F_1(t) = F_1^{(p)}(t)+F_2^{(n)}(t)$, $F_2(t) = F_2^{(p)}(t)+F_2^{(n)}(t)$, and $D(s, t)$ is the correction due to the D-state admixture of the ground state of deuteron. The expression of $D(s, t)$ is given in Appendix. Remember that $t = -4k^2$, s is the total energy square in the CM-system, and M is the deuteron mass taken equal to $2m$, where m is the average nucleon mass.

The Dirac and Pauli form factors are normalized as usually, namely

$$F_2^{(p)}(0) = e \qquad F_1^{(n)}(0) = 0$$

$$F_2^{(p)}(0) = \mu_p = (1+\varkappa_p)\frac{e}{2m}\ ; F_2^{(n)}(0) = \mu_n = \varkappa_n \frac{e}{2m},$$

where $\varkappa_p = 1.79$, and $\varkappa_n = -1.91$. The electric and magnetic form factors $G_E(t)$ and $G_M(t)$ are connected with $F_1(t)$ and $F_2(t)$ by the following relations

$$G_E^{(p,n)}(t) = F_1^{(p,n)}(t) + \frac{t}{2m} F_2^{(p,n)}(t)$$

$$G_M^{(p,n)}(t) = F_2^{(p,n)}(t) + \frac{1}{2m} F_1^{(p,n)}(t).$$

Thus the form factors $F_1(t)$ and $F_2(t)$ which enter into the formula (3.15) are connected with the electric and magnetic form factors of nucleons by the following relations

$$F_1(t) = \left(1 - \frac{t}{4m^2}\right)^{-1} \left[G_E^{(p)}(t)+G_E^{(n)}(t) - \frac{t}{2m}\left(G_M^{(p)}(t)+G_M^{(n)}(t)\right)\right]$$

$$F_2(t) = \left(1- \frac{t}{4m^2}\right)^{-1} \left[G_M^{(p)}(t)+G_M^{(n)}(t) - \frac{1}{2m}\left(G_E^{(p)}(t)+G_E^{(n)}(t)\right)\right]. \qquad (3.16)$$

If one postulates the scaling law which says that (19)

$$\frac{G_E^{(p)}(t)}{e} = \frac{G_M^{(p)}(t)}{\mu_p} = \frac{G_M^{(n)}(t)}{\mu_n} = F(t),$$

then the form factor $F(t)$ is quite well approximated by the "dipole" fit

$$F(t) = (1-t/a)^{-2} \quad \text{with} \quad a = 0.71\ (\text{Gev})^2. \qquad (3.17)$$

4. Remarks on form factors

Within the four-dimensional Lorentz geometry one deals with the Lorentz-invariant form factors, *e. g.* $F(x_\mu^2)$, which are determined for all four-points x_μ. Consequently, the Fourier transform $F(p_\mu^2)$ of $F(x_\mu^2)$ is also determined for all four-momenta p_μ, *i. e.* for p_μ space-like ($p_\mu^2 > 0$), as well as time-like ($p_\mu^2 < 0$). From the dynamical point of view this should mean that the same analytical function which for space-like p_μ determines the scattering amplitude, for time-like p_μ determines the corresponding annihilation amplitude [18]. Since the PG framework deals with the absolute three-dimensional space, and absolute (internal) time, it does not imply this type of symmetry.

On the other hand, it is a well known fact that the phenomenological form factors determined from scattering problems, *i. e.* for $t < 0$, when continued analytically onto the time-like region ($t > 0$) become unreasonable functions. The "dipole" fit of the nucleon form factors — *cf.* (3.17) — or form factors resulting from the Gaussian functions — *cf.* (2C.4) — provide us with examples of this fact. Therefore it is tempting to regard the symmetry which tells that the same analytical function is responsible for the scattering and annihilation processes, as a criterion that the interacting particles have no internal structure. This is really the case in the electron interactions.

From this point of view it is interesting to note that the "dipole" fit describing the nucleon internal structure results in a simple and natural way from the PG framework, if one assumes that nucleon is a composite particle which has the internal Lorentz-absolute wave function in the PG continuum. Let us assume that this wave function is of the form $\psi \sim \exp(-r/2b)$ which accounts properly for the asymptotic behaviour of ψ at infinity. Then $u(r) = (2b^3)^{-1/2}\, r \exp(-r/2b)$, and the corresponding form factors $U_n(k)$ can be identified with the nucleon form factor F in Eq. (3.17). Since all form factors $U_n(k)$ have for each n the same asymptotic behaviour, let us consider the form factor $U_0(k)$. We have

$$U_0(k) = \int_0^\infty dr u^2(r) j_0(kr) = (1+b^2k^2)^{-2} = \left(1 - \frac{1}{4}\, b^2 t\right)^{-2} = F(t). \qquad (4.1)$$

This is nothing else than the "dipole" fit. In the last equation we have used the relation $t = -\frac{1}{4}k^2$ characteristic for two-body systems of equal masses, like deuteron. This relation is by no means justified in the case of nucleon. Nevertheless, this relation is of secondary importance as it does not influence the structure of the factor which coincides with the "dipole" fit.

In this particularly simple example we see that the internal structure of the system is responsible for the form factor whose time-like behaviour is unreasonable, as it has the dipole singularity.

The problem of composition of a physical system is intimately connected with the additivity assumption 4°, and with the opposite procedure of the decomposition of the system (particle) into more "elementary" constituents. The quark model of hadrons gives a fashionable illustration of this problem. The point is that the scattering amplitude can be estimated on the basis of the additivity assumption, whereas the annihilation process is not an additive one. Thus the space-like form factors determined from the scattering processes are not relevant in describing the corresponding annihilation processes.

At the end we would like to point out that the PG scheme can be as well applied to inelastic collisions, like is the disintegration of nuclei. It also gives the basis for the generalization onto the "relativistic" momentum transfers the Glauber type of analysis of the collision processes, which turned out to be so fruitful in the domain of the "nonrelativistic" recoils [4], [5].

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APPENDIX

The correction $D(s, t)$ in the formula (3.15) is due, as we said, to the admixture of the D-state in the ground state wave function of deuteron, thus it vanishes for $w(r) = 0$. We have

$$D(s, t) = \frac{e^2}{3\pi^2} \frac{\varepsilon'^2}{M^2t^2(s-M^2)^2} T(s, t),$$

where

$$\begin{aligned} T(s, t) = & F_1^2(t)[D^{(1)}A_{11}+D^{(2)}B_{11}+D^{(3)}C_{11}+D^{(4)}E_{11}+D^{(5)}G_{11}]+ \\ & + F_1(t)\, F_2(t)\, [D^{(1)}A_{12}+D^{(2)}B_{12}+D^{(3)}C_{12}+D^{(4)}E_{12}+D^{(5)}G_{12}]+ \\ & + F_2^2(t)\, [D^{(1)}A_{22}+D^{(2)}B_{22}+D^{(3)}C_{22}+D^{(4)}E_{22}+D^{(5)}G_{22}]. \end{aligned}$$

Here $F_1(t)$ and $F_2(t)$ are the same form factors as in (3.16), $D^{(J)}/J = 1, \ldots, 5)$ are the deuteron form factors, and $A_{11}, \ldots, G_{22}$ are the invariant factors due to the structure of the interaction. Thus we have

$$D^{(1)} = 6U_0W_0+12V_2^2-6\sqrt{2}\,V_2W_2+3W_0^2+\frac{3}{2}W_2^2,$$

$$D^{(2)} = 2\sqrt{2}\,U_0V_2-2U_0W_0+2U_0W_2+V_2^2-\sqrt{2}\,V_2W_0+\sqrt{2}V_2W_2+ \\ +\frac{1}{2}W_0^2-W_0W_2+\frac{1}{2}W_2^2,$$

$$D^{(3)} = 4\sqrt{2}\,U_0V_2+2U_0W_0-2U_0W_2+8V_2^2+\frac{11\sqrt{2}}{5}V_2W_0-\frac{19\sqrt{2}}{7}V_2W_2+ \\ +\frac{108\sqrt{2}}{35}V_2W_4+W_0^2-\frac{11}{10}W_0W_2+\frac{5}{14}W_2^2-\frac{54}{35}W_2W_4,$$

$$D^{(4)} = \frac{6}{5}U_0W_0-\frac{12}{7}U_0W_2+\frac{108}{35}U_0W_4+12V_2^2+\frac{6\sqrt{2}}{5}V_2W_0- \\ -\frac{12\sqrt{2}}{7}V_2W_2+\frac{108\sqrt{2}}{35}V_2W_4+\frac{9}{25}W_0^2-\frac{27}{35}W_0W_2+\frac{54}{175}W_0W_4+ \\ +\frac{45}{98}W_2^2-\frac{216}{245}W_2W_4+\frac{2106}{1225}W_4^2,$$

$$D^{(5)}\ -4\sqrt{2}\,U_0V_2+\frac{2}{5}U_0W_0-\frac{4}{7}U_0W_2+\frac{36}{35}U_0W_4+4V_2^2-\frac{2\sqrt{2}}{5}V_2W_0+ \\ +\frac{4\sqrt{2}}{7}V_2W_2-\frac{36\sqrt{2}}{35}V_2W_4+\frac{1}{50}W_0^2-\frac{2}{35}W_0W_2+\frac{18}{175}W_0W_4+ \\ +\frac{2}{49}W_2^2-\frac{36}{245}W_2W_4+\frac{162}{1225}W_4^2.$$

The form factors $U_n(t)$, $W_n(t)$ and $V_n(t)$ $(t = -\frac{1}{4}k^2)$ are given in Eq. (3.5). We see that $D^{(J)}$ vanish for $w(r) = 0$.

Let us introduce two auxiliary variables $z = s^{1/2}$ and $z = w+M$. Then the invariant functions $A_{11}, \ldots, G_{22}$ take the following form

$$A_{11} = k^0 \left[\frac{1}{4} M^2w^4 - \frac{1}{2} M^4w^2 + \frac{1}{4} M^6\right] + k^2[Mw^3 - 4M^2w^2 + 3M^3w - $$
$$- M^4] + k^4z^{-2} [w^4 - 10Mw^3 + 14M^2w^2 - 6M^3w + M^4] + k^6z^{-4} \times$$
$$\times [-8w^4 + 28Mw^3 - 16M^2w^2 + 4M^3w] + k^8z^{-6} [20w^4 - 24Mw^3 +$$
$$+ 4M^2w^2] + k^{10}z^{-8} [-16w^4],$$

$$A_{12} = k^2[-2M^2w^3 + 2M^3w^2 + 2M^4w - 2M^5] + k^4z^{-1} [-8Mw^3 + 28M^2w^2 -$$
$$- 16M^3w + 4M^4] + k^6z^{-3} [-8w^4 + 64Mw^3 - 56M^2w^2 + 16M^3w] +$$
$$+ k^8z^{-5} [48w^4 - 96Mw^3 + 16M^2w^2] + k^{10}z^{-7} [-64w^4],$$

$$A_{22} = k^4[4M^2(w-M)^2] + k^6z^{-2} [16Mw^3 - 48M^2w^2 + 16M^3w] + k^8z^{-4} \times$$
$$\times [16w^4 - 96Mw^3 + 16M^2w^2] + k^{10}z^{-6} [-64w^4];$$

$$B_{11} = k^2 \left[\frac{1}{4} w^4 - 2Mw^3 + \frac{11}{2} M^2w^2 - 6M^3w + \frac{9}{4} M^4\right] + k^4z^{-2}[-5w^4 +$$
$$+ 26Mw^3 - 39M^2w^2 + 24M^3w - 2M^4] + k^6z^{-4} [33w^4 - 88Mw^3 +$$
$$+ 78M^2w^2 - 8M^3w + M^4] + k^8z^{-6} [-76w^4 + 104Mw^3 - 12M^2w^2] +$$
$$+ k^{10}z^{-8} [64w^4],$$

$$B_{12} = k^2 [-Mw^4 + 4M^2w^3 - 2M^3w^2 - 4M^4 + 3M^5] + k^4z^{-1} [-4w^4 +$$
$$+ 36Mw^3 - 76M^2w^2 + 56M^3w - 4M^4] + k^6z^{-3} [56w^4 - 228Mw^3 +$$
$$+ 228M^2w^2 - 28M^3w + 4M^4] + k^8z^{-5} [-224w^4 + 368Mw^3 - 48M^2w^2] +$$
$$+ k^{10}z^{-7} [256w^4],$$

$$B_{22} = k^2[M^2(w^2 - M^2)^2] + k^4[8Mw^3 - 28M^2w^2 + 24M^3w] + k^6z^{-2}$$
$$[16w^4 - 112Mw^3 + 148M^2w^2 - 24M^3w + 4M^4] + k^8z^{-4} [-128w^4 +$$
$$+ 320Mw^3 - 48M^2w^2] + k^{10}z^{-6} [256w^4];$$

$$C_{11} = k^2[M(w-M)^3] + k^4z^{-2} [2w^4 - 14Mw^3 + 20M^2w^2 - 10M^3w + 2M^4] +$$
$$+ k^6z^{-4} [-16w^4 + 48Mw^3 - 32M^2w^2 + 8M^3w] + k^8z^{-6} [40w^4 -$$
$$- 48Mw^3 + 8M^2w^2] + k^{10}z^{-8} [-32w^4],$$

$$C_{12} = k^2 [Mw^4 - 2M^2w^3 + 2M^4w - M^5] + k^4z^{-1} [2w^4 - 20Mw^3 + 36M^2w^2 - 24M^3w + 6M^4] + k^6z^{-3} [-28w^4 + 116Mw^3 - 100M^2w^2 + 28M^3w] + k^8z^{-5} [112w^4 - 176Mw^3 + 32M^2w^2] + k^{10}z^{-7} [-128w^4],$$

$$C_{22} = k^4[-4Mw^3 + 12M^2w^2 - 12M^3w + 4M^4) + k^6z^{-2}[-8w^4 + 56Mw^3 - 72M^2w^2 + 24M^3w] + k^8z^{-4} [64w^4 - 160Mw^3 + 32M^2w^2] + k^{10}z^{-6} [-128w^4];$$

$$E_{11} = k^4z^{-2}[(w-M)^4] + k^6z^{-4}[-8w^4 + 20Mw^3 - 16M^2w^2 + 4M^3w] + k^8z^{-6}[20w^4 - 24Mw^3 + 4M^2w^2] + k^{10}z^{-8}[-16w^4],$$

$$E_{12} = k^4z^{-1} [2w^4 - 8Mw^3 + 12M^2w^2 - 8M^3w + 2M^4] + k^6z^{-3} [-20w^4 + 52Mw^3 - 44M^2w^2 + 12M^3w] + k^8z^{-5} [64w^4 - 80Mw^3 + 16M^2w^2] + k^{10}z^{-7} [-64w^4],$$

$$E_{22} = k^4[(w-M)^4] + k^6z^{-2} [-12w^4 + 32Mw^3 - 28M^2w^2 + 8M^3w] + k^8z^{-4} [48w^4 - 64Mw^3 + 16M^2w^2] + k^{10}z^{-6} [-64w^4];$$

$$G_{11} = k^6z^{-2} [(w-M)^2] + k^8z^{-4} [-4w^2],$$

$$G_{12} = k^6z^{-2} [4M(w-M)^2] + k^8z^{-4} [-16Mw^2],$$

$$G_{22} = k^6z^{-2} [4M^2(w-M)^2] + k^8z^{-4} [-16M^2w^2].$$

In the absence of the D-state the deuteron is characterized by the single form factor $U_0(t)$ which is the multiplicative factor of the cross-section — *cf.* (3.15). Therefore in the pure S-state approximation of the deuteron structure the cross-section vanishes for vanishing U_0. On the other hand, the most important models of the ground state wave function account for the "hard-core" [22], while the "hard-core" implies that $U_0(t)$ vanishes in the neighbourhood of $t = -(\text{Gev})^2$. Thus the admixture of the D-state is very essential, as it makes that the cross-section becomes a smooth function of t, and it does not vanish in the mentioned region of t, which is the experimental fact [23].

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