Vol. A37 (1970) ACTA PHYSICA POLONICA Fasc. 5

DYNAMICAL EQUATIONS FOR SPINLESS, NEUTRAL BOSON SYSTEM PART I — ON THE MASS-SHELL FORMULATION

By W. Garczyński

Institute for Theoretical Physics, University of Wrocław*

(Received February 25, 1969 and in final form received December 6, 1969)

The S-matrix theory is proposed in terms of observable quantities — *i.e.*, in terms of matrix elements lying on the mass-shell only. A quantum Markovian process is constructed for an infinite system of spinless neutral particles and the corresponding dynamical equations are derived with the use of functionals technique.

1. *Introduction*

In the present paper we demonstrate an application of our idea of treating quantum theory as a quantum Markovian process [1]–[4] to a simplest relativistic system of massive, neutral and spinless bosons. The theory permits generalization to realistic situations without serious difficulties.

Our theory is in fact the S-matrix type theory [6] and does not make any use of the concepts of quantum field theory. This remark concerns mostly the first part of the paper where one deals with scattering amplitudes on the mass-shell only. In the second part one deals with the extrapolated elements of the S-matrix, off the mass-shell, and the connection with the conventional field-theoretical approach can be established, although it is not necessary since the formalism is autonomic.

Let $\mathscr{X}$ be the phase-space of a particle with mass $m > 0$ *i.e.*, a three-dimensional Euclidian space, or its part, formed by momenta $\boldsymbol{p}$, attainable by the particle. Since there is no reasonable restriction on the magnitude of $\boldsymbol{p}$, we assume that $\mathscr{X} = \{\boldsymbol{p}\}$ coincides with R^3. Clearly, to a system of n particles the direct product $R^3 \otimes \ldots \otimes R^3$ correspond.

A dynamical description of a system with an undetermined number of particles is given by an infinite family of complex functions

$$(\sigma; \boldsymbol{p}_1, \ldots, \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_1, \ldots, \boldsymbol{q}_n), \ (m, n = 0, 1, 2, \ldots)$$

denoting the transition probability density amplitudes from the state of m-particles on the space-like surface σ to the state with n-particle on the surface σ'. Energies of particles

* Address: Instytut Fizyki Teoretycznej, Uniwersytet Wrocławski, Wrocław, Cybulskiego 36, Polska.

are $p_k^0 = \sqrt{\boldsymbol{p}_k^2+m^2}$ so the amplitudes are on the mass-shell, [5], [6]. They satisfy several conditions and among them the quantum Markovian requirement. For that reason we call this set of amplitudes a quantum Markovian process [10]–[13] for spinless neutral bosons.

2. *The postulates*

Let $(\sigma; \boldsymbol{p}_1, \ldots, \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_1, \ldots, \boldsymbol{q}_n) \equiv (\sigma; \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_n)$ be a transition amplitude in the sense that

$$|(\sigma; \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_n)|^2 d\boldsymbol{q}_1 \ldots d\boldsymbol{q}_n \tag{1.2}$$

is the probability of finding n-particles with the momenta $\boldsymbol{q}_1, \ldots, \boldsymbol{q}_n$ in the intervals $\boldsymbol{q}_k, \boldsymbol{q}_k + d\boldsymbol{q}_k$ for all k at the surface σ' if it is known that at $\sigma < \sigma'$ m-particles with momenta $\boldsymbol{p}_2, \ldots, \boldsymbol{p}_m$ were observed. The sign $<$ means "earlier".

The following conditions are imposed on the amplitudes:

$$(i) \quad (\sigma, \boldsymbol{p}_1, \ldots, \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_1, \ldots, \boldsymbol{q}_n) = (\sigma; \boldsymbol{p}_{j_1}, \ldots, \boldsymbol{p}_{jm}|\sigma'; \boldsymbol{q}_{k_1}, \ldots, \boldsymbol{q}_{k_n})$$

where $(j_1, \ldots, j_m)$, $(k_1, \ldots, k_n)$ are permutations of $(1, \ldots, m)$ and $(1, \ldots, n)$ respectively. Clearly this is the symmetry principle suitable for bosons.

$$(ii) \quad (\sigma; \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_n) = (\sigma'; \boldsymbol{q}_n|\sigma; \boldsymbol{p}_m)^* \qquad \sigma < \sigma'.$$

We call this property motion reversal invariance.

$$(iii) \quad \lim_{\sigma \to \sigma'} (\sigma; \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_n) = \delta_{mn}\delta(\boldsymbol{p}_m; \boldsymbol{q}_m).$$

This is the continuity property. Here $\delta(\boldsymbol{p}_m; \boldsymbol{q}_m)$ stands for the symmetric δ-function, [7],

$$\delta(\boldsymbol{p}_m; \boldsymbol{q}_m) = \frac{1}{m!} \sum_{\pi \in S_m} \delta(\boldsymbol{p}_1 - \boldsymbol{q}_{\pi(1)}) \ldots \delta(\boldsymbol{p}_m - \boldsymbol{q}_{\pi(m)}). \tag{2.2}$$

$$(iv) \quad \sum_{l=0}^{\infty} \int d\boldsymbol{k}_l (\sigma; \boldsymbol{p}_m|\sigma'; \boldsymbol{k}_l)(\sigma'; \boldsymbol{k}_l|\sigma; \boldsymbol{q}_n) = \delta_{mn}\delta(\boldsymbol{p}_m; \boldsymbol{q}_m).$$

This is the unitarity property, which, together with the continuity property reflects the internal completeness and autonomy of the theory of one kind of particles.

$$(v) \quad \sum_{l=0}^{\infty} \int d\boldsymbol{k}_l (\sigma'; \boldsymbol{p}_m|\sigma; \boldsymbol{k}_l)\, (\sigma; \boldsymbol{k}_l|\sigma''; \boldsymbol{q}_n) = (\sigma'; \boldsymbol{p}_m|\sigma''; \boldsymbol{q}_n).$$

This is the quantum causality condition. We shall consider it as a central dynamical postulate.

$$(vi) \quad (\sigma; |\sigma'; \boldsymbol{q}_n) = 0 \qquad n > 0$$

$$(\sigma; |\sigma';) = 1.$$

The vacuum stability condition.

$$(\sigma; \boldsymbol{p}|\sigma'; \boldsymbol{q}_n) = 0 \qquad n > 1$$

$$(\sigma; \boldsymbol{p}|\, \sigma'; \boldsymbol{q}) = \delta(\boldsymbol{p}-\boldsymbol{q}).$$

The one-particle stability condition.

$$(vii) \quad (\sigma; \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_n) = (\sigma+a; L\boldsymbol{p}_m|\sigma'+a; L\boldsymbol{q}_n) \exp ia(\sum_{k=1}^{m} \boldsymbol{p}_k - \sum_{j=1}^{n} \boldsymbol{q}_j)$$

$$P_k^0 = \omega(\boldsymbol{p}_k) = \sqrt{\boldsymbol{p}_k^2+m^2}, \qquad q_j^0 = \sqrt{\boldsymbol{q}_j^2+m^2}$$

a — is an arbitrary vector, L — any real proper Lorentz transformation, $L \in L_+^\uparrow(R)$, m — is the mass of particles under consideration. Clearly, this is the relativistic invariance postulate.

In the next sections we shall use the old functional technique of handling an infinite set of functions which was developed further by Rzewuski [7], [8] and Berezin [9]. Since functional methods cannot be considered as highly popular among physicists we quote the results with all details.

3. *Generating functional for the amplitudes*

Instead of the infinite family of amplitudes one may introduce a generating functional in the known way

$$S[\sigma, \sigma'; \alpha, \beta] = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (m!n!)^{-1/2} \int d\boldsymbol{p}_m \int d\boldsymbol{q}_n (\sigma; \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_n) e[\alpha; \boldsymbol{p}_m] e[\beta; \boldsymbol{q}_n] \tag{1.3}$$

where the abbreviations are

$$\int d\boldsymbol{p}_m = \prod_{k=1}^{m} \int d\boldsymbol{p}_k$$

$$e[\alpha; \boldsymbol{p}_m] = \prod_{k=1}^{m} \alpha(\boldsymbol{p}_k) \tag{2.3}$$

$\alpha(\boldsymbol{p})$, $\beta(\boldsymbol{p})$ — are real and belong to the $\mathscr{S}_p$ class of Schwartz test functions space, [14].

In the sequel we shall frequently use the following obvious formulae:

$$e\left[\frac{\delta}{\delta\varphi}; y_m\right] e[\varphi; x_n]_{|\varphi=0} = n!\, \delta_{nm} \delta(y_m; x_m) \tag{3.3}$$

$$\int dx_n f(x_n)\, \delta(x_n; y_n) = f(y_n) \tag{4.3}$$

if $f(x_n) \equiv f(x_1, \ldots, x_n)$ is a symmetric function. Using them we may derive the basic formula

$$(\sigma; \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_n) = (m!\, n!)^{-1/2} e\left[\frac{\delta}{\delta\alpha}; \boldsymbol{p}_m\right] e\left[\frac{\delta}{\delta\beta}; \boldsymbol{q}_n\right] S[\sigma, \sigma'; \alpha, \beta]_{|\alpha=\beta=0}. \tag{5.3}$$

The functional derivatives of analytic functionals are

$$\frac{\delta}{\delta\varphi(y)} F[\varphi] = \lim_{\varepsilon\to 0} \varepsilon^{-1}\{F[\varphi+\varepsilon\delta_y] - F[\varphi]\}. \tag{6.3}$$

One may transform all the postulates onto the generating functional using (1.3) and (5.3).

Namely, the property (*i*) enables us to establish the one to one correspondence between $\{(\sigma; \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_n)\}$ and $S[\sigma, \sigma'; \alpha, \beta)$. Any non-symmetrical terms in the amplitudes would be cut off in $S[\sigma, \sigma'; \alpha, \beta]$. Thus we have the first postulate

(*i*) $\{(\sigma; \boldsymbol{p}_m|\sigma'; \boldsymbol{q}_n)\} \leftrightarrow S[\sigma, \sigma'; \alpha, \beta]$ — analytical functional.

From (*ii*) we obtain simply

(*ii*) $(S[\sigma, \sigma'; \alpha, \beta])^* = S[\sigma', \sigma; \beta, \alpha]$.

The condition (*iii*) reads

$$\lim_{\sigma\to\sigma'} S[\sigma, \sigma'; \alpha, \beta] = \exp(\alpha\cdot\beta)$$

$$\alpha\cdot\beta = \int d\boldsymbol{p}\alpha(\boldsymbol{p})\beta(\boldsymbol{p}).$$

In order to express the unitarity condition we use the following formula which is valid for any analytical functional

$$F[u+v] = \exp\left\{u\frac{\delta}{\delta v}\right\}F[v]. \tag{7.3}$$

Using (5.3) and the last formula we obtain after simple calculations, (*cf. e.g.* [7]),

(*iv*)
$$S\left[\sigma, \sigma'; \alpha, \frac{\delta}{\delta\gamma}\right] S^{+}[\sigma, \sigma'; \gamma, \beta]|_{\gamma=0} = \exp(\alpha\cdot\beta) \tag{8.3}$$

where

$$S^{+}[\sigma, \sigma'; \alpha, \beta] = S^{*}[\sigma, \sigma'; \beta, \alpha].$$

Analogously, the central quantum causality condition becomes

(*v*)
$$S\left[\sigma', \sigma; \alpha, \frac{\delta}{\delta\gamma}\right] S[\sigma, \sigma''; \gamma, \beta]|_{\gamma=0} = S[\sigma', \sigma''; \alpha, \beta].$$

The stability conditions are:

$$e\left[\frac{\delta}{\delta\beta}; q_n\right] S[\sigma, \sigma'; \alpha, \beta]|_{\alpha=\beta=0} = 0, \qquad n > 0$$

$$S[\sigma, \sigma'; 0, 0] = 1$$

(*vi*)

$$\frac{\delta}{\delta\alpha(\boldsymbol{p})} e\left[\frac{\delta}{\delta\beta}; \boldsymbol{q}_n\right] S[\sigma, \sigma'; \alpha, \beta]|_{\alpha=\beta=0} = 0, \qquad n > 1$$

$$\frac{\delta}{\delta\alpha(\boldsymbol{p})} \frac{\delta}{\delta\beta(\boldsymbol{q})} S[\sigma, \sigma'; \alpha, \beta]|_{\alpha=\beta=0} = \delta(\boldsymbol{p}-\boldsymbol{q}).$$

The Lorentz invariance principle has the form

$$\tilde{S}[\sigma, \sigma'; \alpha, \beta] = \tilde{S}[\sigma+a, \sigma'+a; \alpha_{(a,L)}, \beta_{(a,L)}]$$

$$\tilde{S}[\sigma, \sigma'; \alpha, \beta] = S\left[\sigma, \sigma', \frac{\alpha}{\sqrt{2\omega}}, \frac{\beta}{\sqrt{2\omega}}\right]$$

$$\alpha_{(a,L)}(\boldsymbol{p}) = \alpha(L^{-1}p) \exp(iap)|_{p^0=\omega(\boldsymbol{p})}$$

$$\beta_{(a,L)}(\boldsymbol{p}) = \beta(L^{-1}p) \exp(-iap)|_{p^0=\omega(\boldsymbol{p})}.$$

4. *The matrix notation*

Let $\{M(\boldsymbol{p}_m; \boldsymbol{q}_n); m, n = 0, 1, \ldots\}$ be any set of functions symmetric in the $\boldsymbol{p}_k$ and $\boldsymbol{q}_j$ variables. We may attach to it a generating functional $M[\alpha, \beta]$, which can be considered as an "element α, β" of the functional matrix $M = ||M[\alpha, \beta]||$

$$M[\alpha, \beta] = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (m!\, n!)^{-1/2} \int d\boldsymbol{p}_m \int d\boldsymbol{q}_n M(\boldsymbol{p}_m; \boldsymbol{q}_n) e[\alpha; \boldsymbol{p}_m] e[\beta; \boldsymbol{q}_n]. \tag{1.4}$$

The multiplication by numbers and the addition operations are usual

$$(\lambda M)[\alpha, \beta] = \lambda \cdot M[\alpha, \beta]$$

$$(M_1+M_2)[\alpha, \beta] = M_1[\alpha, \beta] + M_2[\alpha, \beta]. \tag{2.4}$$

The functional image of the usual multiplication law for the matrices $||M(\boldsymbol{p}_m; \boldsymbol{q}_n)||$ is

$$(M_1 * M_2)[\alpha, \beta] = M_1\left[\alpha, \frac{\delta}{\delta\varrho}\right] \cdot M_2[\varrho, \beta]|_{\varrho=0}. \tag{3.4}$$

Corresponding to this multiplication law the unit matrix $1[\alpha, \beta]$ which satisfies the relations

$$(1*M)[\alpha, \beta] = (M*1)[\alpha, \beta] = M[\alpha, \beta] \tag{4.4}$$

is

$$1[\alpha, \beta] = \exp(\alpha \cdot \beta). \tag{5.4}$$

This follows easily from (7.3) and the formula

$$F\left[\frac{\delta}{\delta u}\right] \exp(u \cdot v) = \exp(u \cdot v)\, F\left[v + \frac{\delta}{\delta u}\right] \tag{6.4}$$

which holds for any analytical functional $F[u]$.

The inverse matrix M^{-1} and the hermicity conjugation operation are defined as usual

$$M^{+}[\alpha, \beta] = M^{*}[\beta, \alpha] \tag{7.4}$$

$$M * M^{-1} = M^{-1} * M = 1. \tag{8.4}$$

As a particular case we define the row matrix U and the column matrix V as follows:

$$U[\alpha] = \sum_{m=0}^{\infty} (m!)^{-\frac{1}{2}} \int d\boldsymbol{p}_m u(\boldsymbol{p}_m) e[\alpha; \boldsymbol{p}_m]$$

$$V[\beta] = \sum_{n=0}^{\infty} (n!)^{-\frac{1}{2}} \int d\boldsymbol{q}_n v(\boldsymbol{q}_n) e[\beta; \boldsymbol{q}_n] \tag{9.4}$$

$u(\boldsymbol{p}_m)$, $v(\boldsymbol{q}_n)$ — symmetric functions.

Multiplying U by M and M by V we obtain

$$(U * M)\,[\beta] = U\left[\frac{\delta}{\delta \varrho}\right] M[\varrho, \beta]|_{\varrho=0}$$

$$= \sum_{n=0}^{\infty} (n!)^{-\frac{1}{2}} \int d\boldsymbol{q}_n \{ \sum_{m=0}^{\infty} \int d\boldsymbol{p}_m u(\boldsymbol{p}_m) M(\boldsymbol{p}_m; \boldsymbol{q}_n)\} e[\beta; \boldsymbol{q}_n]\,, \tag{10.4}$$

$$(M * V)\,[\alpha] = M\left[\alpha, \frac{\delta}{\delta \varrho}\right] V[\varrho]|_{\varrho=0}$$

$$= \sum_{m=0}^{\infty} (m!)^{-\frac{1}{2}} \int d\boldsymbol{p}_m \{ \sum_{n=0}^{\infty} \int d\boldsymbol{q}_n M(\boldsymbol{p}_m; \boldsymbol{q}_n) v(\boldsymbol{q}_n)\} e[\alpha; \boldsymbol{p}_m]. \tag{11.4}$$

Using this matrix notation one may express the basic postulates in a compact form as follows

(*i*) $S[\sigma, \sigma']$ — the functional matrix

(*ii*) $S^{+}[\sigma, \sigma'] = S[\sigma', \sigma]$

(*iii*) $\lim_{\sigma \to \sigma'} S[\sigma, \sigma'] = 1$

(*iv*) $S[\sigma, \sigma'] * S^{+}[\sigma, \sigma'] = 1$

(*v*) $S[\sigma', \sigma] * S[\sigma, \sigma''] = S[\sigma', \sigma''], \quad \sigma' < \sigma < \sigma''$

(*vi*) $\Omega[\sigma] * S[\sigma, \sigma'] = \Omega[\sigma]$,

$V_1[\sigma] * S[\sigma, \sigma'] = V_1[\sigma]$.

Here the vacuum functional $\Omega[\sigma]$ is

$$\Omega[\sigma; \alpha] = 1 \quad \text{for any } \alpha(\boldsymbol{p}) \in \mathscr{S}_{\boldsymbol{p}}, \tag{12.4}$$

and the one-particle functional $V_1[\sigma]$

$$V_1[\sigma; \alpha] + \int d\boldsymbol{p} v(\sigma; \boldsymbol{p}) \alpha(\boldsymbol{p}) \tag{13.4}$$

(*vii*) $\tilde{S}[\sigma, \sigma'] = \tilde{S}_{(a, L)}[\sigma + a, \sigma' + a]$

$$\tilde{S}_{(a, L)}[\sigma, \sigma'; \alpha, \beta] = \tilde{S}[\sigma, \sigma'; \alpha_{(a, L)}), \beta_{(a, L)})].$$

5. *The differential equations for generating functional*

The unitarity condition (*iv*) and the property (*ii*) enables us to extend the causality condition

$$S[\sigma', \sigma] * S[\sigma, \sigma''] = S[\sigma', \sigma''] \qquad \sigma' < \sigma < \sigma'' \tag{1.5}$$

to any intermediate surface σ. Indeed, if *e.g.*, the surface σ is earliear than σ' then we have

$$S[\sigma', \sigma] * S[\sigma, \sigma''] = S[\sigma', \sigma] * S[\sigma, \sigma'] * S[\sigma', \sigma''] = S[\sigma', \sigma''] \tag{2.5}$$

since according to (*ii*) first terms compensate each other. Similarly, the equation (1.5) is valid when $\sigma > \sigma''$.

Let us assume that the limit

$$\lim_{\sigma' \to -\infty} S[\sigma', \sigma] = S[-\infty, \sigma] \equiv S[\sigma] \tag{3.5}$$

exists. We have from (*ii*) and (1.5)

$$\begin{aligned} S^{+}[\sigma] &= S^{-1}[\sigma] = S[\sigma, -\infty] \\ S^{+}[\sigma'] * S[\sigma''] &= S[\sigma', \sigma'']. \end{aligned} \tag{4.5}$$

This representation will be useful in the discussion of functional differential equations which follow from the causality condition (1.5).

Differentiation of (1.5) over $\sigma'(x)$ yields

$$\frac{\delta S[\sigma', \sigma'']}{\delta \sigma'(x)} = \frac{\delta S[\sigma', \sigma]}{\delta \sigma'(x)} * S^{+}[\sigma', \sigma] * S'[\sigma', \sigma'']. \tag{5.5}$$

Similarly

$$\frac{\delta S[\sigma', \sigma'']}{\delta \sigma''(y)} = S[\sigma', \sigma''] * S^{+}[\sigma, \sigma''] * \frac{\delta S[\sigma, \sigma'']}{\delta \sigma''(y)}. \tag{6.5}$$

It follows from (4.5) that the matrices

$$\mathscr{K}[\sigma'; x] = -\frac{\delta S[\sigma', \sigma]}{\delta \sigma'(x)} * S^{+}[\sigma', \sigma] \tag{7.5}$$

$$\mathscr{L}[\sigma''; y] = S^{+}[\sigma, \sigma''] * \frac{\delta S[\sigma, \sigma'']}{\delta \sigma''(y)} \tag{8.5}$$

do not depend on σ. In fact we have simply the expressions

$$\mathscr{K}[\sigma'; x] = -\frac{\delta S^{+}[\sigma']}{\delta \sigma'(x)} * S[\sigma'] \tag{9.5}$$

$$\mathscr{L}[\sigma''; y] = S^{+}[\sigma''] * \frac{\delta S[\sigma'']}{\delta \sigma''(y)}. \tag{10.5}$$

Equations (5.5) and (6.5) take the form

$$\frac{\delta S[\sigma', \sigma'']}{\delta \sigma'(x)} = -\mathscr{K}[\sigma'; x] * S[\sigma', \sigma''] \tag{11.5}$$

$$\frac{\delta S[\sigma', \sigma'']}{\delta \sigma''(y)} = S[\sigma', \sigma''] * \mathscr{L}[\sigma''; y]. \tag{12.5}$$

From the unitarity condition (*iv*) we obtain

$$\mathscr{K}[\sigma'; x] + \mathscr{K}^{+}[\sigma'; x] = 0$$
$$\mathscr{L}[\sigma''; y] + \mathscr{L}^{+}[\sigma''; y] = 0. \tag{13.5}$$

From time reversal invariance (*ii*) we derive

$$\frac{\delta S^{+}[\sigma, \sigma']}{\delta\sigma(x)} = \frac{\delta S[\sigma', \sigma]}{\delta\sigma(x)} \tag{14.5}$$

or equivalently

$$(-\mathscr{K}[\sigma; x] * S[\sigma, \sigma'])^{+} = S[\sigma', \sigma] * \mathscr{L}[\sigma; x]. \tag{15.5}$$

Using (*ii*) and the antihermicity property (13.5) of $\mathscr{K}$ we get

$$S[\sigma'; \sigma] * \mathscr{K}[\sigma; x] = S[\sigma'; \sigma] * \mathscr{L}[\sigma; x] \tag{16.5}$$

for any σ, σ'.
Passing to $\sigma' \to \sigma$ we conclude using (*iii*) that

$$\mathscr{K}[\sigma; x] = \mathscr{L}[\sigma; x]. \tag{17.5}$$

Let us denote by $H[\sigma; x]$ the hermitian matrix

$$H[\sigma; x] = i\hbar\mathscr{K}[\sigma; x]. \tag{18.5}$$

We shall call this matrix — Hamiltonian on the mass-shell.

We then may write the equation of motion

$$i\hbar\frac{\delta S[\sigma, \sigma']}{\delta\sigma'(x)} = S[\sigma, \sigma'] * H[\sigma'; x]. \tag{19.5}$$

The second equation follows easily by the hermitian conjugation operation. These differential equations may be considered as a base of a theory formulated directly for observable quantities, *i.e.*, scattering amplitudes on the mass-shell.

It is well known at present [8] that it is possible to formulate a theory in the language of a generating functional depending on only one instead of two variables α, β on which $S[\sigma, \sigma']$ depends. However, in this case unphysical off mass-shell extrapolations of scattering amplitudes are present inherently.

The second part of this paper is devoted to a discussion of the role of the causality condition written for off mass-shell amplitudes.

REFERENCES

[1] W. Garczyński, Quantum Mechanics as a Quantum Markovian Process, *Acta Phys. Polon.*, **35**, 479 (1969).

[2] W. Garczyński, Quantum Markovian Process for One Nonrelativistic Spinless Particle, *Bull. Acad. Polon. Sci.*, **17**, 251 (1969).

[3] W. Garczyński, Quantum Markovian Process for a Nonrelativistic Particle with Spin, *Bul. Acad. Polon. Sci.*, **17**, 257 (1969).

[4] W. Garczyński, Quantum Markovian Process with Denumerable Set of States, *Bull. Acad. Polon. Sci.*, **17**, 517 (1969).

[5] W. Heisenberg, *Z. Phys.*, **120**, 513, 673 (1943).
[6] G. F. Chew, *S-Matrix Theory of Strong Interactions*, Benjamin, INC, New York 1962.
[7] J. Rzewuski, *Field Theory*, Part II, PWN — Polish Scientific Publishers, Warszawa.
[8] J. Rzewuski, *Report at the IV-th Winter School of Theoretical Physics in Karpacz*, February 1967, Proceedings, Vol. I p. 1–42.
[9] F. A. Berezin, *Metod vtoričnogo kvantovanija* (in Russian), Moscow 1965.
[10] A. Markow, *Izv. Akad. Nauk SSSR*, **1**, 61 (1907).
[11] M. Smoluchowski, *Bull. Int. Acad. Polonaise Sci. Lett.*, A, 418, (1913).
[12] A. Kolmogorov, *Math. Ann.*, **104**, 415 (1931).
[13] J. L. Doob, *Stochastic Processes*, New York 1953.
[14] L. Schwartz, *Théorie des distributions*, Hermann, Paris, Part I, 1957.