Vol. A37 (1970) *ACTA PHYSICA POLONICA* Fasc. 4

DETERMINATION OF MAGNETIC STRUCTURES IN hcp CRYSTALS. PART III

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(Received July 23, 1969)

On the base of the Landau and Lifshitz phase transition theory, all possible magnetic structures arising in the nearest vicinity of the transition point from the paramagnetic to the magnetic phase are considered in crystals having the magnetic space group D_{6h}^4R in the paramagnetic phase. The resulting structures related to irreducible representations of D_{6h}^4R belonging to the stars $\{\boldsymbol{k}_2\}$, $\{\boldsymbol{k}_3\}$, $\{\boldsymbol{k}_4\}$, $\{\boldsymbol{k}_6\}$, $\{\boldsymbol{k}_7\}$, $\{\boldsymbol{k}_8\}$, $\{\boldsymbol{k}_9\}$, $\{\boldsymbol{k}_{10}\}$, $\{\boldsymbol{k}_{11}\}$, $\{\boldsymbol{k}_{14}\}$, $\{\boldsymbol{k}_{15}\}$ and $\{\boldsymbol{k}_{17}\}$ are discussed.

As known, just below the disorder-order phase transition point, a magnetic moment density function $\boldsymbol{m}(\boldsymbol{r})$ transforms under the magnetic symmetry operators according to a single irreducible representation of the disorder phase magnetic space group.

Let us first discuss the magnetic orderings arising in the neighbourhood of the transition point in connection with representations of the stars $\{\boldsymbol{k}_{14}\}$, $\{\boldsymbol{k}_{15}\}$, and $\{\boldsymbol{k}_{17}\}$.

The star $\{\boldsymbol{k}_{14}\}$ consists of the vectors: $1/2\,(\boldsymbol{b}_1+\boldsymbol{b}_3)$, $1/2\,(\boldsymbol{b}_2+\boldsymbol{b}_3)$ and $1/2\,(\boldsymbol{b}_1+\boldsymbol{b}_2+\boldsymbol{b}_3)$ (see: Kovalev 1961), where $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, $\boldsymbol{b}_3$ are the basic vectors of the reciprocal hcp lattice. Searching for the minimum value of the thermodynamical potential with respect to a magnetic moment density function just below the Curie point, we obtain information on magnetic orderings arising in the crystal. Two magnetic structures described by the magnetic space groups: $D_{3d}^3(\boldsymbol{a}_3|\hat{R})$ and $D_{3h}^2(\boldsymbol{a}_3|\hat{R})$ respectively, are expected to appear ($\hat{R}$ is the time-inversion operator). Both magnetic unit cells have the basic vectors: $2\boldsymbol{a}_1$, $2\boldsymbol{a}_2$, $2\boldsymbol{a}_3$, where $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$ are the hexagonal basic vectors. Such a magnetic unit cell consists of 16 magnetic sites. The components of magnetic moments attached to them are given below in Table I.

In Table I and, similarly, in further Tables of the present paper, the odd magnetic sites: 1, 3, 5, 7, are localized in a plane perpendicular to the hexagonal axis $\boldsymbol{a}_3$; the even magnetic sites: 2, 4, 6, 8 are localized in a parallel plane displaced along the $\boldsymbol{a}_3$ vector by $1/2|\boldsymbol{a}_3|$; the sites: 9, 10, 11,..., 16 are displaced parallelly by $\boldsymbol{a}_3$ with regard to the sites: 1, 2, 3, ..., 8, respectively. Moreover, it should be noted that the unit axial vector $\boldsymbol{\sigma}_y$ can be directed along one of the hexagonal axes: $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ or $-(\boldsymbol{a}_1+\boldsymbol{a}_2)$ ($\sphericalangle\,(\boldsymbol{a}_1, \boldsymbol{a}_2) = 120°$ and $\sphericalangle\,(\boldsymbol{\sigma}_x, \boldsymbol{\sigma}_y) = 90°$),

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TABLE I

Number of the magnetic structure	Number of the site	m_x, m_y, m_z	Number of the site	m_x, m_y, m_z
1	1	$0, m, 0$	9	$0, -m, 0$
	2	$0, -{}^1/_2m, 0$	10	$0, {}^1/_2m, 0$
	3	$0, -{}^1/_2m, 0$	11	$0, {}^1/_2m, 0$
	4	$0, m, 0$	12	$0, -m, 0$
	5	$0, -{}^1/_2m, 0$	13	$0, {}^1/_2m, 0$
	6	$0, -{}^1/_2m, 0$	14	$0, {}^1/_2m, 0$
	7	$0, 0, 0$	15	$0, 0, 0$
	8	$0, 0, 0$	16	$0, 0, 0$
2	1	$0, 0, m$	9	$0, 0, -m$
	2	$0, 0, l$	10	$0, 0, -l$
	3	$0, 0, m$	11	$0, 0, -m$
	4	$0, 0, l$	12	$0, 0, -l$
	5	$0, 0, m$	13	$0, 0, -m$
	6	$0, 0, l$	14	$0, 0, -l$
	7	$0, 0, n$	15	$0, 0, -n$
	8	$0, 0, p$	16	$0, 0, -p$

whereas $\boldsymbol{\sigma}_z$ is always directed along $\boldsymbol{a}_3$. From Table I, one can conclude that both structures represent the collinear antiferromagnetic type. In structure No. 1, the magnetic moments attached to the particular magnetic sites take two distinct values including zero, whereas in structure No. 2 they exhibit four distinct values.

The star $\{\boldsymbol{k}_{15}\}$ consists of two vectors: $1/3\ (\boldsymbol{b}_1+\boldsymbol{b}_2)+1/2\boldsymbol{b}_3$ and $-1/3\ (\boldsymbol{b}_1+\boldsymbol{b}_2)+1/2\,\boldsymbol{b}_3$. In relation with the irreducible representations of the star $\{\boldsymbol{k}_{15}\}$, the following magnetic orderings are expected to arise in the immediate vicinity of the Curie point:

1. Magnetic space group: $D_{3d}^2(\boldsymbol{a}_3|\hat{R})$. Basic vectors of magnetic unit cell: $\boldsymbol{a}_1-\boldsymbol{a}_2$, $\boldsymbol{a}_1+$ $+2\boldsymbol{a}_2$, $2\boldsymbol{a}_3$. The magnetic unit cell consists of 12 magnetic sites. The ensuing Table gives the components of magnetic moment axial vectors localized in the particular magnetic sites.

It is easily seen that the statistical average magnetic moment vectors are perpendicular to the basic vector $\boldsymbol{a}_3$, and moreover their values are all equal. This structure is antiferromagnetic.

2. Magnetic space group: $D_{3h}^2(\boldsymbol{a}_3|\hat{R})$. Basic vectors of magnetic unit cell: $\boldsymbol{a}_1-\boldsymbol{a}_2$, $\boldsymbol{a}_1+2\boldsymbol{a}_2$, $2\boldsymbol{a}_3$.

3. Magnetic space group: $C_{3h}^1(\boldsymbol{a}_3|\hat{R})$. Basic vectors of magnetic unit cell: $\boldsymbol{a}_1+2\boldsymbol{a}_2$, $-2\boldsymbol{a}_1-\boldsymbol{a}_2$, $2\boldsymbol{a}_3$. In cases *2* and *3*, the magnetic unit cell consists of 12 magnetic sites, 8 of which have non-zero, equal magnetic moments attached to them and directed along the hexagonal axis $\boldsymbol{a}_3$. Hence, one can predict mutually parallel ferrimagnetic planes perpendicular to the vector $\boldsymbol{a}_3$ having ordering $++--\ldots$ in the direction of $\boldsymbol{a}_3$. These ferrimagnetic planes are mutually distant by $1/2|\boldsymbol{a}_3|$. Zero magnetic moments can be attached to the sites: 1, 4, 7 and 10 (*cf.* Table II). Finally, one concludes that both structures are of the antiferromagnetic type.

TABLE II

Number of the site	m_x, m_y, m_z	Number of the site	m_x, m_y, m_z
1	$0,\ m,\ 0$	7	$0,\ -m,\ 0$
2	$\frac{\sqrt{3}}{2}m,\ -\frac{1}{2}m,\ 0$	8	$-\frac{\sqrt{3}}{2}m,\ \frac{1}{2}m,\ 0$
3	$-\frac{\sqrt{3}}{2}m,\ -\frac{1}{2}m,\ 0$	9	$\frac{\sqrt{3}}{2}m,\ \frac{1}{2}m,\ 0$
4	$0,\ m,\ 0$	10	$0,\ -m,\ 0$
5	$\frac{\sqrt{3}}{2}m,\ -\frac{1}{2}m,\ 0$	11	$-\frac{\sqrt{3}}{2}m,\ \frac{1}{2}m,\ 0$
6	$-\frac{\sqrt{3}}{2}m,\ -\frac{1}{2}m,\ 0$	12	$\frac{\sqrt{3}}{2}m,\ \frac{1}{2}m,\ 0$

4. Magnetic space group: $C_{2h}^3(\boldsymbol{a}_3|\hat{R})$. Basic vectors of magnetic unit cell: $2\boldsymbol{a}_3$, $2\boldsymbol{a}_1+\boldsymbol{a}_2$, $\boldsymbol{a}_1+2\boldsymbol{a}_2$. The components of the magnetic moments localized in 12 sites of the magnetic unit cell are given in Table III.

TABLE III

Number of the site	m_x, m_y, m_z	Number of the site	m_x, m_y, m_z
1	$0,\ m,\ 0$	7	$0,\ -m,\ 0$
2	$-l_1,\ l_2,\ -l_3$	8	$l_1,\ -l_2,\ l_3$
3	$l_1,\ l_2,\ l_3$	9	$-l_1,\ -l_2,\ -l_3$
4	$0,\ m,\ 0$	10	$0,\ -m,\ 0$
5	$-l_1,\ l_2,\ -l_3$	11	$l_1,\ -l_2,\ l_3$
6	$l_1,\ l_2,\ l_3$	12	$-l_1,\ -l_2,\ -l_3$

As readily seen, this magnetic structure belongs to the antiferrimagnetic type, too. The magnetic moments localized in planes distant by $|\boldsymbol{a}_3|$ have mutually opposite directions.

Let us now discuss in short magnetic structures exhibited by the crystal in the immediate vicinity of the Curie point in relation with the star $\{\boldsymbol{k}_{17}\}$ consisting of only a single vector $1/2\ \boldsymbol{b}_3$. From our analysis of the minimum value of free energy in the nearest neighbourhood of the Curie point, three possible magnetic structures result.

1. Magnetic space group: $D_{3d}^4(\boldsymbol{a}_3|\hat{R})$. The magnetic unit cell has basic vectors: $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ and $2\boldsymbol{a}_3$, and thus the cell contains 4 magnetic points. The structure is of the antiferromagnetic type. All the magnetic moments are equal in magnitude and are directed along the $\boldsymbol{a}_3$-vector. In the direction perpendicular to the basic verctor $\boldsymbol{a}_3$, ferromagnetic mutually parallel planes are predictable.

2. *Magnetic space group*: $C_{3v}^3(\boldsymbol{a}_3|\hat{R})$. The magnetic unit cell presents basic vectors: $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $2\boldsymbol{a}_3$. This structure is antiferromagnetic. The magnetic moments attached to all four magnetic sites of the unit cell take two distinct values. By analogy to the previous structure, ferromagnetic planes perpendicular to the $\boldsymbol{a}_3$-vector are found, having ordering: $++--\ldots$ The magnetic moments are directed along $\boldsymbol{a}_3$.

3. *Magnetic space group*: $C_{2h}^1(\boldsymbol{a}_3|\hat{R})$. The magnetic unit cell has basic vectors: $\boldsymbol{a}_1-\boldsymbol{a}_2$, $\boldsymbol{a}_1+\boldsymbol{a}_2$, $2\boldsymbol{a}_3$ and contains 8 magnetic points. This magnetic ordering is of an antiferrimagnetic type with magnetic moment vectors in the $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ or $-(\boldsymbol{a}_1+\boldsymbol{a}_2)$ -direction. Two distinct lengths of the magnetic moments are predictable within the magnetic unit cell. The ferrimagnetic mutually parallel planes are perpendicular to $\boldsymbol{a}_3$-axis.

Let us now discuss in brief the magnetic structures arising from the disordered phase in connection with the representations of the group D_{6h}^4R related to the stars: $\{\boldsymbol{k}_2\}$, $\{\boldsymbol{k}_3\}$, $\{\boldsymbol{k}_4\}$, $\{\boldsymbol{k}_6\}$, $\{\boldsymbol{k}_7\}$, $\{\boldsymbol{k}_8\}$, $\{\boldsymbol{k}_9\}$, $\{\boldsymbol{k}_{10}\}$ and $\{\boldsymbol{k}_{11}\}$. These magnetically ordered states represent either a spiral or a periodically modulated structure. One has to keep in mind that magnetic structures continuously varying throughout the crystal cannot be univocally described in terms of magnetic space groups. Thus, we shall consider the basis functions of our representations in more detail. Within the framework of the model of magnetic moments strictly localized at crystal sites, a function of an arbitrary basis can be assumed to have the form:

$$\psi_1(\boldsymbol{r}) = e^{i(\boldsymbol{k}_1,\boldsymbol{r})}\delta(\boldsymbol{r}-\boldsymbol{r}_i), \tag{1.1}$$

where $\boldsymbol{k}_1$ is the first vector of a given star; $\delta(\boldsymbol{r}-\boldsymbol{r}_i)$ is the Dirac delta function, where $\boldsymbol{r}_i=\boldsymbol{r}_{AB}+n_j\boldsymbol{a}_j$ ($j=1, 2, 3$; $\boldsymbol{a}_j$ — the hexagonal basis axis; $n_j=0, 1, 2,$ and $\boldsymbol{r}_{AB} = 1/3\,(\boldsymbol{a}_1-\boldsymbol{a}_2)+1/2\boldsymbol{a}_3$). Other basic vectors can be obtained if we have in mind that

$$\psi_j(\boldsymbol{r}) = \psi_1(\hat{g}_j^{-1}\boldsymbol{r}), \tag{1.2}$$

where $\hat{g}_j \in D_{6h}^4R$; j labels the basic vectors and, moreover, under an arbitrary translation operator $\hat{t}_a$ each basis function $\psi_j(\boldsymbol{r})$ transforms as follows:

$$\hat{t}_a\psi_j(\boldsymbol{r}) = e^{i(\boldsymbol{k}_j,\boldsymbol{r})}\psi_j(\boldsymbol{r}),$$

where $\boldsymbol{k}_j$ is the j-th vector of the star. As known, in the neighbourhood of the Curie point a statistical average magnetic moment density $\boldsymbol{m}(\boldsymbol{r})$ can be expanded as follows:

$$\boldsymbol{m}(\boldsymbol{r}) = \boldsymbol{\sigma}_\alpha\eta_\alpha e^{i\lambda_j^\alpha}\psi_j^\alpha(\boldsymbol{r}), \tag{2}$$

where $\boldsymbol{\sigma}_\alpha$ is the axial unit vector ($\alpha = x, y, z$); η_α accounts for the temperature-dependence of $\boldsymbol{m}(\boldsymbol{r})$. From the physical point of view, we have to require the axial vector function $\boldsymbol{m}(\boldsymbol{r})$ to take only real values when transforming under the elements of D_{6h}^4R. This requirement is fulfilled by imposing adequate conjugate conditions on the basis functions.

Consider first the magnetic ordering with magnetic moment density periodically modulated along the hexagonal axis $\boldsymbol{a}_3$ (or z-axis), with no components in the plane perpendicular to $\boldsymbol{a}_3$. Such ordering can arise in connection with the star $\{\boldsymbol{k}_{11}\}$, consisting of two vectors: $\boldsymbol{k}_1=\mu\boldsymbol{b}_3$ and $\boldsymbol{k}_2=-\boldsymbol{k}_1$, where $0<\mu<1/2$; μ is fixed for a given crystal. The magnetic moment density function $\boldsymbol{m}(\boldsymbol{r})$ can be expressed in the form:

$$\boldsymbol{m}_{1,2}(\boldsymbol{r}) = \pm\boldsymbol{\sigma}_z\eta_{1,2}\cos\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda^{(1,2)}\right)\delta(\boldsymbol{r}-\boldsymbol{r}_i), \tag{3.1}$$

where $\lambda^{(1)}$ and $\lambda^{(2)}$ are the arbitrary phase constant. Both functions $\boldsymbol{m}_{1,2}(\boldsymbol{r})$ exhibit invariance with respect to the translations $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. Moreover, the function $\boldsymbol{m}(\boldsymbol{r})$ is invariant under the operator $(\hat{\sigma}|\hat{R})$ ($\hat{\sigma}$ — the reflection plane containing the $\boldsymbol{a}_3$-axis, $\hat{R}$ — the time-inversion operator), and the function $\boldsymbol{m}_2(\boldsymbol{r})$ invariant under the operator $(\hat{I}|\hat{R})$ ($\hat{I}$ — the inversion operator). This case leads to (001) ferromagnetic planes with magnetic moments lying along the $\boldsymbol{a}_3$-axis. The values of the magnetic moments vary sinusoidally from one (001) plane to another.

Let us next consider the magnetic ordering with axial vector magnetic moment density having only a z-component. This ordering is expected to appear in relation with the star $\{\boldsymbol{k}_6\}$. The set $\{\boldsymbol{k}_6)$ contains the following six vectors: $\boldsymbol{k}_1 = \mu(\boldsymbol{b}_1+\boldsymbol{b}_2)$, $\boldsymbol{k}_2 = -2\mu\boldsymbol{b}_1+\mu\boldsymbol{b}_2$, $\boldsymbol{k}_3 = \mu\boldsymbol{b}_1-2\mu\boldsymbol{b}_2$, $\boldsymbol{k}_4 = -\boldsymbol{k}_1$, $\boldsymbol{k}_5 = -\boldsymbol{k}_2$, $\boldsymbol{k}_6 = -\boldsymbol{k}_3$, where $0<\mu<1/2$. The magnetic moment density is of the form:

$$\boldsymbol{m}_1(\boldsymbol{r}) = \eta_1\boldsymbol{\sigma}_z[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(1)})+\cos((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_{2j}^{(1)}-2\pi\mu)+ \\ +\cos((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(1)}+4\pi\mu)]\,\delta(\boldsymbol{r}-\boldsymbol{r}_i), \tag{3.2}$$

where the coefficient η_1 accounts for the temperature dependence of $\boldsymbol{m}_1(\boldsymbol{r})$, and $\lambda_1^{(1)}, \lambda_2^{(1)}, \lambda_3^{(1)}$ are arbitrary constants. The magnetic ordering is of the periodically modulated type. It exhibits invariance with respect to the translation $\boldsymbol{a}_3$ as well as to the operator $\hat{\sigma}_h$ (reflection in the plane perpendicular to the hexagonal axis $\boldsymbol{a}_3$). The magnetic moments attached to the particular magnetic sites lie along the hexagonal axis $\boldsymbol{a}_3$; their direction is fixed, whereas within each (001) plane their modules vary periodically.

A similar result is obtained in connection with the star $\{\boldsymbol{k}_9\}$ consisting of the vectors:

$$\boldsymbol{k}_1 = \frac{1}{2}\,\boldsymbol{b}_1+\mu\boldsymbol{b}_3, \quad \boldsymbol{k}_2 = \frac{1}{2}\,(-\boldsymbol{b}_1+\boldsymbol{b}_2)+\mu\boldsymbol{b}_3, \quad \boldsymbol{k}_3 = \frac{1}{2}\,\boldsymbol{b}_2+\mu\boldsymbol{b}_3,$$
$$\boldsymbol{k}_4 = -\boldsymbol{k}_1, \quad \boldsymbol{k}_5 = -\boldsymbol{k}_2,\ \boldsymbol{k}_6 = -\boldsymbol{k}_3.$$

The magnetic moment density function is found in the form:

$$\boldsymbol{m}_1(\boldsymbol{r}) = \eta_1\boldsymbol{\sigma}_y[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(1)})-\cos((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(1)})- \\ -\cos((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(1)})]\,\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i). \tag{4}$$

One can show that the magnetic moments localized at magnetic sites lie along one of the hexagonal axes: $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ or $-(\boldsymbol{a}_1+\boldsymbol{a}_2)$ in accordance with the position of $\boldsymbol{\sigma}_y$, and moreover that their direction remains fixed as the temperature lowers. However, their modules vary periodically. It is readily noticed that this magnetic ordering is invariant under the translations: $2\boldsymbol{a}_1$ and $2\boldsymbol{a}_2$. Within each plane perpendicular to the hexagonal axis $\boldsymbol{a}_3$, the magnetic moments attached to particular sites take two distinct values.

We consider next such magnetic structures in which the directions of magnetic moments are free to rotate in an arbitrary plane. Such magnetic ordering is expected to arise at temperatures close to the transition point, in relation with the star $\{\boldsymbol{k}_{11}\}$. The magnetic moment density can be expanded in the following form:

$$\boldsymbol{m}_3(\boldsymbol{r}) = (\eta_x^{(3)}\boldsymbol{\sigma}_x \pm \eta_y^{(3)}\boldsymbol{\sigma}_y)\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda^{(3)})\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i). \tag{5.1}$$

As easily seen, this magnetic ordering is of the sinusoidally modulated type. But the direction of magnetic moments is not fixed and they can rotate in the plane perpendicular to the hexagonal $\boldsymbol{a}_3$-axis as the temperature is lowered from the Curie point. This results from the temperature-dependence of the coefficients $\eta_x^{(3)}$ and $\eta_y^{(3)}$. The structure exhibits invariance with respect to the translations: $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. Analogously, a magnetic structure with magnetic moment axial vectors lying in the plane perpendicular to the axis $\boldsymbol{a}_3$ is expected to appear in connection with the star $\{\boldsymbol{k}_{10}\}$. The star $\{\boldsymbol{k}_{10}\}$ contains the following vectors:

$$\boldsymbol{k}_1 = \frac{1}{3}(\boldsymbol{b}_1+\boldsymbol{b}_2)+\mu\boldsymbol{b}_3, \quad \boldsymbol{k}_2 = -\frac{1}{3}(\boldsymbol{b}_1+\boldsymbol{b}_2)+\mu\boldsymbol{b}_3, \quad \boldsymbol{k}_3 = -\boldsymbol{k}_1$$

and $\boldsymbol{k}_4 = -\boldsymbol{k}_2$, where $0 < \mu < 1/2$. In this case the magnetic moment density can be expressed as follows:

$$\boldsymbol{m}_1(\boldsymbol{r}) = (\eta_x^{(1)}\boldsymbol{\sigma}_x \pm \eta_y^{(1)}\boldsymbol{\sigma}_y)[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(1)})+ \\ +\cos((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(1)}+4/3\pi+\mu\pi)]\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i). \quad (5.2)$$

On the basis of the previous considerations, one easily concludes that the magnetic structure discussed here also exihibits a space periodically modulated type, with magnetic moment axial vectors rotating within the plane perpendicular to the $\boldsymbol{a}_3$-axis when the temperature is lowered. This structure is invariant under the following translations: $\boldsymbol{a}_1-\boldsymbol{a}_2$, $2\boldsymbol{a}_1+\boldsymbol{a}_2$, $\boldsymbol{a}_1+2\boldsymbol{a}_2$.

In connection with the star $\{\boldsymbol{k}_9\}$, it is possible to obtain the magnetic structure with magnetic moment axial vectors whose direction can rotate in the (100) or (010) or (110) hexagonal plane. The corresponding magnetic moment density is obtained in the form:

$$\boldsymbol{m}_2(\boldsymbol{r}) = (\eta_x^{(2)}\boldsymbol{\sigma}_x \pm \eta_z^{(2)}\boldsymbol{\sigma}_z)[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(2)})-\cos((\boldsymbol{k}_2,\boldsymbol{r})+ \\ +\lambda_2^{(2)})-\cos((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(2)})]\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i). \quad (5.3)$$

The function $\boldsymbol{m}_2(\boldsymbol{r})$ exhibits invariance under the translations: $2\boldsymbol{a}_1$ and $2\boldsymbol{a}_2$, only.

In the case of the star $\{\boldsymbol{k}_7\}$ consisting of the vectors:

$$\boldsymbol{k}_1 = \mu\boldsymbol{b}_1+\frac{1}{2}\boldsymbol{b}_3, \quad \boldsymbol{k}_2 = \mu\boldsymbol{b}_2+\frac{1}{2}\boldsymbol{b}_3, \quad \boldsymbol{k}_3 = \mu(\boldsymbol{b}_1-\boldsymbol{b}_2)+\frac{1}{2}\boldsymbol{b}_3,$$
$$\boldsymbol{k}_4 = -\boldsymbol{k}_1, \quad \boldsymbol{k}_5 = -\boldsymbol{k}_2, \quad \boldsymbol{k}_6 = -\boldsymbol{k}_3$$

magnetic orderings similar to the previously considered ones result from the disordered phase in the vicinity of the Curie point. The magnetic moment axial vectors attached to particular crystal points lie in the (001) hexagonal plane, or in one of the possible hexagonal planes containing the $\boldsymbol{a}_3$-axis: (100), (010), (110), (210), (120), or $(1\bar{1}0)$. These structures can be represented as magnetic moment densities of the form:

$$\boldsymbol{m}_1(\boldsymbol{r}) = (\eta_x^{(1)}\boldsymbol{\sigma}_x \pm \eta_y^{(1)}\boldsymbol{\sigma}_y)[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(1)})+\cos((\boldsymbol{k}_2,\boldsymbol{r})+ \\ +\lambda_2^{(1)}+2\pi\mu)+\cos((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(1)}+2\pi\mu)]\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i), \quad (5.4)$$

$$\boldsymbol{m}_2(\boldsymbol{r}) = (\eta_x^{(2)}\boldsymbol{\sigma}_x \pm \eta_z^{(2)}\boldsymbol{\sigma}_z)[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(2)})+\cos((\boldsymbol{k}_2,\boldsymbol{r})+ \\ +\lambda_2^{(2)}+2\pi\mu)+\cos((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(2)}+2\pi\mu)]\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i), \quad (5.5)$$

$$m_3(r) = (\eta_y^{(3)}\sigma_y \pm \eta_z^{(3)}\sigma_z)[\cos((k_1, r)+\lambda_1^{(3)})+\cos((k_1, r)+ +\lambda_2^{(3)}+2\pi\mu)+\cos((k_3, r)+\lambda_3^{(3)}+2\pi\mu)]\delta(r-r_i). \quad (5.6)$$

Thus, one obtains analogous magnetic structures with magnetic moments that can rotate in the above-mentioned hexagonal planes, respectively, as the temperature decreases below the transition. These structures are invariant under the translation $2a_3$ and moreover, under the operator $(\hat{t}_{a_3}|\hat{R})$. The latter implies a mutual antiferromagnetic ordering of the (001) hexagonal planes.

In connection with the star $\{k_6\}$, one can expect a magnetic structure of the same type. The statistical average magnetic moment density expansion is of the form:

$$m_2(r) = (\eta_x^{(2)}\sigma_x \pm \eta_y^{(2)}\sigma_y)[\cos((k_1, r)+\lambda_1^{(2)})+\cos((k_2, r)+ +\lambda_2^{(2)}+2\pi\mu)+\cos((k_3, r)+\lambda_3^{(2)}+4\pi\mu)]\delta(r-r_i). \quad (5.7)$$

Invariance under the operator $(\sigma_\lambda|R)$ as well as under the translation a_3 is obtained. With decreasing temperature, the magnetic moments can change their direction within the (001) hexagonal plane, whereas their modules vary periodically throughout this plane. In this case, the mutual ordering of (001) hexagonal planes is ferromagnetic.

We now consider in brief magnetic orderings, in which the magnetic moments attached to magnetic sites can rotate in space, according to the temperature dependence of the ratio: $(\eta_x : \eta_y : \eta_z)$ as the temperature is lowered from the Curie point. Such orderings are determined by an axial vector magnetic moment density having components along three axes.

A magnetic ordering of this type is expected to arise below the transition in connection with the star $\{k_{10}\}$. The magnetic moment density is expanded as follows:

$$m_2(r) = (\eta_x^{(2)}\sigma_x \pm \eta_y^{(2)}\sigma_y \pm \eta_z^{(2)}\sigma_z)[\cos((k_1, r)+\lambda_1^{(2)})+ +\cos((k_2, r)+\lambda_2^{(2)}+3/2\pi)+\cos((k_3, r)+\lambda_3^{(2)}+4/3\pi)- -\cos((k_4, r)+\lambda_4^{(2)})]\delta(r-r_i). \quad (6.1)$$

This structure is invariant under the translations: a_1-a_2, $2a_1+a_2$, a_1+2a_2. The direction of magnetic moment axial vectors depends on the temperature as expressed by way of the coefficients $\eta_x^{(2)}$, $\eta_y^{(2)}$ and $\eta_z^{(2)}$. The modules of the magnetic moments vary periodically throughout the crystal.

A magnetic structure of the same type can be shown to appear when the transition related with the star $\{k_8\}$ occurs. The star $\{k_8\}$ contains the following vectors:

$$k_1 = \mu(b_1+b_2)+1/2b_3,\ k_2 = \mu(-2b_1+b_2)+1/2b_3,\ k_3 = \mu(b_1-2b_2)+1/2b_3$$

and $k_4 = -k_1$, $k_5 = -k_2$, $k_6 = -k_3$, where $0 < \mu < 1/2$.

The magnetic moment density expansion is given as:

$$m(r) = (\eta_x\sigma_x \pm \eta_y\sigma_y \pm \eta_z\sigma_z)[\cos((k_1, r)+\lambda_1)+ +\sin((k_1, r)+\lambda_2+2\pi\mu)+\cos((k_2, r)+\lambda_3+4\pi\mu)+\sin((k_2, r)+ +\lambda_4+4\pi\mu)+\cos((k_3, r)+\lambda_5+2\pi\mu)+\sin((k_3, r)+\lambda_6)]\delta(r-r_i). \quad (6.2)$$

The considered structure exhibits invariance with respect to the translation: $2\boldsymbol{a}_3$ as well as to the operator $(\hat{t}_{\boldsymbol{a}_3}|\hat{R})$. Thus one can observe an antiferromagnetic ordering of the (001) hexagonal planes.

Similarly, in the case of the star $\{\boldsymbol{k}_7\}$ one can expect, at temperatures close to the Curie point a "space" periodically modulated magnetic structure. The corresponding magnetic moment density is of the form:

$$\boldsymbol{m}_4(\boldsymbol{r}) = (\eta_x^{(4)}\boldsymbol{\sigma}_x \pm \eta_y^{(4)}\boldsymbol{\sigma}_y \pm \eta_z^{(4)}\boldsymbol{\sigma}_z)[\cos((\boldsymbol{k}_1, \boldsymbol{r}) + \lambda_1^{(4)}) + \\ + \cos((\boldsymbol{k}_2, \boldsymbol{r}) + \lambda_2^{(4)} + 2\pi\mu) + \cos((\boldsymbol{k}_3, \boldsymbol{r}) + \lambda_3^{(4)} + 2\pi\mu)]\boldsymbol{\delta}(\boldsymbol{r} - \boldsymbol{r}_i). \tag{6.3}$$

This magnetic ordering exhibits invariance with respect to the translation $2\boldsymbol{a}_3$ and, moreover, to the operator $(\hat{t}_{\boldsymbol{a}_3}|\hat{R})$. As known, such invariance determines an antiferromagnetic ordering of the (001) planes.

A quite similar result, although in a more complicated form, is obtained when the star $\{\boldsymbol{k}_4\}$ is discussed. The set $\{\boldsymbol{k}_4\}$ contains 12 vectors:

$$\boldsymbol{k}_1 = \mu(\boldsymbol{b}_1 + \boldsymbol{b}_2) + \mu_3\boldsymbol{b}_3,\ \boldsymbol{k}_2 = \mu(-\boldsymbol{b}_1 + 2\boldsymbol{b}_2) + \mu_3\boldsymbol{b}_3,\ \boldsymbol{k}_3 = \mu(-2\boldsymbol{b}_1 + \boldsymbol{b}_2) + \mu_3\boldsymbol{b}_3, \\ \boldsymbol{k}_4 = -\mu(\boldsymbol{b}_1 + \boldsymbol{b}_2) + \mu_3\boldsymbol{b}_3,\ \boldsymbol{k}_5 = \mu(\boldsymbol{b}_1 - 2\boldsymbol{b}_2) + \mu_3\boldsymbol{b}_3, \\ \boldsymbol{k}_6 = \mu(2\boldsymbol{b}_1 - \boldsymbol{b}_2) + \mu_3\boldsymbol{b}_3$$

and

$$\boldsymbol{k}_7 = -\boldsymbol{k}_6,\ \boldsymbol{k}_8 = -\boldsymbol{k}_1,\ \boldsymbol{k}_9 = -\boldsymbol{k}_2,\ \boldsymbol{k}_{10} = -\boldsymbol{k}_3,\ \boldsymbol{k}_{11} = -\boldsymbol{k}_4,\ \boldsymbol{k}_{12} = -\boldsymbol{k}_5,$$

where $0 < \mu \neq \mu_3 < 1/2$.

The magnetic structure can be represented as a magnetic moment density of the form:

$$\boldsymbol{m}_1(\boldsymbol{r}) = (\eta_x^{(1)}\boldsymbol{\sigma}_x \pm \eta_y^{(1)}\boldsymbol{\sigma}_y \pm \eta_z^{(1)}\boldsymbol{\sigma}_z)[\cos((\boldsymbol{k}_1, \boldsymbol{r}) + \lambda_1^{(1)}) + \\ + \cos((\boldsymbol{k}_2, \boldsymbol{r}) + \lambda_2^{(1)} + 2\pi\mu + \pi\mu_3) + \cos((\boldsymbol{k}_3, \boldsymbol{r}) + \lambda_3^{(1)} + 4\pi\mu) + \\ + \cos((\boldsymbol{k}_4, \boldsymbol{r}) + \lambda_4^{(1)} + 4\pi\mu + \pi\mu_3) + \cos((\boldsymbol{k}_5, \boldsymbol{r}) + \lambda_5^{(1)} + 2\pi\mu) + \\ + \cos((\boldsymbol{k}_6, \boldsymbol{r}) + \lambda_6^{(1)} + \pi\mu_3)]\boldsymbol{\delta}(\boldsymbol{r} - \boldsymbol{r}_i). \tag{6.4}$$

It is easy to show that this magnetic ordering does not exhibit invariance with respect to any operator of D_{6h}^4R. The magnetic moments attached to the crystal sites can rotate as the temperature decreases below the Curie point. Similarly in the case of the star $\{\boldsymbol{k}_3\}$ one can expect to obtain a "space" periodically modulated magnetic structure. The star $\{\boldsymbol{k}_3\}$ consists of the vectors:

$$\boldsymbol{k}_1 = \mu_1\boldsymbol{b}_1 + \mu\boldsymbol{b}_3,\ \boldsymbol{k}_2 = \mu_1(-\boldsymbol{b}_1 + \boldsymbol{b}_2) + \mu\boldsymbol{b}_3,\ \boldsymbol{k}_3 = \mu_1\boldsymbol{b}_2 + \mu\boldsymbol{b}_3, \\ \boldsymbol{k}_4 = \mu_1(\boldsymbol{b}_1 - \boldsymbol{b}_2) + \mu\boldsymbol{b}_3,\ \boldsymbol{k}_5 = -\mu_1\boldsymbol{b}_1 + \mu\boldsymbol{b}_3,\ \boldsymbol{k}_6 = \mu_1\boldsymbol{b}_2 + \mu\boldsymbol{b}_3$$

and

$$\boldsymbol{k}_7 = -\boldsymbol{k}_1,\ \boldsymbol{k}_8 = -\boldsymbol{k}_2,\ \boldsymbol{k}_9 = -\boldsymbol{k}_3,\ \boldsymbol{k}_{10} = -\boldsymbol{k}_4,\ \boldsymbol{k}_{11} = -\boldsymbol{k}_5,\ \boldsymbol{k}_{12} = -\boldsymbol{k}_6,$$

where $0 < \mu \neq \mu_1 < 1/2$.

The statistical average magnetic moment density exhibited by the crystal below the transition can be expanded as follows:

$$\boldsymbol{m}_1(\boldsymbol{r}) = (\eta_x^{(1)}\boldsymbol{\sigma}_x \pm \eta_y^{(1)}\boldsymbol{\sigma}_y \pm \eta_z^{(1)}\boldsymbol{\sigma}_z)[\cos((\boldsymbol{k}_1, \boldsymbol{r}) + \lambda_1^{(1)})) + \cos((\boldsymbol{k}_2, \boldsymbol{r}) + \\ + \lambda_2^{(1)} + 2\pi(^1/_3\,\mu_1 + ^1/_2\mu)) + \cos((\boldsymbol{k}_3, \boldsymbol{r}) + \lambda_3^{(1)} + 2\pi\mu_1) + \cos((\boldsymbol{k}_4, \boldsymbol{r}) + \\ + \lambda_4^{(1)} + 2\pi(^4/_3\mu_1 + ^1/_2\,\mu)) + \cos((\boldsymbol{k}_5, \boldsymbol{r}) + \lambda_5^{(1)} + 2\pi\mu_1) + \\ + \cos((\boldsymbol{k}_6, \boldsymbol{r}) + \lambda_6^{(1)} + 2\pi(^1/_3\,\mu_1 + ^1/_2\,\mu))]\boldsymbol{\delta}(\boldsymbol{r} - \boldsymbol{r}_i). \qquad (6.5)$$

As previously, the magnetic moments can rotate as the temperature is lowered.

Let us finally consider the last example of "space" periodically modulated structure, related with the star $\{\boldsymbol{k}_2\}$, consisting of the following vectors:

$$\boldsymbol{k}_1 = \mu_1\boldsymbol{b}_1 + \mu\boldsymbol{b}_2 + ^1/_2\boldsymbol{b}_3,\ \boldsymbol{k}_2 = -\mu\boldsymbol{b}_1 + (\mu_1 + \mu)\boldsymbol{b}_2 + ^1/_2\boldsymbol{b}_3, \\ \boldsymbol{k}_3 = -(\mu_1 + \mu)\boldsymbol{b}_1 + \mu_1\boldsymbol{b}_2 + ^1/_2\boldsymbol{b}_3,\ \boldsymbol{k}_4 = -\mu\boldsymbol{b}_1 - \mu_1\boldsymbol{b}_2 + ^1/_2\boldsymbol{b}_3, \\ \boldsymbol{k}_5 = -(\mu_1 + \mu)\boldsymbol{b}_1 + \mu\boldsymbol{b}_2 + ^1/_2\boldsymbol{b}_3,\ \boldsymbol{k}_6 = \mu_1\boldsymbol{b}_1 - (\mu_1 + \mu)\boldsymbol{b}_2 + ^1/_2\boldsymbol{b}_3, \\ \boldsymbol{k}_7 = -\boldsymbol{k}_1,\ \boldsymbol{k}_8 = -\boldsymbol{k}_2,\ \boldsymbol{k}_9 = -\boldsymbol{k}_3,\ \boldsymbol{k}_{10} = -\boldsymbol{k}_4,\ \boldsymbol{k}_{11} = -\boldsymbol{k}_5,\ \boldsymbol{k}_{12} = -\boldsymbol{k}_6,$$

where $0 < \mu_1 \neq \mu < ^1/_2$.

The magnetic moment density expansion is of the form:

$$\boldsymbol{m}_1(\boldsymbol{r}) = (\eta_x^{(1)}\boldsymbol{\sigma}_x \pm \eta_y^{(1)}\boldsymbol{\sigma}_y \pm \eta_z^{(1)}\boldsymbol{\sigma}_z)[\cos((\boldsymbol{k}_1, \boldsymbol{r}) + \lambda_1^{(1)}) + \\ + \sin((\boldsymbol{k}_2, \boldsymbol{r}) + \lambda_2^{(1)} + 2\pi(^1/_3\,\mu_1 + ^2/_3\,\mu)) + \cos((\boldsymbol{k}_3, \boldsymbol{r}) + \lambda_3^{(1)} + 2\pi(\mu_1 + \mu)) + \\ + \sin((\boldsymbol{k}_4, \boldsymbol{r}) + \lambda_4^{(1)} + 2\pi(^2/_3\,\mu_1 + ^1/_3\mu)) + \cos((\boldsymbol{k}_5, \boldsymbol{r}) + \lambda_5^{(1)} + \\ + 2\pi\mu_1) + \sin((\boldsymbol{k}_6, \boldsymbol{r}) + \lambda_6^{(1)} + 2\pi(^1/_3\,\mu_1 - ^1/_3\,\mu))]\boldsymbol{\delta}(\boldsymbol{r} - \boldsymbol{r}_i). \qquad (6.6)$$

This magnetic ordering is found to exhibit invariance with respect to the translation $2\boldsymbol{a}_3$ and the operator $(\hat{t}_{\boldsymbol{a}_3}|R)$. Therefore, this structure is antiferromagnetically ordered with respect to the hexagonal axis $\boldsymbol{a}_3$.

Let us now proceed to discuss spiral magnetic structures. First, we consider spiral structures, in which the direction of magnetic moments, localized in particular crystal sites, is restricted to an arbitrary plane. Consequently, the magnetic moment density consists of two components. Such magnetic ordering can arise in the neighbourhood of the Curie point in relation with the star $\{\boldsymbol{k}_9\}$. The magnetic moment density describing this spiral structure can be expanded here as follows:

$$\boldsymbol{m}_3(\boldsymbol{r}) = \{\eta_x^{(3)}\boldsymbol{\sigma}_x[\cos((\boldsymbol{k}_1, \boldsymbol{r}) + \lambda_1^{(3)}) - \cos((\boldsymbol{k}_2, \boldsymbol{r}) + \lambda_2^{(3)}) - \\ - \cos((\boldsymbol{k}_3, \boldsymbol{r}) + \lambda_3^{(3)})] \pm \eta_y^{(3)}\boldsymbol{\sigma}_y[\sin((\boldsymbol{k}_1, \boldsymbol{r}) + \lambda_1^{(3)}) - \sin((\boldsymbol{k}_2, \boldsymbol{r}) + \lambda_2^{(3)}) - \\ - \sin((\boldsymbol{k}_3, \boldsymbol{r}) + \lambda_3^{(3)})]\}\boldsymbol{\delta}(\boldsymbol{r} - \boldsymbol{r}_i). \qquad (7.1)$$

By our previous considerations, this magnetic structure is invariant under the translations $2\boldsymbol{a}_1$ and $2\boldsymbol{a}_2$. The modules are constant with respect to position in the crystal, whereas their direction can rotate freely within the (001) hexagonal plane from one position to another. Rotation of magnetic moments with decreasing temperature is also predicted according to the ratio:

$$(\eta_x^{(3)} : \eta_y^{(3)}).$$

A similar result is expected to obtain in the case of the star $\{\boldsymbol{k}_7\}$. The magnetic structures are represented by one of the magnetic moment densities expanded as follows:

$$\begin{aligned}\boldsymbol{m}_4(\boldsymbol{r}) = \{\eta_x^{(4)}\boldsymbol{\sigma}_x[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(4)})+\cos((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(4)}+2\pi\mu)+\\ +\cos((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(4)}+2\pi\mu)]\pm\eta_y^{(4)}\boldsymbol{\sigma}_y[\sin((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(4)})+\\ +\sin((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(4)}+2\pi\mu)+\sin((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(4)}+2\pi\mu)]\}\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i),\end{aligned}\tag{7.2}$$

$$\begin{aligned}\boldsymbol{m}_5(\boldsymbol{r}) = \{\eta_x^{(5)}\boldsymbol{\sigma}_x[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(5)})+\cos((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(5)}+2\pi\mu)+\\ +\cos((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(5)}+2\pi\mu)]\pm\eta_z^{(5)}\boldsymbol{\sigma}_z[\sin((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(5)})+\\ +\sin((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(5)}+2\pi\mu)+\sin((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(5)}+2\pi\mu)]\}\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i),\end{aligned}\tag{7.3}$$

$$\begin{aligned}\boldsymbol{m}_6(\boldsymbol{r}) = \{\eta_y^{(6)}\boldsymbol{\sigma}_y[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(6)})+\cos((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(6)}+2\pi\mu)+\\ +\cos((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(6)}+2\pi\mu)]\pm\eta_z^{(6)}\boldsymbol{\sigma}_z[\sin((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(6)})+\sin((\boldsymbol{k}_2,\boldsymbol{r})+\\ +\lambda_2^{(6)}+2\pi\mu)+\sin((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(6)}+2\pi\mu)]\}\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i).\end{aligned}\tag{7.4}$$

It should be noted that the corresponding magnetic structures are of the "plane" spiral type in the hexagonal planes: (001) (see (7.2)), (100), (010), (110) (see: (7.3)) and (210), (120), $(1\bar{1}0)$ (see (7.4)), respectively. Invariance with respect to the translation $2\boldsymbol{a}_3$ as well as the operator $(\hat{t}_{\boldsymbol{a}_3}|\hat{R})$ is exhibited by the structures. One can therefore observe an antiferromagnetic ordering of the (001) hexagonal planes.

The "plane" spiral magnetic structures can be found also in connection with the star $\{\boldsymbol{k}_6\}$. In this case, the magnetic moment axial vectors lie in the (001) plane. Their modules are constant with respect to position and can vary with the temperature only. The magnetic moment density determining such ordering is given by the following expansion:

$$\begin{aligned}\boldsymbol{m}_3(\boldsymbol{r}) = \{\eta_x^{(3)}\boldsymbol{\delta}_x[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(3)})+\cos((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(3)}+2\pi\mu)+\\ +\cos((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(3)}+4\pi\mu)]\pm\eta_y^{(3)}\boldsymbol{\sigma}_y[\sin((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(3)})+\\ +\sin((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(3)}+2\pi\mu)+\sin((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(3)}+4\pi\mu)]\}\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i).\end{aligned}\tag{7.5}$$

This magnetic ordering is invariant under the translation $\boldsymbol{a}_3$ as well as under the operator $(\hat{\sigma}_h|\hat{R})$.

Finally, we proceed to consider spiral magnetic configurations described by axial vector magnetic moment density functions having three components. From our previous considerations, we easily conclude that the modules of magnetic moments localized at magnetic points vary throughout the crystal and with temperature decreasing below the Curie point.

We now give the magnetic moment density functions determining such structures in relation with the distinct stars.

Star $\{\boldsymbol{k}_2\}$. Magnetic moment density expansion:

$$\begin{aligned}\boldsymbol{m}_2(\boldsymbol{r}) = \{(\eta_x^{(2)}\boldsymbol{\sigma}_x\pm\eta_y^{(2)}\boldsymbol{\sigma}_y)[\cos((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(2)})+\sin((\boldsymbol{k}_2,\boldsymbol{r})+\\ +\lambda_2^{(2)}+2\pi({}^1/_3\,\mu_1+{}^2/_3\,\mu))+\cos((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(2)}+2\pi(\mu_1+\mu))+\\ +\sin((\boldsymbol{k}_4,\boldsymbol{r})+\lambda_4^{(2)}+2\pi({}^1/_3\,\mu_1+{}^2/_3\,\mu))+\cos((\boldsymbol{k}_5,\boldsymbol{r})+\lambda_5^{(2)}+2\pi\mu_1)+\end{aligned}$$

$$+\sin\left((\boldsymbol{k}_6,\boldsymbol{r})+\lambda_6^{(2)}+2\pi(^1/_3\,\mu_1-{}^1/_3\,\mu)\right)]\pm\eta_z^{(2)}\boldsymbol{\sigma}_z[\sin\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(2)}\right)+$$
$$+\cos\left((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(2)}+2\pi(^1/_3\,\mu_1+{}^2/_3\,\mu)\right)+\sin\left((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(2)}+2\pi(\mu_1+\mu)\right)+$$
$$+\cos\left((\boldsymbol{k}_4,\boldsymbol{r})+\lambda_4^{(2)}+2\pi(^1/_3\,\mu_1+{}^2/_3\,\mu)\right)+\sin\left((\boldsymbol{k}_5,\boldsymbol{r})+\lambda_5^{(2)}+2\pi\mu_1\right)+$$
$$+\cos\left((\boldsymbol{k}_6,\boldsymbol{r})+\lambda_6^{(2)}+2\pi(^1/_3\,\mu_1-{}^1/_3\,\mu)\right)]\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i). \tag{8.1}$$

This ordering is invariant under the translation $2\boldsymbol{a}_3$ and the operator $(\hat{t}_{\boldsymbol{a}_3}|\hat{R})$. The magnetic moments can rotate within a well-defined plane as their position varies throughout the crystal and as the temperature is lowered.

Star $\{\boldsymbol{k}_3\}$. Magnetic moment density expansion:

$$\boldsymbol{m}_2(\boldsymbol{r}) = \{(\eta_x^{(2)}\boldsymbol{\sigma}_x\pm\eta_y^{(2)}\boldsymbol{\sigma}_y)[\cos\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(2)}\right)+$$
$$+\cos\left((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(2)}+2\pi(^1/_3\mu_1+{}^1/_3\mu)\right)+\cos\left((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(2)}+2\pi\mu_1\right)+$$
$$+\cos\left((\boldsymbol{k}_4,\boldsymbol{r})+\lambda_4^{(2)}+2\pi(^4/_3\mu_1+{}^1/_2\mu)\right)+\cos\left((\boldsymbol{k}_5,\boldsymbol{r})+\lambda_5^{(2)}+2\pi\mu_1\right)+$$
$$+\cos\left((\boldsymbol{k}_6,\boldsymbol{r})+\lambda_6^{(2)}+2\pi(^1/_3\mu_1+{}^1/_2\mu)\right)]\pm\eta_z^{(2)}\boldsymbol{\sigma}_z[\sin\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(2)}\right)+$$
$$+\sin\left((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(2)}+2\pi(^1/_3\mu_1+{}^1/_2\mu)\right)+\sin\left((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(2)}+2\pi\mu_1\right)+$$
$$+\sin\left((\boldsymbol{k}_4,\boldsymbol{r})+\lambda_4^{(2)}+2\pi(^4/_3\mu_1+{}^1/_2\mu)\right)+\sin\left((\boldsymbol{k}_5,\boldsymbol{r})+\lambda_5^{(2)}+2\pi\mu_1\right)+$$
$$+\sin\left((\boldsymbol{k}_6,\boldsymbol{r})+\lambda_6^{(2)}+2\pi(^1/_3\mu_1+{}^1/_2\mu)\right)]\}\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i). \tag{8.2}$$

Star $\{\boldsymbol{k}_4\}$. Magnetic moment density expansion:

$$\boldsymbol{m}_2(\boldsymbol{r}) = \{(\eta_x^{(2)}\boldsymbol{\sigma}_x\pm\eta_y^{(2)}\boldsymbol{\sigma}_y)[\cos\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(2)}\right)+$$
$$+\cos\left((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(2)}+2\pi\mu+\pi\mu_3\right)+\cos\left((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(2)}+4\pi\mu\right)+$$
$$+\cos\left((\boldsymbol{k}_4,\boldsymbol{r})+\lambda_4^{(2)}+4\pi\mu+\pi\mu_3\right)+\cos\left((\boldsymbol{k}_5,\boldsymbol{r})+\lambda_5^{(2)}+2\pi\mu\right)+$$
$$+\cos\left((\boldsymbol{k}_6,\boldsymbol{r})+\lambda_6^{(2)}+\pi\mu_3\right)]\pm\eta_z^{(2)}\boldsymbol{\sigma}_z[\sin\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(2)}\right)+$$
$$+\sin\left((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(2)}+2\pi\mu+\pi\mu_3\right)+\sin\left((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(2)}+4\pi\mu\right)+$$
$$+\sin\left((\boldsymbol{k}_4,\boldsymbol{r})+\lambda_4^{(2)}+4\pi\mu+\pi\mu_3\right)+\sin\left((\boldsymbol{k}_5,\boldsymbol{r})+\lambda_5^{(2)}+2\pi\mu\right)+$$
$$+\sin\left((\boldsymbol{k}_6,\boldsymbol{r})+\lambda_6^{(2)}+\pi\mu_3\right)]\}\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i). \tag{8.3}$$

Star $\{\boldsymbol{k}_7\}$. Magnetic moment density expansions:

$$\boldsymbol{m}_4(\boldsymbol{r}) = \{(\eta_x^{(4)}\boldsymbol{\sigma}_x\pm\eta_y^{(4)}\boldsymbol{\sigma}_y)[\cos\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(4)}\right)+$$
$$+\cos\left((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(4)}+2\pi\mu\right)+\cos\left((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(4)}+2\pi\mu\right)]\pm\eta_z^{(4)}\boldsymbol{\sigma}_z[\sin\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(4)}\right)+$$
$$+\sin\left((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(4)}+2\pi\mu\right)+\sin\left(\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(4)}+2\pi\mu\right]\}\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i), \tag{8.4}$$

$$\boldsymbol{m}_5(\boldsymbol{r}) = \{(\eta_x^{(5)}\boldsymbol{\sigma}_x\pm\eta_z^{(5)}\boldsymbol{\sigma}_z)\ [\cos\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(5)}\right)+\cos\left((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(5)}+2\pi\mu\right)+$$
$$+\cos\left((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(5)}+2\pi\mu\right)]\pm\eta_y^{(5)}\boldsymbol{\sigma}_y[\sin\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(5)}\right)+\sin\left((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(5)}+2\pi\mu\right)+$$
$$+\sin\left((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(5)}+2\pi\mu\right)]\}\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}_i), \tag{8.5}$$

$$\boldsymbol{m}_6(\boldsymbol{r}) = \{(\eta_y^{(6)}\boldsymbol{\sigma}_y\pm\eta_z^{(6)}\boldsymbol{\sigma}_z)\ [\cos\left((\boldsymbol{k}_1,\boldsymbol{r})+\lambda_1^{(6)}\right)+$$
$$+\cos\left((\boldsymbol{k}_2,\boldsymbol{r})+\lambda_2^{(6)}+2\pi\mu\right)+\cos\left((\boldsymbol{k}_3,\boldsymbol{r})+\lambda_3^{(6)}+2\pi\mu\right)]\pm$$

$$\pm \eta_x^{(6)} \boldsymbol{\sigma}^x [\sin((\boldsymbol{k}_1, \boldsymbol{r}) + \lambda_1^{(6)}) + \sin((\boldsymbol{k}_2, \boldsymbol{r}) + \lambda_2^{(6)} + 2\pi\mu) + \\ + \sin((\boldsymbol{k}_3, \boldsymbol{r}) + \lambda_3^{(6)} + 2\pi\mu)]\} \boldsymbol{\delta}(\boldsymbol{r} - \boldsymbol{r}_i). \tag{8.6}$$

These three magnetic configurations ((8.4), (8.5), (8.6)) correspond to three distinct positions of the plane within which the magnetic moments can rotate as one proceeds throughout the crystal. Invariance under the translation $2\boldsymbol{a}_3$ as well as under the operator $(\hat{t}_{\boldsymbol{a}_3}|\hat{R})$ is predicted. As known, the magnetic moments can rotate also as the temperature lowers according to the temperature-dependence of η_x, η_y and η_z.

In concluding, let us briefly summarize the information obtained by applying the Landau–Lifshitz theory to a single second-order phase transition from the paramagnetic to the magnetic state. It is possible to specify all the types of magnetic configurations which can exist in the immediate vicinity of the transition point if only the magnetic space group of the disordered paramagnetic phase is known. It is also found that, in the region of temperatures just below the Curie point, conical or ferromagnetic spiral orderings cannot arise from the paramagnetic phase in hcp crystals.

The author wishes to thank Docent Dr L. Kowalewski for critically reading the manusscript as well as for his suggestions. Thanks are also due to Mr Ernö Nagy, M. Sc., for calculating and discussing the results relating to the stars $\{\boldsymbol{k}_{17}\}$ and $\{\boldsymbol{k}_{11}\}$. The author is much indebted to Dr P. Szweykowski for his advice concerning the final results of this work.

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