THE ELECTRODYNAMICS OF THE FERROMAGNETIC FERMI LIQUID FOR LONGITUDINAL MAGNETIC FIELD

By J. Czerwonko

Institute of Mathematics and Theoretical Physics, Technical University of Wrocław*

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In this paper a simple model of band ferromagnetism is discussed. The model is based on the assumption that Fermi surfaces for both spins are spherical, but the quasiparticles interact. The applied method is a modification of Landau's phenomenological approach. We calculate the quasistatic and quasihomogeneous reactions of the system, with the provision that the induced magnetic field is parallel to the axis of magnetization. Some of our results are valid for an arbitrary band structure and for an arbitrary form of Fermi surfaces.

1. Introduction

We shall analyze here a simple model of band ferromagnetism. It will be assumed that we have a single band and that the Fermi surfaces are spherical for both spins. The effective interaction between quasiparticles on the Fermi surface depends only on the angle between the momenta of the quasiparticles. A model such as this was considered by Dzyaloshinskii [1,2], Kondratenko, [3], and this author [4, 5]. The method applied in [1–5] consists in a generalization of the theory of normal Fermi liquids developed by Landau [6]. The main formulae for the ferromagnetic Fermi liquid can be written as follows (see [1–4])

$$N_\alpha = p_\alpha^3/6\pi^2 \tag{1}$$

$$\left(\frac{\partial N}{\partial \mu}\right) = \frac{1}{2\pi^2\tilde{w}_0}\left[Bx_1^2 + Ax_{\bar{1}}^2 - 2Cx_1x_{\bar{1}}\right] \tag{2}$$

$$\frac{\partial(N_1 - N_{\bar{1}})}{\partial\mu} = \frac{1}{2\pi^2\tilde{w}_0}\left[Bx_1^2 - Ax_{\bar{1}}^2\right] = -\frac{1}{\mu_B}\left(\frac{\partial N}{\partial H}\right) \tag{3}$$

$$\chi_N = 2\mu_B^2(x_1x_{\bar{1}})^2\left[\pi^2(Bx_1^2 + Ax_{\bar{1}}^2 - 2Cx_1x_{\bar{1}})\right]^{-1} \tag{4}$$

$$C_v = \frac{T}{6}(x_1^2 + x_{\bar{1}}^2) + 0(T^{3/2}), \qquad T \geqslant 2\mu_B H_0 \tag{5}$$

$$\left(\frac{m}{m_\alpha}\right)(1 + f_{\omega 1}^{\alpha\alpha}) + f_{\omega 1}^{1\bar{1}}\left(\frac{p_{\bar{\alpha}}}{p_\alpha}\right)^{3/2}\left(\frac{m^2}{m_1 m_{\bar{1}}}\right)^{1/2} = 1 \tag{6}$$

* Address: Instytut Matematyki i Fizyki Teoretycznej Politechniki Wrocławskiej, Wrocław, Pl. Grunwaldzki 9, Polska.

Here, p_α and m_α denote the Fermi momentum and the effective mass of electrons with spin $\alpha(\alpha = \pm 1)$, respectively, N_α the number of electrons with spin α, and $N = N_1 + N_{\bar{1}}$, $(\bar{\alpha} \equiv -\alpha)$. Moreover, μ is the chemical potential μ_B is Bohr's magneton, H is the external magnetic field, χ_N the longitudinal susceptibility for constant N, and $x_\alpha \equiv (m_\alpha p_\alpha)^{1/2}$. All derivatives with respect to μ are taken at constant H, and *vice versa*.

We have also

$$\begin{pmatrix} A & C \\ C & B \end{pmatrix} \equiv \delta_{\alpha\beta} + f^{\alpha\beta}_{\omega 0}, \ \tilde{w}_0 = AB - C^2 \tag{7}$$

where $f^{\alpha\beta}_{\omega l}$ are partial wave projections of l-th order for the amplitude representing dimensionless effective interaction of quasiparticles $f^{\alpha\beta}_\omega(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}')$. The partial wave amplitudes are defined as

$$f^{\alpha\beta}_\omega(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}') = \sum_{l=0}^{\infty} (2l+1) f^{\alpha\beta}_{\omega l} P_l(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}') \tag{8}$$

where $\hat{\boldsymbol{p}} \equiv \boldsymbol{p}/|\boldsymbol{p}|$, with $\boldsymbol{p}$ being the momentum vector.

The dimensionless effective interaction is connected with the proper vertex function of two-particle interaction, defined in [7, 8], in the form given earlier in [4]. It should be mentioned that all formulae (1–6) are written in a "system of units" such that $\hbar$, the Boltzmann constant and the volume of the system are equal to unity. For the paramagnetic systems the Landau parameters $f^{\alpha\beta}_{\omega 0}$ and $f^{\alpha\alpha}_{\omega 1} + f^{1\bar{1}}_{\omega 1}$ appearing in (2–6) can be determined from experimental data. We have only four relations (2–5) for five parameters A, B, C, m_1, $m_{\bar{1}}$, Moreover, in the equation (6) for both effective masses there appear three unknown parameters $f^{\alpha\beta}_{\omega 1}$. This shows that for ferromagnetic systems, in contrast with paramagnetic systems, some of these parameters remain undertemined from thermostatic measurements. If we want to determine all these parameters we must compute the reaction of our systems on the electromagnetic field. Our approach will be very similar to the phenomenological one, developed by Landau [9] and Silin [10] and systematized by Pines and Noziéres [11] (see also [7], [8]). It should be emphasized that for the ferromagnetic system the effective interaction of quasiparticles cannot be represented by the sum of spin-direct and spin-exchange terms. The parameters $f^{\alpha\beta}_{\omega l}$ fulfil the following inequalities:

$$\mathrm{Det}\,(\delta_{\alpha\beta} + f^{\alpha\beta}_{\omega l}) \geqslant 0, \qquad 1 + f^{\alpha\alpha}_{\omega l} \geqslant 0, \qquad f^{1\bar{1}}_{\omega 1} \geqslant 0. \tag{9}$$

Since there are too many amplitudes in (2–6) we shall not consider the problem of induction of spin waves by the external electromagnetic field. If we neglect this restriction we obtain some formulae with new, spin-transverse Landau amplitudes. The condition that the field does not induce spin waves is equivalent to saying that the external magnetic field is parallel to the axis of magnetization. The spin-waves can be investigated by means of the method developed in the paper [12]. The results of our present paper can be probably obtained also from the microscopic approach. The proof that results obtained in microscopic and phenomenological ways are equivalent was given for the paramagnetic system in [7]. It may be pointed out that this proof can be applied for the ferromagnetic systems at least as regards induced charges and currents. In this case the proof does not require any modification. The modification is necessary if we want to identify the induced spin currents. As a conclusion we obtain that the results for field-induced charges and currents are in accordance with the microscopic approach. For spin currents such accordance is also very probable.

2. *The kinetic equation*

The main difference between the kinetic equations for paramagnetic and ferromagnetic systems consists in the explicit spin dependence in the latter case. Moreover, the strong magnetic field connected with the magnetization of the system should be taken into account. Hence, the kinetic equation can be obtained in a manner very similar to that used in [7–11] for paramagnetic systems. The effective Hamiltonian for the quasiparticle with momentum $\boldsymbol{k}$ at position $\boldsymbol{x}$ can be written as

$$h(\boldsymbol{k}, \boldsymbol{x}) = E\left(\boldsymbol{k} - \frac{e}{c}\boldsymbol{A}, \boldsymbol{x}\right) + e\varphi - \mu_B(\boldsymbol{\sigma} \operatorname{rot} \boldsymbol{A}) \tag{10}$$

where $[A, c\varphi]$ is the electromagnetic four-potential and

$$E_{\alpha\beta}(\boldsymbol{k}, \boldsymbol{x}) = \delta_{\alpha\beta} E^0_{\boldsymbol{k}\alpha} + \frac{1}{(2\pi)^3} \sum_{\gamma\delta} \int d^3\boldsymbol{k}' F^{\alpha\beta,\gamma\delta}_{\boldsymbol{k},\boldsymbol{k}'} n'_{\gamma\delta}(\boldsymbol{k}', \boldsymbol{x}) \tag{11}$$

Here, $E^0_{\boldsymbol{k}\alpha}$ denotes the excitation energy of the quasiparticle with the momentum $\boldsymbol{k}$ and spin α, whereas $F^{\alpha\beta,\gamma\delta}_{\boldsymbol{k},\boldsymbol{k}'}$ gives the effective interaction between quasiparticles, and $n'_{\gamma\delta}(\boldsymbol{k}, \boldsymbol{x})$ is the variation of the equilibrium occupation numbers for quasiparticles. The corresponding Boltzmann equation has the form

$$\frac{\partial n(\boldsymbol{k}, \boldsymbol{x})}{\partial t} + \frac{1}{2}\sum_{a=1}^{3}\left[\left\{\frac{\partial n}{\partial x_a}, \frac{\partial h}{\partial k_a}\right\} - \left\{\frac{\partial n}{\partial k_a}, \frac{\partial h}{\partial x_a}\right\}\right] + $$
$$+ i[h, n] + \left(\frac{\partial n(\boldsymbol{k}, \boldsymbol{x})}{\partial t}\right)_{\text{collisions}} = 0 \tag{12}$$

where {,}, [,] denote the respective anticommutators and commutators, and $n_{\alpha\beta}$ $(\boldsymbol{k}, \boldsymbol{x})$ is the occupation number, $n_{\alpha\beta}$ $(\boldsymbol{k}, \boldsymbol{x}) = \delta_{\alpha\beta}\Theta(\mu - E^0_{\boldsymbol{k}\alpha}) + n'_{\alpha\beta}(\boldsymbol{k}, \boldsymbol{x})$, with Θ representing Heaviside's step function. If rot $\boldsymbol{A} \uparrow\uparrow Oz$, then $h(\boldsymbol{k}, \boldsymbol{x})$ is spin-diagonal, provided that n' has the same property. This assumption can be proved because the interaction $F^{\alpha\beta\gamma\delta}$ is spin-conserving and, in particular, $F^{\alpha\beta\gamma\gamma} \sim \delta_{\alpha\beta}$. Substituting spin-diagonal n into (12) we find that $\{A, B\} = 2AB$ and $[h, n] = 0$. Choosing $n'_\alpha(\boldsymbol{k}, \boldsymbol{x}) = \delta(E^0_{\boldsymbol{k}\alpha} - \mu)\, g_\alpha(\boldsymbol{k}, \boldsymbol{x})$ and neglecting the terms smaller than linear with respect to induced fields we obtain from (12)

$$\frac{\partial g_\alpha}{\partial t} + (\boldsymbol{V}_{\boldsymbol{k}\alpha}\nabla_x)\bar{g}_\alpha + \frac{e}{c}\,[(\boldsymbol{V}_{\boldsymbol{k}\alpha} \times \boldsymbol{B}_0)\nabla_k]\bar{g}_\alpha = e\boldsymbol{E}\boldsymbol{V}_{\boldsymbol{k}\alpha} + \mu_B\alpha(\boldsymbol{V}_{\boldsymbol{k}\alpha}\nabla_x)B + I(\bar{g}_\alpha) \tag{13}$$

Here, $\boldsymbol{V}_{\boldsymbol{k}\alpha} = (\nabla_k E^0_{\boldsymbol{k}\alpha})_{F,S}$, where "F. S." is a subscript denoting that the given quantity is taken on the Fermi surface, $\boldsymbol{B}_0 = [0, 0, H + 4\pi M]$ and B is the z-th component of the vector of magnetic induction $\boldsymbol{B}$. It should be noted that $\boldsymbol{B} = [0, 0, B]$ and $M = \mu_B(N_1 - N_{\bar{1}})$.

Also $\delta(E^0_{k\alpha}-\mu)\,\bar{g}_\alpha(\boldsymbol{k},\boldsymbol{x})\equiv \bar{n}'_\alpha(\boldsymbol{k},\boldsymbol{x})$ denotes the variation of the occupation number from the local equilibrium values and is determined by

$$\bar{g}_\alpha(\boldsymbol{k},\boldsymbol{x}) = g_\alpha(\boldsymbol{k},\boldsymbol{x}) + \frac{1}{(2\pi)^3}\sum_\beta \int d^3\boldsymbol{k}' F^{\alpha\beta}_{\boldsymbol{kk}'}\delta(E^0_{k'\beta}-\mu)g_\beta(\boldsymbol{k}',\boldsymbol{x}), \tag{14}$$

whereas $I(\bar{g}_\alpha)$ is the collision integral. We have also $F^{\alpha\beta}_{\boldsymbol{kk}'}\equiv F^{\alpha\alpha,\beta\beta}_{\boldsymbol{kk}'}$, and the relation between $f^{\alpha\beta}_\omega(\hat{\boldsymbol{k}}\hat{\boldsymbol{k}}')$ and $F^{\alpha\beta'}_{\boldsymbol{kk}'}$ is given by

$$f^{\alpha\beta}_\omega = x_\alpha x_\beta F^{\alpha\beta}/2\pi^2 \tag{15}$$

It can be verified using (13) that the current of particles with spin α is expressed by

$$\boldsymbol{I}^\alpha(\boldsymbol{x}) = \frac{e}{(2\pi)^3}\int d^3\boldsymbol{k}\,\boldsymbol{V}_{k\alpha}\delta(E^0_{k\alpha}-\mu)\bar{g}_\alpha(\boldsymbol{k},\boldsymbol{x}). \tag{16}$$

It can be shown that this quantity fulfils the continuity equation, provided that the collision integral is spin-conserving. All previous considerations of this chapter are valid for an arbitrary band structure and Fermi surface. Without loss of generality we can assume that $\boldsymbol{E}$, $\boldsymbol{B}$ and g_α, $\bar{g}_\alpha$ are monochromatic, *i. e.* $\boldsymbol{E}=\boldsymbol{E}_{q\omega}\exp i(\boldsymbol{qx}-\omega t)$, *etc.* Passing to the assumed simple model when $\boldsymbol{V}_{k\alpha}=V_\alpha\hat{\boldsymbol{k}}$ we can rewrite (13) for monochromatic quantities in the form

$$-\omega m_\alpha g_\alpha + (\boldsymbol{kq})\bar{g}_\alpha + i\omega_c m\frac{\partial \bar{g}_\alpha}{\partial\varphi} + ie\boldsymbol{Ek} - \alpha\alpha_B B(\boldsymbol{kq}) = -im_\alpha I(\bar{g}_\alpha) \tag{17}$$

where $\omega_c\equiv eB_0/cm$ and the subscripts "$q\omega$" near $\boldsymbol{E}$, $\boldsymbol{B}$, g_α, $\bar{g}_\alpha$ are omitted for simplicity. The variable φ denotes the azimuthal angle in the momentum space. The magnetic induction $\boldsymbol{B}$ is connected with $\boldsymbol{E}$ by the Maxwell equation. We have $\omega\boldsymbol{B}=c(\boldsymbol{q}\times\boldsymbol{E})$ with $\boldsymbol{B}//Oz$. The functions g_α and $\bar{g}_\alpha$ can be expanded into spherical harmonics in momentum space. Assuming that $r_\alpha(\boldsymbol{k})/_{F,S}=\Sigma_{lm}r^\alpha_{lm}y_{lm}(\hat{\boldsymbol{k}})$, where r denotes g or $\bar{g}$, we find using (8), (14) and (15)

$$\bar{g}^\alpha_{lm} = (1+f^{\alpha\alpha}_{\omega l})\,g^\alpha_{lm} + f^{1\bar{1}}_{\omega l}(x_{\bar{\alpha}}/x_\alpha)\,g^{\bar{\alpha}}_{lm} \tag{18}$$

Hence,

$$g^\alpha_{lm} = [(1+f^{\bar{\alpha}\bar{\alpha}}_{\omega l})\bar{g}^\alpha_l - f^{1\bar{1}}_{\omega l}(x_{\bar{\alpha}}/x_\alpha)\bar{g}^{\bar{\alpha}}_{lm}]\,\tilde{w}_l^{-1} \tag{19}$$

where

$$\tilde{w}_l = (1+f^{11}_{\omega l})(1+f^{\bar{1}\bar{1}}_{\omega l}) - (f^{1\bar{1}}_{\omega l})^2$$

If we define

$$\begin{Bmatrix}\delta N_\alpha\\ \delta\bar{N}_\alpha\end{Bmatrix} = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{k}\,\delta(E^0_{k\alpha}-\mu)\begin{Bmatrix}g_\alpha(\boldsymbol{k})\\ \bar{g}_\alpha(\boldsymbol{k})\end{Bmatrix} \tag{20}$$

then, by virtue of (8), (14) and (15), we find

$$\delta\bar{N}_\alpha = (1+f^{\alpha\alpha}_{\omega 0})\,\delta N_\alpha + f^{1\bar{1}}_{\omega 0}(x_\alpha/x_{\bar{\alpha}})\,\delta N_{\bar{\alpha}} \tag{21}$$

or

$$\delta N_\alpha = [(1+f^{\bar{\alpha}\bar{\alpha}}_{\omega 0})\,\delta\bar{N}_\alpha - f^{1\bar{1}}_{\omega 0}(x_{\bar{\alpha}}/x_\alpha)\,\delta\bar{N}_{\bar{\alpha}}]\,\tilde{w}_0^{-1} \tag{22}$$

In the formulae written above δN_α denotes the variation of the number of particles with spin α. Taking into account the discussion of the collision integral performed by Silin [10] (also *cf.* [11]) it can be written as

$$m_\alpha I(\bar{g}_\alpha) = -m \sum_{\substack{lm \\ l>0}} [\bar{g}^\alpha_{lm} Y_{lm}(\hat{k})/\tau_{\alpha l}],\ \tau_{\alpha l} > 0 \tag{23}$$

Our proof of the kinetic equation is valid only for the system with short-range forces. It can be easily verified that the kinetic equation is valid also for a system where forces have the long-range Coulomb component. In this case $F^{\alpha\beta}_{kk'}$ denotes the screened quasiparticle interaction and the electric field $\boldsymbol{E}$ contains the contribution from the electric polarization of the system (*cf.* [10] and [11]).

3. *The quasistatic reaction*

The equation (17) cannot be effectively solved unless we restrict ourselves to the consideration of suitably simplified quasiparticle interaction. In the simplified interaction all Legendre amplitudes $f^{\alpha\beta}_{\omega l}$ vanish for $l > l_0$, where l_0 is some small integer. Such systems will not be considered here. On the other hand, the equation (17) can be solved in two particular cases:

i) when $qV \gg |\omega|$ (the quasistatic field),

ii) when $|\omega| \gg qV$ (the quasihomogeneous field).

Let us consider the case *i*) and the longitudinal electric field. Then $B = 0$ and the solution of the equation (17) for $\omega = 0$ can be chosen as follows: $\bar{g}^{(0)}_\alpha = -ie\mathscr{E}/q$ with $\boldsymbol{E} = \mathscr{E}\hat{\boldsymbol{q}}$. It can be easily seen that this solution is determined uniquely by the condition $\bar{g}^{(0)}_\alpha(\varphi+2\pi) = \bar{g}^{(0)}_\alpha(\varphi)$. Assuming that $\bar{g} \approx \bar{g}^{(0)} + \bar{g}^{(1)}$, where $\bar{g}^{(1)}$ is of order (ω/qv), we obtain from (17)

$$-\omega m_\alpha g^{(0)}_\alpha + (\boldsymbol{kq})\bar{g}^{(1)}_\alpha + im\omega_c \frac{\partial \bar{g}^{(1)}_\alpha}{\partial \varphi} = 0 \tag{24}$$

where, according to (19) and the solution for $\bar{g}^{(0)}$,

$$g^{(0)}_\alpha = -\frac{ie\varepsilon}{q(AB-C^2)}[1 + f^{\bar{\alpha}\bar{\alpha}}_{\omega 0} - Cx_{\bar{\alpha}}/x_\alpha] \tag{25}$$

(cf. (7)). The collision integral was neglected in (24). This is possible in the "collisionless region" when $qv \gg \operatorname{Max}_{l,\alpha}(\tau^{-1}_{\alpha l})$ or $|\omega| \gg \operatorname{Max}_{l,\alpha}(\tau^{-1}_{\alpha l})$. It is clear that for the quasistatic field only the first condition is important. Taking into account (23) we find that $(\boldsymbol{kq})$ in (24) can be considered as $(\boldsymbol{k}\,\boldsymbol{q}) - i0$. This is connected with the causality condition. The symmetry of our problem allows us to put $q_y = 0$ without loss of generality. Then $(\boldsymbol{kq})$ in (24) can be replaced by $p_\alpha(q_x \sin\vartheta \cos\varphi + q_z \cos\vartheta)$. It can be verified that the unique solution of (24) which is φ-periodic with the period 2π can be expressed by

$$\bar{g}^{(1)}_\alpha = -\frac{i\omega m_\alpha g^{(0)}_\alpha}{\omega_c m}[\exp(-2\pi i q_z p_\alpha \cos\vartheta/m\omega_c) - 1]^{-1} \times$$

$$\times \int_\varphi^{\varphi+2\pi} d\varphi' \exp\left\{\frac{ip_\alpha}{m\omega_c}[q_x \sin\vartheta(\sin\varphi - \sin\varphi') + q_z \cos\vartheta(\varphi - \varphi')]\right\}. \tag{26}$$

Substituting this quantity into (16) we obtain the induced current of electrons with spin α. Analogously, the induced charges can be obtained by means of (20) and (22). One should remember that in (26) $q_z \cos \vartheta$ has the meaning of $q_z \cos \vartheta - i\eta$, $\eta = +0$. The solution (26) resembles to some extent that obtained by Rodriguez, [13], [14], for the problem of the cyclotron resonance in metals. Our solution is more complicated, *e. g.*, under the integral there appears $\cos \vartheta$ instead of 1 (see [14]). The solution (26) has a singularity for q_z tending to zero. This singularity disappears if the relaxation times $\tau_{\alpha l}$ are finite (in our case all $\tau_{\alpha l}$ tend to infinity) and has a simple physical meaning. The inegrals in (16) and (20), if we substitute (26) in them, can be computed only in a numerical way, provided that $q_x \neq 0$. If $q_x = 0$, then

$$\bar{g}_\alpha^{(1)} = - \frac{\omega m_\alpha g_\alpha^{(0)}}{q p_\alpha (\cos \vartheta - i\eta)} = - \frac{\omega m_\alpha g_\alpha^{(0)}}{p_\alpha q} \left[i\pi\delta(\cos \vartheta) + P\left(\frac{1}{\cos \vartheta}\right)\right] \tag{27}$$

and in $\delta \bar{N}_\alpha$ only the first term in the square bracket is effective. Substituting $\bar{g}_\alpha^{(0)} + \bar{g}_\alpha^{(1)}$ into (20) we find

$$\delta \bar{N}_\alpha = - \frac{i\mathscr{E} e}{q} \cdot \frac{x_\alpha^2}{2\pi^2} \left\{ 1 + \frac{\pi i}{2} \cdot \frac{\omega}{q v_\alpha} \left[1 + f_{\omega 0}^{\bar{\alpha}\bar{\alpha}} - C x_{\bar{\alpha}}/x_\alpha\right]\right\}. \tag{28}$$

Applying Eq. (22) and taking into account that $4\pi i \delta(Ne)/q\mathscr{E} = \varepsilon(q\omega) - 1$, where $\varepsilon(q\omega)$ denotes the dielectric constant, we find

$$\varepsilon(q\omega) = 1 + \frac{4\pi e^2}{q^2} \left(\frac{\partial N}{\partial \mu}\right) + \frac{i e^2 \omega}{4\pi \tilde{w}_0^2 q^3} \left[(B x_1 - C x_{\bar{1}})^2 v_1^{-1} + (A x_{\bar{1}} - C x_1)^2 v_{\bar{1}}^{-1})\right] \tag{29}$$

On the other hand,

$$\delta(N_1 - N_{\bar{1}}) = - \left(\frac{ie\mathscr{E}}{q}\right) \left\{ \frac{\partial (N_1 - N_{\bar{1}})}{\partial \mu} + \frac{i\omega}{4\pi \tilde{w}_0^2 q} \left[(B^2 x_1^2 - C^2 x_{\bar{1}}^2) v_1^{-1} - (A^2 x_{\bar{1}}^2 - C^2 x_1^2) v_{\bar{1}}^{-1}\right]\right\}. \tag{30}$$

Both results are valid only for an electric field directed along the magnetization axis, but for the static field ($\omega = 0$) such restrictions are not important. Then the results can be expressed in the form $\delta(N_1 + \alpha N_{\bar{1}}) = (-ie\mathscr{E}/q)\,[\partial(N_1 + \alpha N_{\bar{1}})/\partial\mu]$ ($\alpha = \pm 1$) and have a simple thermodynamic interpretation. Namely, for the weak static field, depending weakly on space variables (*i. e.* for $q \ll p_\alpha$), there appears a new, space-inhomogeneous equilibrium such that $\mu + e\varphi = \text{const.}$ In this case the reaction of the system can be given by the formula above because φ for longitudinal fields corresponds to $i\mathscr{E}/q$. The formula (29) for $\omega = 0$ can be also interpreted in terms of compressibility [11]. Hence, the screening length can be also computed. This quantity, denoted by λ_s, is equal to s/ω_p, where ω_p is the classical plasma frequency and $s^2 = (N/m)\,(\partial\mu/\partial N)$. The quantity s can be interpreted as the "electron velocity of sound". Due to electron-phonon interaction s cannot be measured directly in metals. It should be noted that (29) with (2) to (5) make it possible in principle, to obtain the parameters A, B, C, and both effective masses from experimental data.

Applying the methods developed in [11] (cf. [15]) we can compute the effective electron-ion interaction, provided it can be treated as static. This quantity can be also determined as a scattering potential for quasiparticles with the impurity atom [15]. We find that the effective electron-ion interaction is spin-dependent for ferromagnetics. The considered quantity is given by

$$\mathcal{V}^{\mathrm{eff}}_{\boldsymbol{q},\boldsymbol{k},\alpha} = 2\pi^2 Z[1+f^{\alpha\alpha}_{\omega 0} - Cx_{\bar{\alpha}}/x_{\alpha}]\,[Bx_1^2+Ax_{\bar{1}}^2-2Cx_1x_{\bar{1}}]^{-1} \tag{31}$$

where $-Ze$ is the ionic charge. It should be noted that interionic effective interaction is given by

$$\mathcal{V}^{\mathrm{eff}}_{\boldsymbol{q}} = \frac{4\pi ZZ'e^2}{q^2\varepsilon(\boldsymbol{q}\ 0)} \to ZZ'(\partial\mu/\partial N). \tag{32}$$

The formula (32) can be obtained directly from the microscopic approach. Applying here the methods developed by Heine, Noziéres and Wilkins [15] we obtain

$$\mathcal{V}^{\mathrm{eff}}_{\boldsymbol{q},\boldsymbol{k},\alpha} = \frac{[Z^{\alpha}_{\boldsymbol{k}-\boldsymbol{q}/2}Z^{\alpha}_{\boldsymbol{k}+\boldsymbol{q}/2}]^{1/2}\tilde{T}^{0}_{\boldsymbol{k}\alpha}(\boldsymbol{q})V(\boldsymbol{q})}{1-u\tilde{S}^{00}(\boldsymbol{q})}$$

where $Z^{\alpha}_{\boldsymbol{k}}$ denotes the discontinuity of the occupation numbers on the Fermi surface for particles with momentum k and spin α, and $u_q = 4\pi e^2/q^2$, $V(\boldsymbol{q}) \to -4\pi Ze^2/q^2$ for small q. Also $\tilde{T}^{0}_{\boldsymbol{k}\alpha}(q)$ denotes the proper vertex function such that for vanishing interelectron interaction $\tilde{T}^0 = 1$, and $\tilde{S}^{00}(q)$ denotes the proper correlation function of these vertices. These quantities are determined in detail in the papers [15] and [4]. It should be pointed out that the formula is also valid for unspherical Fermi surfaces. Tending with q to zero we find by virtue of the Ward identities (see [4], [15])

$$\mathcal{V}^{\mathrm{eff}}_{\boldsymbol{q},\boldsymbol{k},\alpha} = -\ Zv_{\boldsymbol{k}\alpha}\frac{\partial p_{\boldsymbol{k}\alpha}}{\partial\mu}\left(\frac{\partial\mu}{\partial N}\right) \tag{34}$$

where $p_{\boldsymbol{k}\alpha} = |\boldsymbol{k}|_{F.S.\alpha}$ depends on the space direction. Using (1) to (3) here we find in the isotropic case after some manipulations the equation (31) (the suitable formula, ready for substitution into (34), is given in [4]). Even in the anisotropic case we have

$$\frac{1}{(2\pi)^3}\int d^3\boldsymbol{k}\mathcal{V}^{\mathrm{eff}}_{\boldsymbol{q},\boldsymbol{k},\alpha}\delta(E^0_{\boldsymbol{k}\alpha}-\mu) = -Z\left(\frac{\partial N_\alpha}{\partial\mu}\right)\left(\frac{\partial\mu}{\partial N}\right). \tag{35}$$

The summation of these formula over α gives $-Z$, according to the result for the paramagnetic system.

It can be verified that the solution of Eqs. (17) for $\omega = 0$ and the transverse electric field cannot be φ-periodic for arbitrary q and ϑ. The explicit spin-dependence in unimportant here; we have the same situation for the paramagnetic system in the strong static magnetic field.

4. *The quasihomogeneous reaction; concluding remarks*

Let us consider the system (17) for $qv \ll \tau_{\alpha l}^{-1}$. Then all terms in (17) containing $(\boldsymbol{kq})$ are negligibly small. Taking into account (23) we find

$$-\omega m_\alpha g_\alpha + im\omega_c\frac{\partial g_\alpha}{\partial\varphi} + i\boldsymbol{Ek} = im\sum_{\substack{lm\\ l>0}}\left[\frac{\bar{g}^{\alpha}_{lm}}{\tau_{\alpha l}}\,Y_{lm}(\hat{\boldsymbol{k}})\right]. \tag{36}$$

The solutions of the above equations can be chosen in the form $g_\alpha = \boldsymbol{A}_\alpha \hat{\boldsymbol{k}}$, whereas $\bar{g}_\alpha = \bar{\boldsymbol{A}}_\alpha \hat{\boldsymbol{k}}$. For solutions in this form $\bar{g}^\alpha_{lm}$ vanishes unless $l = 1$. Hence, using (18) we get

$$\bar{\boldsymbol{A}}_\alpha = (1 + f^{\alpha\alpha}_{\omega 1})\, \boldsymbol{A}_\alpha + f^{1\bar{1}}_{\omega 1}(x_{\bar{\alpha}}/x_\alpha)\boldsymbol{A}_{\bar{\alpha}} \tag{37}$$

The geometry of our system allows us to choose $\boldsymbol{E}$ in the form $\boldsymbol{E} = [E_\perp, 0, E_{||}]$ without any loss of generality. Substituting g and $\bar{g}$ in the chosen form into (36) we find

$$-\omega m_\alpha A^\alpha_{||} + ieE_{||} p_\alpha = im\, \bar{A}^\alpha_{||}/\tau_{\alpha 1} \tag{38a}$$

$$-\omega m_\alpha A^\alpha_x + im\omega_c \bar{A}^\alpha_y = im\, \bar{A}^\alpha_x/\tau_{\alpha 1} - iep_\alpha E_\perp \tag{38b}$$

$$-\omega m_\alpha A^\alpha_y - im\omega_c \bar{A}^\alpha_x = im\bar{A}^\alpha_y/\tau_{\alpha 1} \tag{38c}$$

where $A_{||} \equiv A_z$, $\bar{A}_{||} \equiv \bar{A}_z$. Note that by virtue of (16) we have

$$\boldsymbol{I}^\alpha = \lim_{q \to 0} \boldsymbol{I}^\alpha_{q\omega} = \frac{eN_\alpha}{p_\alpha} \bar{\boldsymbol{A}}_\alpha. \tag{39}$$

From (37) and (38a), after rather long but relatively simple calculations, we obtain using (39)

$$I^\alpha_z = \frac{ie^2 N_\alpha}{m\omega w(\omega)} (1 + iR/\omega\tau_{\bar{\alpha}}) E_{||} \tag{40}$$

where

$$R = 1 - f(t_1 + t_{\bar{1}}),\ t_\alpha = (N_{\bar{\alpha}}/N_\alpha)^{\frac{1}{2}},\ f = f^{1\bar{1}}_{\omega 1} m (m_1\, m_{\bar{1}})^{-\frac{1}{2}},$$
$$w(\omega) = 1 + \sum_\alpha [i(1 - ft_\alpha)/\omega\tau_\alpha] - R/\omega^2 \tau_1 \tau_{\bar{1}} \tag{41}$$

and the τ_α in our present notation coincides with($\tau_{\alpha 1}$. Note that the parameter f is nonnegative (see (9)) and that in the proof of the formula (40) we have applied (1) and (6). If $\tau_1 = \tau_{\bar{1}} = \tau$ then $w(\omega) = (1 + i/\omega\tau)(1 + iR/\omega\tau)$ and the formula (40) can be highly simplified. We have then

$$I_z = \sum_\alpha I^\alpha_z = \frac{ie^2 N}{m(\omega + i/\tau)} E_{||} \tag{42}$$

and obtain a condictivity such as for paramagnetic metal. If $\tau_1 \neq \tau_{\bar{1}}$ then these quantities as well as f can be in principle determined by means of (40) from the experimental data. Since the thermostatic quantities (2) to (5) and the quasistatic measurements allow to determine A, B, C and both effective masses we can thus obtain together all amplitudes $f^{\alpha\beta}_{\omega l}$, $l = 0, 1$.

Let us solve the system of equations described by (37) and (38b, *c*). With the help of (1), (6) and (39) we can write after performing some long calculations

$$I^\alpha_x = \frac{ie^2 N_\alpha}{m\omega\, U(\omega)} [1 + i/\omega\tau_{\bar{\alpha}} - Z(\omega)\, (1 + i/\omega\tau_\alpha)] E_\perp \tag{43}$$

$$I^\alpha_y = \frac{\omega_c e^2 N_\alpha}{m\omega^2 U(\omega) w(\alpha)} \{(1 + i/\omega\tau_{\bar{\alpha}})\, [1 - ft_\alpha(1 - Z(\omega)) + iR/\omega\tau_{\bar{\alpha}}] -$$
$$- (1 + i/\omega\tau_\alpha)\, [(1 - ft_\alpha) Z(\omega) + iRZ(\omega)/\omega\tau_\alpha - ft_\alpha]\} E_\perp \tag{44}$$

where

$$Z(\omega) = R^2\omega_c^2/\omega^2 w(\omega)$$

$$U(\omega) = w(\omega) - Z(\omega)\,[\sum_\alpha (1+i/\omega\tau_\alpha)^2 - 1 + R^{-2} + (2f/R)\sum_\alpha (it_\alpha/\omega\tau_\alpha) - (\omega_c/\omega)^2] \quad (45)$$

and the remaining symbols are defined in (41). Taking into account the symmetry of our problem we can write the conductivity tensor in the form

$$\sigma^{zz} = I_z/E_{||},\ \sigma^{xx} = \sigma^{yy} = I_x/E_\perp,$$
$$\sigma^{yx} = -\sigma^{xy} = I_y/E_\perp,\ I_a \equiv \sum_\alpha I_a^\alpha \quad (46)$$

where we have used Onsager's reciprocal relations. The remaining components of the conductivity tensor vanish. It should be emphasized that the assumption $\tau_1 = \tau_{\bar{1}}$ does not simplify (43) and (44) markedly. On the other hand, the results (40), (43) and (44) are valid for the longitudinal as well as transversal fields. If $\omega \gg \tau_\alpha$ (*i. e.* in the collisionless region), then

$$\sigma^{zz} = i\omega_p^2/4\pi\omega,\ \sigma^{xx} = \sigma^{yy} = (i\omega_p^2/4\pi\omega)\,[1-(\omega_c/\omega)^2]^{-1},$$
$$\sigma^{yx} = -\sigma^{xy} = (\omega_c\omega_p^2/4\pi\omega^2)\,[1-(\omega_c/\omega)^2]^{-1} \quad (47)$$

were ω_p denotes the classical plasma frequency. In this limit the conductivity tensor does not depend on Landau parameters.

Our present considerations resemble the earlier ones of Lifshitz, Azbel and Kaganov [16]. In contrast with this paper we consider only spherical Fermi surfaces; hence, *e. g.*, open orbits are beyond the scope of our investigation. On the other hand, we consider interacting quasiparticles, whereas in [16] they are free. All our investigations in this paper, with the exception of the problem of effective potentials, are restricted to such a simple case. It can be shown that the static reaction on the longitudinal field can be expressed in the form (29) and (30) for $\omega = 0$, independently of the details of the band structure. For the proof of the above statement let us remark that the solution of the equation (13) for the monochromatic $\boldsymbol{E}$ with $\omega = 0$ can be expressed by $\bar{g} = -ie\mathscr{E}/q$, $(\boldsymbol{E} = \mathscr{E}\hat{\boldsymbol{q}})$. The relation reciprocal to (14) can be written according to [11] as follows:

$$g_\alpha(\boldsymbol{k}) = \bar{g}_\alpha(\boldsymbol{k}) - \frac{1}{(2\pi)^3}\sum\int d^3\boldsymbol{k}' D^{\alpha\beta}_{\boldsymbol{kk}'}\delta(E^0_{\boldsymbol{k}'\beta}-\mu)\bar{g}_\beta(\boldsymbol{k}') \quad (48)$$

where D is defined by

$$D^{\alpha\beta}_{\boldsymbol{kk}'} = F^{\alpha\beta}_{\boldsymbol{kk}'} - \frac{1}{(2\pi)^3}\sum_\gamma\int d^3\boldsymbol{k}'' F^{\alpha\gamma}_{\boldsymbol{kk}''}\delta(E^0_{\boldsymbol{k}''\beta}-\mu)D^{\gamma\beta}_{\boldsymbol{k}''\boldsymbol{k}'} \quad (49)$$

and plays the role of the forward scattering amplitude of quasiparticles [11]. The function D is connected with the function f_k appearing in the microscopic approach by the formula in the form (15). Substituting into (48) the static solution for $\bar{g}$ and using (20) we find after some manipulations

$$\delta\begin{Bmatrix} N \\ N_1 - N_{\bar{1}} \end{Bmatrix} = \left(-\frac{ie\mathscr{E}}{q}\right)\frac{1}{(2\pi)^3}\sum_{\alpha}\int d^3\boldsymbol{k}\begin{Bmatrix}1\\ \alpha\end{Bmatrix}\delta(E^0_{\boldsymbol{k}\alpha}-\mu)\times$$

$$\times\left[1-\frac{1}{(2\pi)^3}\sum_{\beta}\int d^3\boldsymbol{k}' D^{\alpha\beta}_{\boldsymbol{k}\boldsymbol{k}'}\delta(E^0_{\boldsymbol{k}'\beta}-\mu)\right]. \tag{50}$$

Taking into account the identities for correlation functions introduced in [7] and discussed in [4] we find that the formula (50) can be expressed as $\delta(N_1+\alpha N_{\bar{1}}) = (-ie\mathscr{E}/q)\,\partial(N_1+ +\alpha N_{\bar{1}})/\partial\mu$. With this formula we have our proof. It should be noted that the identities for correlation functions mentioned above are proved without any use of the translational invariance of the system. Oh the other hand, the indices of Brillouin zones can be included quite formally into (48) to (50) and, hence, our proof is general.

In our considerations the effects of electron-phonon interaction are not included. These effect should be investigated because the phonon renormalizations are very important for ferromagnets. The inclusion of the electron-phonon interaction into the considered scheme is a necessary step in achieving further progress. It should be pointed out that our results are valid also for systems with electron-phonon interaction provided that the frequency ω is much greater than the Debye frequency (*i. e.* as usual $\sim 10^{13}$ sec^{-1}).

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